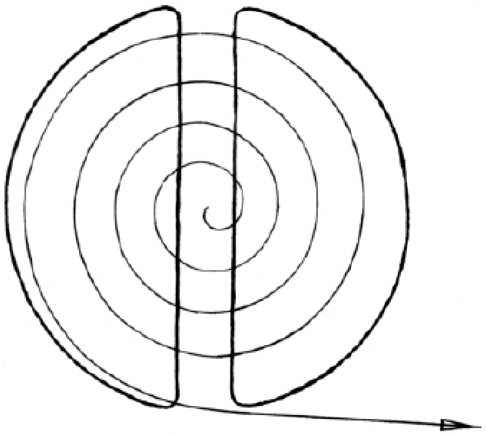
Non-Scaling Fixed Field Gradient Accelerator (FFAG) Design for the Proton and Carbon Therapy

- **Introduction: The scaling and non-scaling FFAG?**
- **A little bit of chronology: Lawrence, Thompson,…**
- **Proton and Carbon Cancer Therapy.**
- **The non-scaling FFAG:**
  - **Particle orbits**
  - **Lattice functions**
  - **Acceleration**
  - **Update on the lattice design**
- **Conclusions**

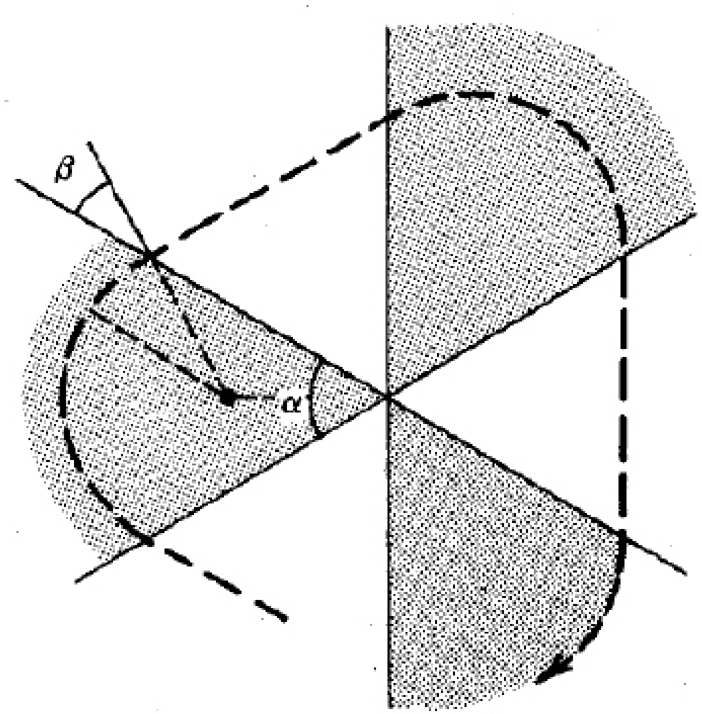
Slides from the Rick Baartman presentation: 'Cyclotrons: Classic to FFAG' - 2002

Invention (Lawrence, 1930)

