Global structure of ITG turbulence-zonal mode system in tokamak plasmas

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Abstract. Using a global Landau fluid code in toroidal geometry, an ion temperature gradient (ITG) driven turbulence-zonal mode system is investigated. Two types of zonal flows, stationary zonal flows in a low q (safety factor) region and oscillatory ones in a high q region which are called geodesic acoustic modes (GAM), are found to be simultaneously excited in a torus. Energy loop between the ITG turbulence and the zonal flows in a low beta plasma is identified. The stationary flows suppress turbulent transport effectively, while the suppression effect on the ITG turbulence by the oscillatory zonal flows is weak compared to the stationary ones. Therefore the zonal flows are dominant over the ITG turbulence in the low q region where the zonal flows are stationary. On the other hand, the ITG turbulence remains active in the high q region where the zonal flows are oscillatory. Control of the turbulent transport may be possible through the control of the zonal flow behaviour by the q profile.

1 Introduction

Suppression of anomalous transport or formation of transport barriers are necessary for advanced tokamak operation with good confinement properties. Drift wave turbulence including ion temperature gradient (ITG) turbulence has been studied extensively in relation to the anomalous transport. The studies revealed that the drift wave turbulence nonlinearly generates zonal flows which can suppress the turbulent transport. If the zonal flows are stationary, they suppress the turbulence effectively. Recent local fluid simulations in toroidal geometry[1, 2, 3] and tokamak experiments[4, 5, 6] have shown that the zonal flows near the edge oscillate. The oscillation is called geodesic acoustic mode (GAM) which can appear in toroidal plasmas[7]. The time dependent zonal flows are less effective in suppressing the turbulence than the stationary ones[8]. Hence, it is important in understanding the formation of transport barrier and controlling the turbulent transport to investigate global variation of the zonal flow behaviour in tokamak plasmas. In this paper, the zonal flow behaviour and nonlinear interactions between the ITG turbulence and the zonal flows including both stationary and oscillatory modes in tokamak plasmas are investigated in detail by a global electromagnetic Landau fluid code.

2 Model Equations

In the code, five-field (density n, electrostatic potential ϕ , parallel component of magnetic vector potential A, parallel ion velocity v and ion temperature T) Landau fluid equation system is applied to describe the global electromagnetic turbulence in tokamak plasmas. The highest moments (electron temperature and ion parallel heat flux) are approximated by lower moments based on the Hammett-Perkins closure[9, 10]. In the electrostatic limit with adiabatic electrons the five-field model reduces to the three-field ion Landau fluid model. Nonlinear evolution equations for these fields consist of a continuity equation

$$\frac{dn}{dt} = a \frac{dn_{eq}}{dr} \nabla_{\theta} \phi - n_{eq} \nabla_{\parallel} v_{\parallel} + \nabla_{\parallel} j_{\parallel} + \omega_d (n_{eq} \phi - p_e) + D_n \nabla_{\perp}^2 n, \tag{1}$$

a vorticity equation

$$\frac{d}{dt}\nabla_{\perp}^{2}\phi = -T_{eq}\frac{a}{n_{eq}}\frac{dn_{eq}}{dr}(1+\eta_{i})\nabla_{\theta}\nabla_{\perp}^{2}\phi + \frac{1}{n_{eq}}\nabla_{\parallel}j_{\parallel} - \omega_{d}\left(T_{i} + \frac{T_{eq}}{n_{eq}}n + \frac{p_{e}}{n_{eq}}\right) + D_{U}\nabla_{\perp}^{4}\phi, \quad (2)$$

an equation of motion for the ion fluid in the parallel direction

$$\frac{dv_{\parallel}}{dt} = -\nabla_{\parallel}T_i - (1+\tau)\frac{T_{eq}}{n_{eq}}\nabla_{\parallel}n - \beta T_{eq}\frac{a}{n_{eq}}\frac{dn_{eq}}{dr}(1+\eta_i+\tau)\nabla_{\theta}A_{\parallel} + D_v\nabla_{\perp}^2v_{\parallel}, \qquad (3)$$

an equation of motion for the electron fluid in the parallel direction or Ohm's law