$mv^2/r = qvB$, so $m\omega_0 = qB$, with $r = v/\omega_0$

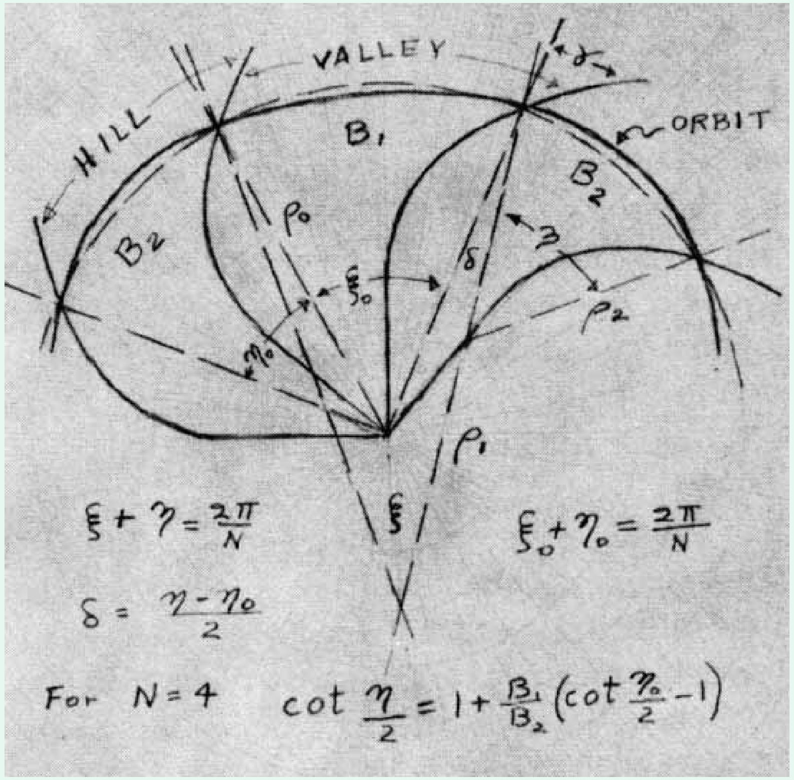


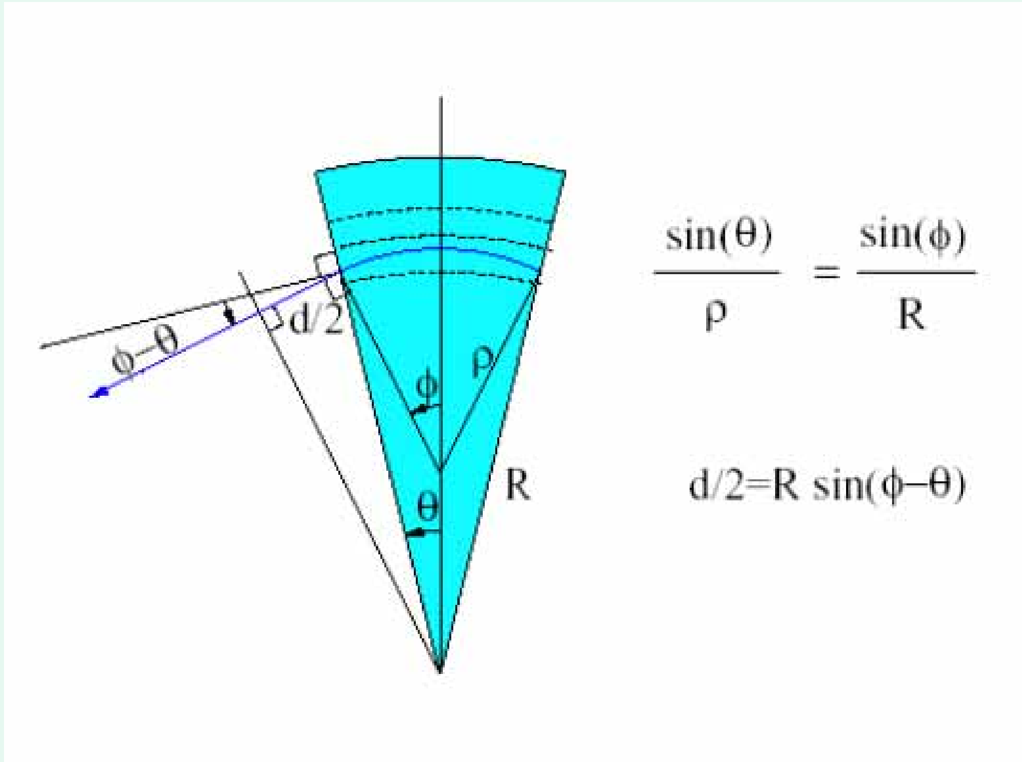
With B constant in time and uniform in space, as particles gain energy from the rf system, they stay in synchronism, but spiral outward in r.



Thomas focusing and later the Okhawa-Symon-Kolomenski FFAG

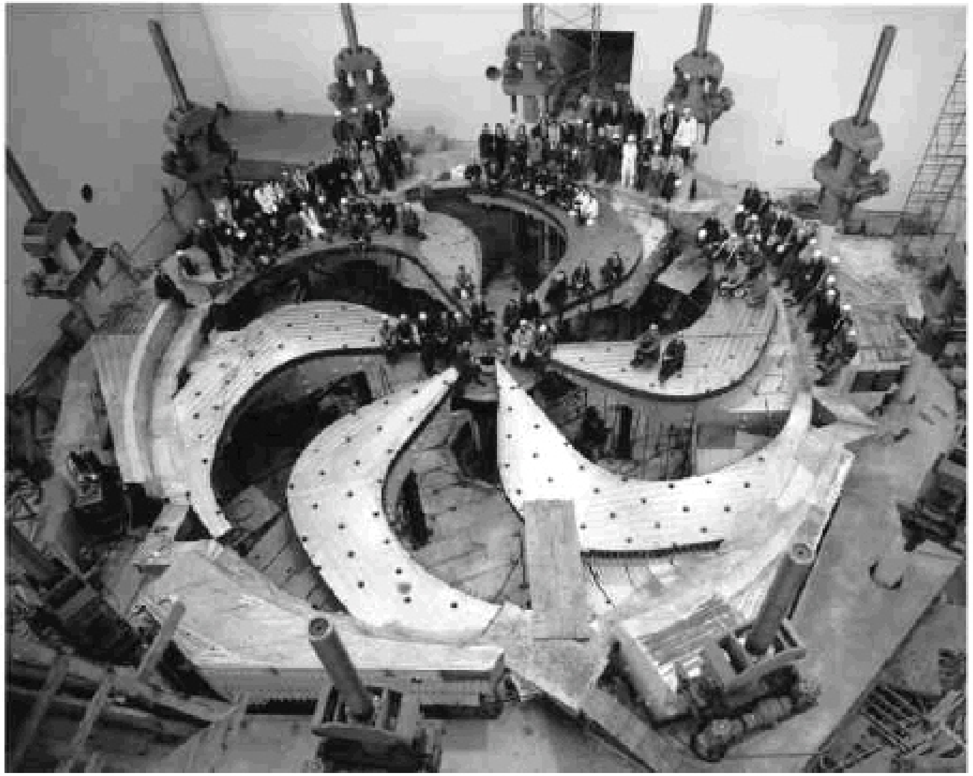
From CYCLOTRONS to FFAG's:





Slides from the Rick Baartman presentation:

Example: TRIUMF cyclotron



Energy	R	$\beta\gamma$	ξ	$1 + 2\tan^2\xi$	F^2	ν_z
100 MeV	175 in.	0.47	0°	0.0	0.30	0.28
250 MeV	251 in.	0.78	47°	3.3	0.20	0.24
505 MeV	311 in.	1.17	72°	20.0	0.07	0.24

The first SCALING FFAG

MURA-KRS-6 Phys.Rev. **103,** 1837 (1956)
November 12, 1954
K. R. Symon: The FFAG SYNCHROTRON – MARK I