$$\beta \frac{\partial A_{\parallel}}{\partial t} = -\nabla_{\parallel} \phi + \tau \frac{T_{eq}}{n_{eq}} \nabla_{\parallel} n + \beta \tau T_{eq} \frac{a}{n_{eq}} \frac{dn_{eq}}{dr} \nabla_{\theta} A_{\parallel} + \sqrt{\frac{\pi}{2}} \tau \frac{m_e}{m_i} |\nabla_{\parallel}| \left(v_{\parallel} - \frac{j_{\parallel}}{n_{eq}} \right) - \eta j_{\parallel}, \quad (4)$$

an ion temperature equation

$$\frac{dT_i}{dt} = T_{eq} \frac{a}{n_{eq}} \frac{dn_{eq}}{dr} \eta_i \nabla_\theta \phi - (\Gamma - 1) T_{eq} \nabla_{\parallel} v_{\parallel} - (\Gamma - 1) \sqrt{\frac{8T_{eq}}{\pi}} |\nabla_{\parallel}| T_i
+ T_{eq} \omega_d \left((\Gamma - 1) \phi + (2\Gamma - 1) T_i + (\Gamma - 1) \frac{T_{eq}}{n_{eq}} n \right) + D_T \nabla_{\perp}^2 T_i,$$
(5)

and parallel current is related with the magnetic potential through the Ampère's law

$$j_{\parallel} = -\nabla_{\perp}^2 A_{\parallel},\tag{6}$$

where, $p_e = \tau T_{eq}n$, n_{eq} (T_{eq}) is an equilibrium density (ion temperature) normalized by the central value n_c (T_c), $\tau = T_{e0}/T_{i0}$ is a ratio of electron and ion equilibrium temperatures, $\beta = (n_c T_c)/(B_0^2/\mu_0)$ is a half of beta value evaluated on the plasma center, $\eta_i = d \ln T_{eq}/d \ln n_{eq}$, B_0 is a toroidal magnetic field on the magnetic axis and $\Gamma = 5/3$ is a ratio of specific heats. We assume a circular tokamak geometry (r, θ, ζ), where r is a radius of magnetic surface, θ and ζ are poloidal and toroidal angles, respectively. Then operators are defined as

$$\frac{df}{dt} = \partial_t f + [\phi, f], \quad \nabla_{\parallel} f = \epsilon \partial_{\zeta} f - \beta [A_{\parallel}, f],$$
$$\omega_d \cdot f = 2 \frac{a}{R_0} [r \cos \theta, f], \quad [f, g] = \frac{1}{r} \left(\frac{\partial f}{\partial r} \frac{\partial g}{\partial \theta} - \frac{\partial f}{\partial \theta} \frac{\partial g}{\partial r} \right)$$

where a and R_0 are minor and major radii, respectively. Here the normalizations are $tv_{ti}/a \to t, r/\rho_i \to r, \rho_i \nabla_{\perp} \to \nabla_{\perp}, a \nabla_{\parallel} \to \nabla_{\parallel},$

$$\frac{a}{\rho_i}\left(\frac{n}{n_c}, \frac{e\phi}{T_c}, \frac{v_{\parallel}}{v_{ti}}, \frac{A_{\parallel}}{\beta B_0 \rho_i}, \frac{T_i}{T_c}\right) \to (n, \phi, v_{\parallel}, A_{\parallel}, T_i),$$

where $v_{ti} = \sqrt{T_c/m_i}$, $\rho_i = v_{ti}/\omega_{ci}$, $\omega_{ci} = eB_0/m_i$. Artificial dissipations (D_n, D_U, D_v, D_T) are included to damp the small scale fluctuations.

3 Global Structure of Zonal Flows in Tokamak Plasmas

Normalized equations describing the zonal flow behaviour in low β tokamak plasmas consist of a zonal flow equation,

$$\frac{\partial \langle v_E \rangle}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \langle \tilde{v}_{Er} \tilde{v}_{E\theta} \rangle - \frac{2}{n_{eq}} \frac{a}{R} \langle p \sin \theta \rangle, \tag{7}$$

a (1,0)-pressure equation,

$$\frac{\partial}{\partial t} \langle p \sin \theta \rangle = -\langle [\tilde{\phi}, \tilde{p}] \sin \theta \rangle + (\Gamma + \tau) p_{eq} \frac{a}{qR} \langle v_{\parallel} \cos \theta \rangle + (\Gamma + \tau) p_{eq} \frac{a}{R} \langle v_E \rangle, \tag{8}$$