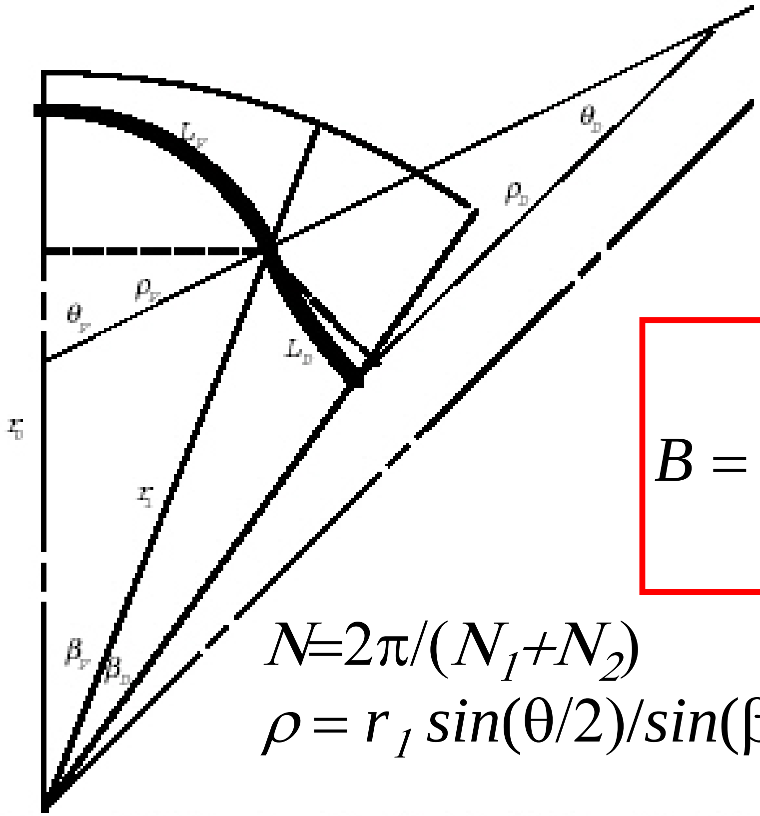
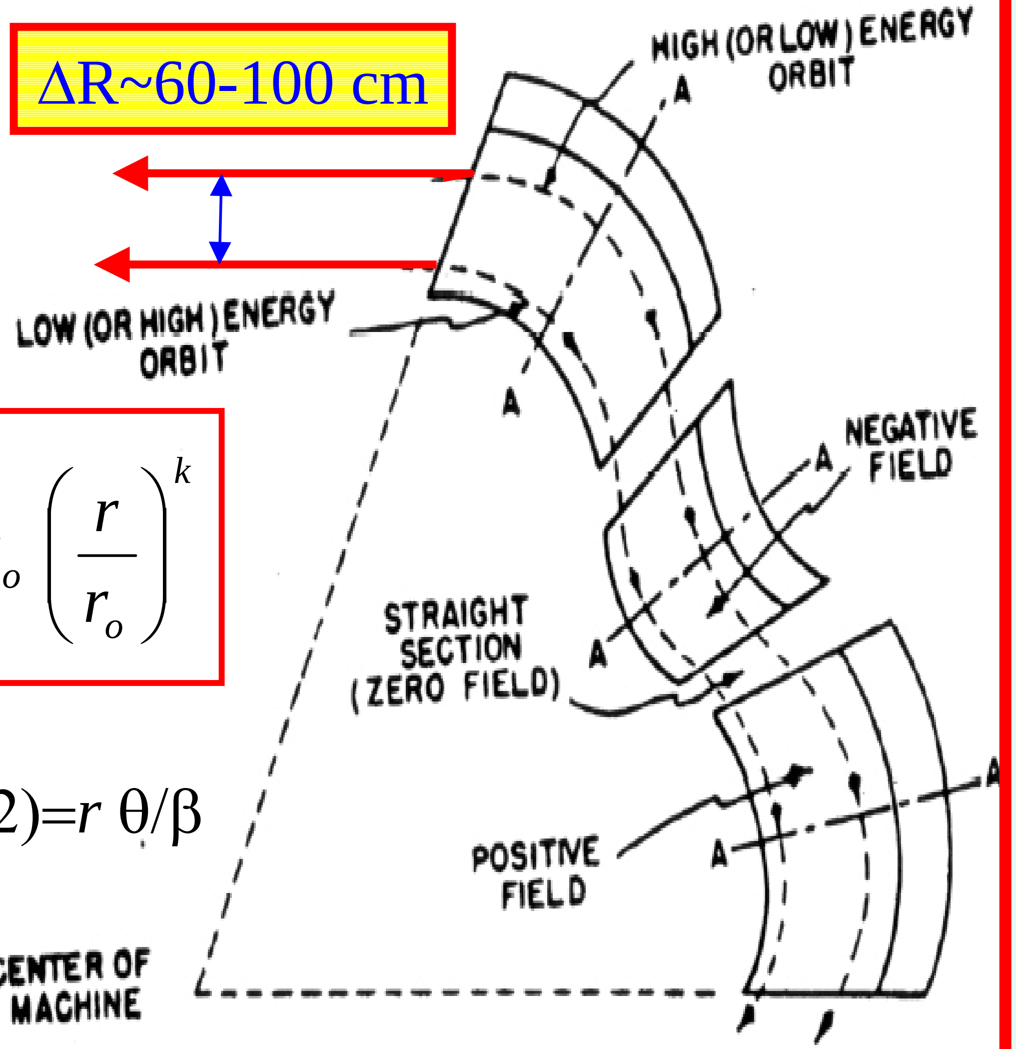


Figure 1: Closed orbit of a triplet focusing FF
half cell; a half of F magnet, D magnet of one
half straight section, is depicted.

ΔR~60-100 cm



$$B = B_o \left(\frac{r}{r_o} \right)^k$$

$$N = 2\pi/(N_1+N_2)$$

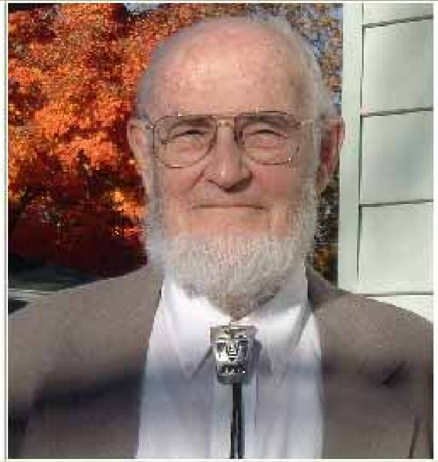
$$\rho = r_1 \sin(\theta/2)/\sin(\beta/2) = r\,\theta/\beta$$

Originator of the SCALING FFAG ?

PAC05 Particle Accelerator Conference
Knoxville, Tennessee, USA • May 16-20, 2005

Wednesday, May 18

2005 Wilson Prize to
Keith Randolph Symon
University of Wisconsin



13:00	**Wednesday Afternoon Oral Sessions** (no parallel sessions) **Ballroom D-G** **WOPA - Special Session: Accelerator Science and Technology Awards** *Chair: N. Phinney (SLAC)*
13:00	**WOPA001 - Wilson Prize Talk** ***K. Symon (Univ. of Wisconsin–Madison)*** **2005 APS Robert R. Wilson Prize**—"For fundamental contributions to accelerator science including the FFAG concept and the invention of the RF phase manipulation technique that was essential to the success of the ISR and all subsequent hadron colliders."

Citation:

"For fundamental contributions to accelerator science including the FFAG concept and the invention of the RF phase manipulation technique that was essential to the success of the ISR and all subsequent hadron colliders."

Background:

Harvard University in 1948. He taught at Wayne University in Detroit from 1947 to 1955, and at the University of Wisconsin-Madison from 1955 until he retired in 1989. From 1956 to 1967 he was a staff member of the Midwestern Universities Research Association, and its Technical Director in 1957-60. He chaired the Argonne Accelerator Users Group in 1961-62, was Acting Director of the Madison Academic Computing Center in 1982-83, and Acting Director of the UW-Madison Synchrotron Radiation Center from 1983 to 1985.
His research areas are the theory and design of particle accelerators and theoretical plasma physics. He invented the FFAG accelerator design, and contributed to the theory of radio-frequency acceleration, collective instabilities, and colliding beam techniques. He developed bit-pushing and distribution-pushing techniques for the numerical solution of the Vlasov equation for the study of collective instabilities in plasmas and accelerator beams. He wrote a textbook Mechanics, 1953, third edition, 1971.
He is a Fellow of the American Physical Society and of the American Association for the Advancement of Science, and a member of the American Association of Physics Teachers. He is a recipient of the 2003 Particle Accelerator and Technology Award of the IEEE, and a Distinguished Faculty Fellow of the Physics Department of the University of Wisconsin-Madison.