and a (1,0)-parallel ion velocity equation,

$$\frac{\partial}{\partial t} \langle v_{\parallel} \cos \theta \rangle = -\langle [\tilde{\phi}, \tilde{v}_{\parallel}] \cos \theta \rangle - \frac{1}{n_{eq}} \frac{a}{qR} \langle p \sin \theta \rangle, \tag{9}$$

where $\langle \cdot \rangle$ denotes the flux surface average, $\langle v_E \rangle = \frac{\partial \phi_0}{\partial r}$ is the zonal flow, $\tilde{v}_{Er} = -\frac{1}{r} \frac{\partial \tilde{\phi}}{\partial \theta}$ and $\tilde{v}_{E\theta} = \frac{\partial \tilde{\phi}}{\partial r}$ are E×B drift velocities in radial and poloidal directions, respectively, $\langle p \sin \theta \rangle$ is the (1,0) pressure perturbation, $p = p_i + p_e = n_{eq}T + T_{eq}n + \tau T_{eq}n$ is total pressure and $\langle v_{\parallel} \cos \theta \rangle$ is the (1,0) parallel ion velocity. Two modes of zonal flows are obtained from the above equations without nonlinear terms. Neglecting nonlinear terms and time derivatives in the above equations yields a stationary solution,

$$\langle v_E \rangle = -\langle v_{\parallel} \cos \theta \rangle / q, \quad \langle p \sin \theta \rangle = 0.$$
 (10)

The stationary zonal flows are accompanied with the parallel flows. In a low q region the zonal flow term in Eq. (8) can balance with the parallel flow term and the stationary zonal flow becomes dominant. On the other hand, the parallel flow term is very small in a high q region. In this case the zonal flows oscillate with (1,0) pressure perturbations. Eliminating $\langle p \sin \theta \rangle$ from Eqs. (7) and (8) in the $q \gg 1$ limit yields the oscillation equation,

$$\frac{\partial^2 \langle v_E \rangle}{\partial t^2} = -2(\Gamma + \tau) T_{eq} \left(\frac{a}{R}\right)^2 \langle v_E \rangle.$$
(11)

From the above equation, the pure GAM frequency,

$$\omega_{\text{GAM}} = \sqrt{2(\Gamma + \tau)T_{eq}}\frac{a}{R},\tag{12}$$

is obtained. Another important frequency is the parallel sound frequency of the (1,0) mode,

$$\omega_{\text{sound}} = \sqrt{(\Gamma + \tau)T_{eq}} \frac{a}{qR},\tag{13}$$

which is obtained from Eqs. (8) and (9) in the $q \ll 1$ limit. This frequency represents the timescale of the dynamics parallel to the magnetic field for the (1,0) mode. Nonlinear simulations show that the frequency plays an important role in determining the zonal flow behaviour.

Using the developed global Landau-fluid code, we have performed electromagnetic ITG turbulence simulations. Parameters used in the calculations are R/a=4, $\rho_i/a=0.0125$,

 $T_e=T_i, \beta=0.1\%, n_{eq}=0.8+0.2e^{-2(r/a)^2}, T_{eq}=0.35+0.65(1-(r/a)^2)^2, q=1.05+2(r/a)^2$. The density and temperature profiles are fixed in the calculations. In these parameters a dominant linear instability is the ITG mode. The numerical calculations are done by Fourier mode expansion in the poloidal and toroidal directions and finite difference in the radial direction. The Fourier modes included in the calculations are ones having resonant surfaces between 0.2 < r/a < 0.8 in the range of $m \leq 80$ and $n \leq 50$, and nonresonant (m, n) = (0, 0), (1, 0) components, where m and n are poloidal and toroidal mode numbers, respectively. Only even toroidal modes are kept to reduce computational time. The number of radial grid is 256. The artificial dissipations for finite (m, n) modes are $D_n = D_U = D_v = D_T = 9.375 \times 10^{-8}m^4$ in order to make the high (m, n) modes like $k_{\theta}\rho_i > 1(m > 50)$ linearly stable, 8×10^{-3} for the (0,0) mode and the resistivity is $\eta = 5 \times 10^{-7}$.

Figure 1 shows radial variation of the zonal flow frequency spectra. The frequency change of the zonal flows is clearly seen in FIG. 1, in which the pure GAM frequency $f_{\text{GAM}} = \omega_{\text{GAM}}/2\pi$ and the pure parallel sound frequency of the $(m,n)=(1,0) \mod f_{\text{sound}} = \omega_{\text{sound}}/2\pi$ are also plotted. The stationary zonal flows are dominant in the inner low q region $(r/a \lesssim 0.45)$ and the oscillatory ones are dominant in the outer high q region $(r/a \gtrsim$ 0.5). Main peaks of the oscillatory zonal flows deviate from f_{GAM} line. Besides the oscillatory zonal flows have the same frequency at a different radius. The zonal flow behaviour, stationary or oscillatory, depends on the relation between the frequency of the oscillatory zonal flows $(f_{\rm ZF})$ and f_{sound} . The (1,0) pressure perturba-



FIG. 1: Radial variation of zonal flow frequency spectra. The pure GAM frequency $f_{\text{GAM}} = \omega_{\text{GAM}}/2\pi$ and the pure parallel sound wave frequency of the (1,0) mode $f_{\text{sound}} = \omega_{\text{sound}}/2\pi$ are also plotted.

tions are necessary for the zonal flow oscillation. When f_{sound} approaches f_{ZF} , the (1,0) pressure perturbations relax along the magnetic field before they act on the zonal flows. Then the stationary zonal flows become dominant in the low q region.

4 Interaction between ITG Turbulence and Zonal Flows

It is important to identify the energy transfer channel for the zonal flows. Energy of both types of zonal flows are supplied from the ITG turbulence via the Reynolds stress which is the first term in the right hand side of Eq. (7). Meanwhile, the geodesic transfer due to the coupling with the (1,0) pressure perturabations $\langle p \sin \theta \rangle$ is a sink for the zonal flow energy in the present parameters. The Maxwell stress drive is small compared to the other drives because the beta is very small. The Maxwell stress may play a role in zonal flow energetics in high β plasmas. The two kinds of zonal flows differ in the destination



FIG. 2: Time averaged $\langle p \sin \theta \rangle$ energy drives as a function of radius.

of $\langle p \sin \theta \rangle$ energy. Figure 2 shows time averaged $\langle p \sin \theta \rangle$ energy drives as a function of radius. In the stationary zonal flow region $(r/a \leq 0.45)$ most of the energy transferred from the zonal flows goes to the (1,0) parallel ion flow $\langle v_{\parallel} \cos \theta \rangle$. The parallel flows saturate by the nonliear energy transfer to the ITG turbulence and then the stationary zonal flows also saturate[12]. On the other hand, the energy flow to $\langle v_{\parallel} \cos \theta \rangle$ in the oscillatory zonal flow or GAM region $(r/a \gtrsim 0.5)$ is much smaller than that in the stationary zonal flow region. Instead nonlinear energy transfer to the ITG turbulence is dominant. This is the same result as the drift-Alfvén turbulence simulation in Ref. [2].

When timescale of the zonal flows is much longer than that of the turbulence, suppression of the turbulence by the zonal flows Therefore the stationary zonal is strong. flows suppress the turbulence effectively and can dominate over the turbulence. Figure 3 shows a ratio of the zonal flow energy to the total E×B kinetic energy $|\nabla \phi_0|^2 / |\nabla \phi|^2$ as a function of radius and time. The ratio in the inner low q region $(r/a \lesssim 0.45)$ is high because the stationary zonal flows are dominant. On the other hand, the ratio in the outer high q region is small. The ITG turbulence in the region is not suppressed strongly because the timescale of the oscillatory zonal flows is of the same order as that of ITG modes. It is noted that FIG. 3 was obtained from the simulation including the



FIG. 3: Ratio of zonal flow energy to total $E \times B$ kinetic energy $|\nabla \phi_0|^2 / |\nabla \tilde{\phi}|^2$ as a function of radius and time.

n = 0 modes with higher poloidal modes like (2,0), (3,0) and so on. In this case the energy of the (1,0) pressure perturbations partly goes to the (2,0) mode. Then the zonal flows, especially the oscillatory flows, are more reduced. Thus the stationary zonal flows in the low q region are favourable for the suppression of the turbulence. Figure 4 shows time averaged heat flux as a function of radius for $q = 1.05 + 2(r/a)^{3.5}$ (solid line) and $q = 1.05 + 2(r/a)^2$ (dashed line). The stationary zonal flow region for $q = 1.05 + 2(r/a)^{3.5}$ (solid line) and $(r/a \leq 0.6)$ is wider than that in the previous case $(r/a \leq 0.45)$. The expansion of the stationary zonal flow region decreases the heat transport as shown in FIG. 4.