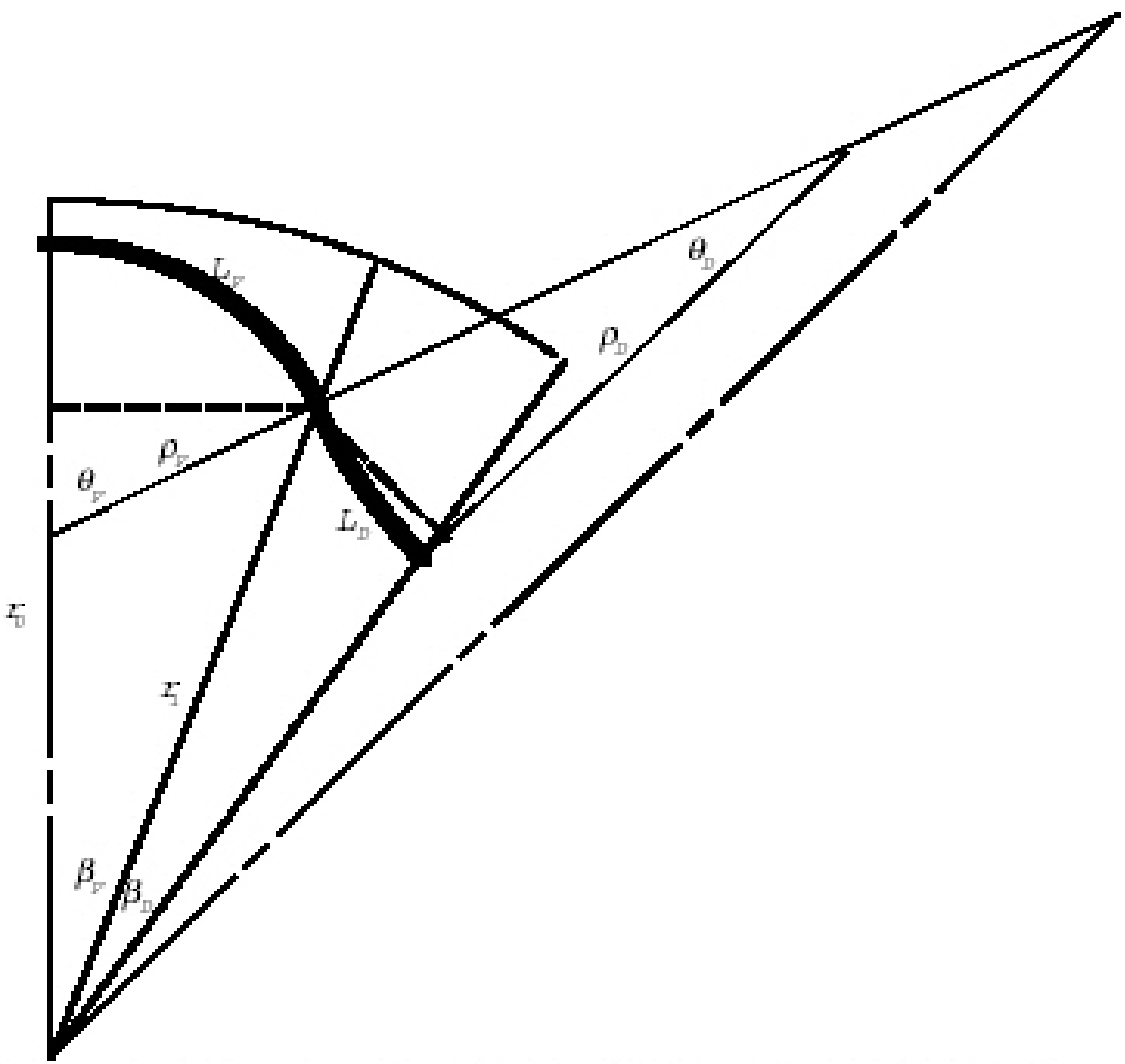


Figure 1: Closed orbit of a triplet focusing FFAG. Only a half cell; a half of F magnet, D magnet of one side, and a half straight section, is depicted.

Original FFAG Phys. Rev. article (1956)

PHYSICAL REVIEW VOLUME 103, NUMBER 6 SEPTEMBER 15, 1956

Fixed-Field Alternating-Gradient Particle Accelerators*

K. R. SYMON,† D. W. KERST,‡ L. W. JONES,§ L. J. LASLETT,|| AND K. M. TERWILLIGER§
Midwestern Universities Research Association

(Received June 6, 1956)

It is possible, by using alternating-gradient focusing, to design circular accelerators with magnetic guide fields which are constant in time, and which can accommodate stable orbits at all energies from injection to output energy. Such accelerators are in some respects simpler to construct and operate, and moreover, they show promise of greater output currents than conventional synchrotrons and synchrocyclotrons. Two important types of magnetic field patterns are described, the radial-sector and spiral-sector patterns, the former being easier to understand and simpler to construct, the latter resulting in a much smaller accelerator for a given energy. A theory of orbits in fixed-field alternating-gradient accelerators has been worked out in linear approximation, which yields approximate general relationships between machine parameters, as well as more accurate formulas which can be used for design purposes. There are promising applications of these principles to the design of fixed-field synchrotrons, betatrons, and high-energy cyclotrons.

INTRODUCTION

ALTERNATING-GRADIENT (AG) focusing[1] provides a high degree of stability for both radial and vertical modes of betatron oscillations in circular magnets vary in the same way with radius but with alternating signs (or in certain cases alternating magnitudes). Since the orbit in the reverse field magnet bends away from the center, the machine is considerably

Original FFAG Phys. Rev. article (1956)

In Part I of this paper we discuss the radial- and spiral-sector types of FFAG accelerator in detail. In Part II the theory of particle trajectories in FFAG machines is developed. Part III contains a description of a 10-Bev radial-sector synchrotron, a 20-Bev spiral-sector synchrotron, and FFAG betatrons and cyclotrons.

I. TYPES OF FFAG DESIGN

1. Radial-Sector Type

Circular particle accelerators with radial sectors can be built with the high-energy orbits at the outer edge of the machine and the injection orbits at the inside edge, or vice versa. This discussion assumes that the is of course determined by the necessity for preserving stability of the vertical betatron oscillations. Some vertical focusing and radial defocusing occur because the orbits are scalloped and do not cross the magnet edges at right angles. In machines in which the number of sectors is large and the effects of orbit scalloping small, the negative-field magnet can be made no shorter than about $\frac{2}{3}$ of the positive-field magnet if we wish to preserve vertical stability. This means that, neglecting

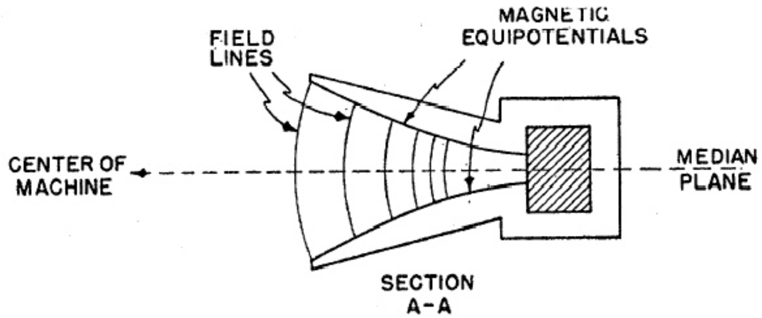


FIG. 1. Vertical section through positive or negative radial-sector magnets.

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[4] Terwilliger, Jones, Kerst, and Symon, Phys. Rev. 98, 1153(A) (1955). This had been pointed out independently by G. Miyamoto, Tokyo University, Tokyo, Japan, at a meeting of the Physical Society of Japan in April, 1952 (private communication).

[5] L. H. Thomas, Phys. Rev. 54, 580, 588 (1938).