FIG. 4: Time averaged heat flux for $q = 1.05 + 2(r/a)^{3.5}$ (solid line) and $q = 1.05 + 2(r/a)^2$ (dashed line).



FIG. 5: Reversed q profiles



FIG. 6: Frequency spectra of zonal flows in reversed shear plasmas for (a) $q=2-3(r/a)^2+4(r/a)^4$, (b) $q=1.9-3(r/a)^2+4(r/a)^4$ and (c) $q=1.8-3(r/a)^2+4(r/a)^4$.



FIG. 7: Heat flux as a function of radius and time in the cases (a) $q=2-3(r/a)^2+4(r/a)^4$ and (b) $q=1.8-3(r/a)^2+4(r/a)^4$, and (c) the time averaged heat flux.

5 Zonal Flows in Reversed Shear Plasmas

In tokamak experiments with a reversed magnetic shear configuration, internal transport barriers (ITBs) were observed near the minimum q surface. The formation of ITB by a rarefaction of resonant surfaces in the q_{\min} region was reported based on global ITG simulations without nonresonant modes[13]. On the other hand, linear analysis including the nonresonant modes showed that the slablike ITG mode which has the significant nonresonant mode may appear in the q_{min} region [14]. Another simulations with nonresonant modes also showed that no gap of the turbulent transport exists in the q_{\min} region[15]. From the point of view of the zonal flow behaviour, it is expected that the staionary zonal flows are easily excited in the q_{\min} region where f_{sound} is high. In this section we investigate the zonal flow behaviour in the reversed shear tokamaks. We take the electrostatic limit ($\beta = 0$) and the electron response is assumed to be adiabatic here. The other parameters except the q profile shown in FIG. 5 used in the calculations are the same as those of the previous case. Nonresonant modes are included in all calculations because it is confirmed by linear calculations that the nonresonant modes have large amplitude in the q_{\min} region as reported in Refs. [14, 15]. Figure 6 shows frequency spectra of zonal flows in the reversed shear plasmas. In FIG. 6 (a) the oscillatory zonal flows are dominant. The frequency peaks deviate from f_{GAM} and some peaks are located at the same frequency over a wide radial region. It is seen that increase of f_{sound} by decreasing q makes the stationary zonal flows dominant like the positive shear case. Figure 7 shows heat flux as a function of radius and time in the cases (a) $q=2-3(r/a)^2+4(r/a)^4$ and (b) $q=1.8-3(r/a)^2+4(r/a)^4$, and (c) the time averaged heat flux. The heat flux shown in FIG. 7(a) and its time average (red line in FIG. 7(c)) is large over a broad radial region because the oscillatory zonal flows are dominant. When the stationary zonal flows are dominant, the heat flux is reduced as shown in FIG. 7(b) and FIG. 7(c). It is noted that the heat flux on the minimum q surface $(r \simeq 0.61)$ is large and the heat flux near the maximum f_{sound} surface $(r \simeq 0.5)$ is relatively small.

6 Summary

We have performed the ITG turbulence simulations in tokamak plasmas using the developed global Landau-fluid code. Two types of zonal flows, stationary and oscillatory modes, are possible in tokamak plasmas. The zonal flow behaviour depends on the relation between the parallel sound frequency of the (1,0) mode f_{sound} and the frequency of the oscillatory zonal flows f_{ZF} . In the low q region where $f_{\text{sound}} \sim f_{\text{ZF}}$, the stationary zonal flows are dominant. The stationary zonal flows suppress the turbulence effectively and become dominant over the turbulence. On the other hand, the zonal flows in the high q region oscillate with the (1,0) pressure perturbations $\langle p \sin \theta \rangle$ at $f_{\text{ZF}}(> f_{\text{sound}})$. The oscillatory zonal flows are less effective in suppressing the turbulence. The two kinds of zonal flows differ in the energy loop between the ITG turbulence and the zonal flows. For the stationary zonal flows, the energy loop is $\text{ITG} \rightarrow \text{ZF} \rightarrow (\langle p \sin \theta \rangle) \rightarrow \langle v_{\parallel} \cos \theta \rangle \rightarrow \text{ITG}$. On the other hand, the energy loop for the oscillatory zonal flows is $\text{ITG} \rightarrow \text{ZF} \rightarrow \langle p \sin \theta \rangle \rightarrow \text{ITG}$. The turbulent transport can be controlled through the control of the zonal flow behaviour by the q profile.

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