K.R. Symon

$C = 2\pi r_o$~612 m

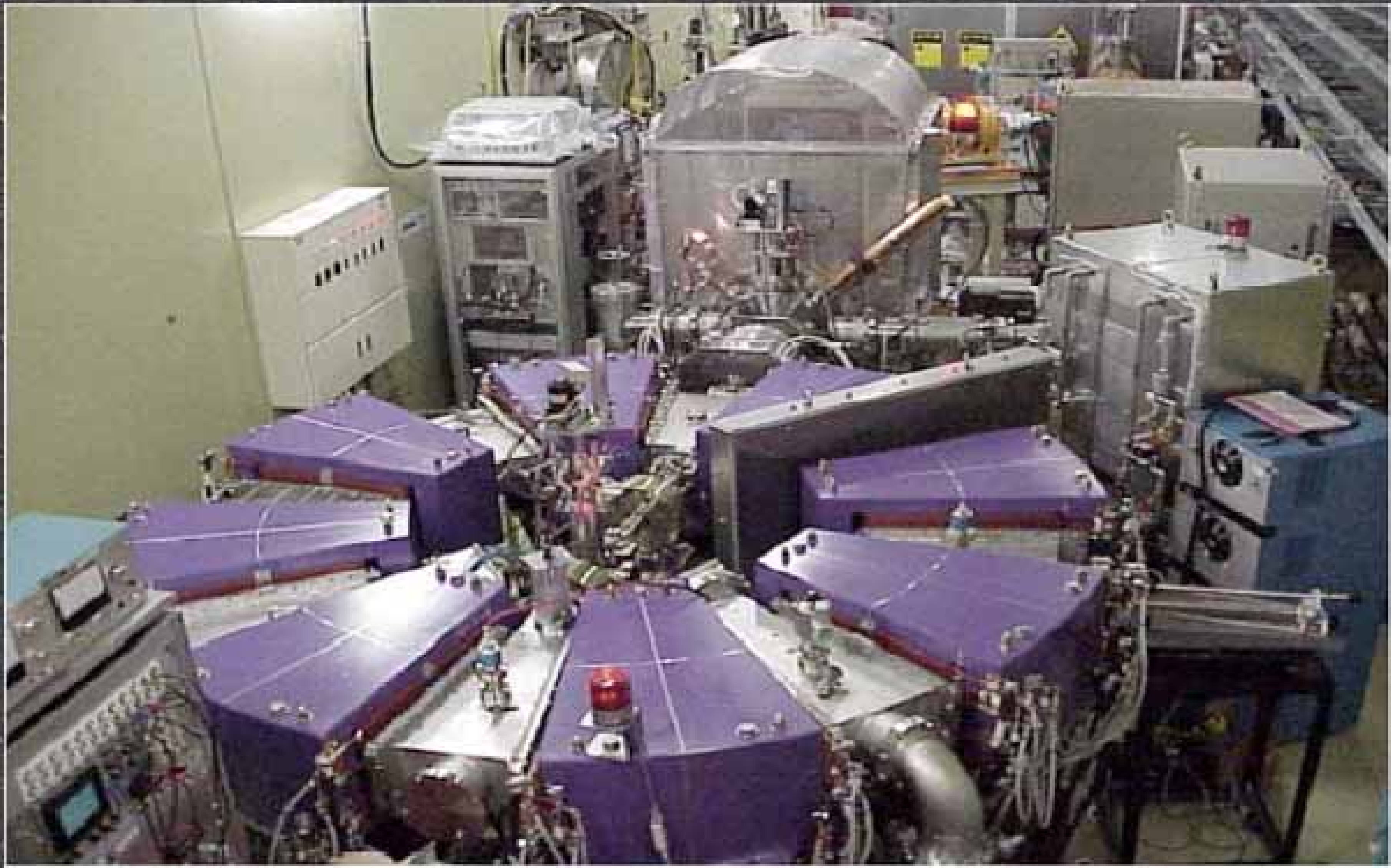
p_{end} = 10.9 GeV/c
$p_{initial}$= 97.0 MeV/c
p_c= 5.5 GeV/c

$\delta p/p_c$= +/–98 %

TABLE II. Physical dimensions of a radial sector accelerator. Subscript 0 refers to maximum energy, subscript i refers to injection.

$E_0=10$ Bev (GeV)	$E_i=5$ Mev	proton kinetic energy
$r_0=97.3$ m	$r_i=95.0$ m	synchrotron radius
$B_0=20\,000$ gauss	$B_i=200$ gauss	magnet guide field
$\rho_0=18.2$ m	$\rho_i=17.8$ m	radius of curvature
$Z_0=3.0$ cm	$Z_i=15.0$ cm	vertical semiaperture
$r_0-r_i=2.3$ m	radial aperture ← **Orbit offsets**	
$E_t=12$ Bev	transition energy	
$Z_i=2.5$ cm	vertical semiheight of injected beam	
$\delta_i=\pm0.001$ radian	angular spread of injected beam	
$p=5\times10^{-6}$ mm Hg	pressure in the vacuum chamber	

150 MeV FFAG at KEK



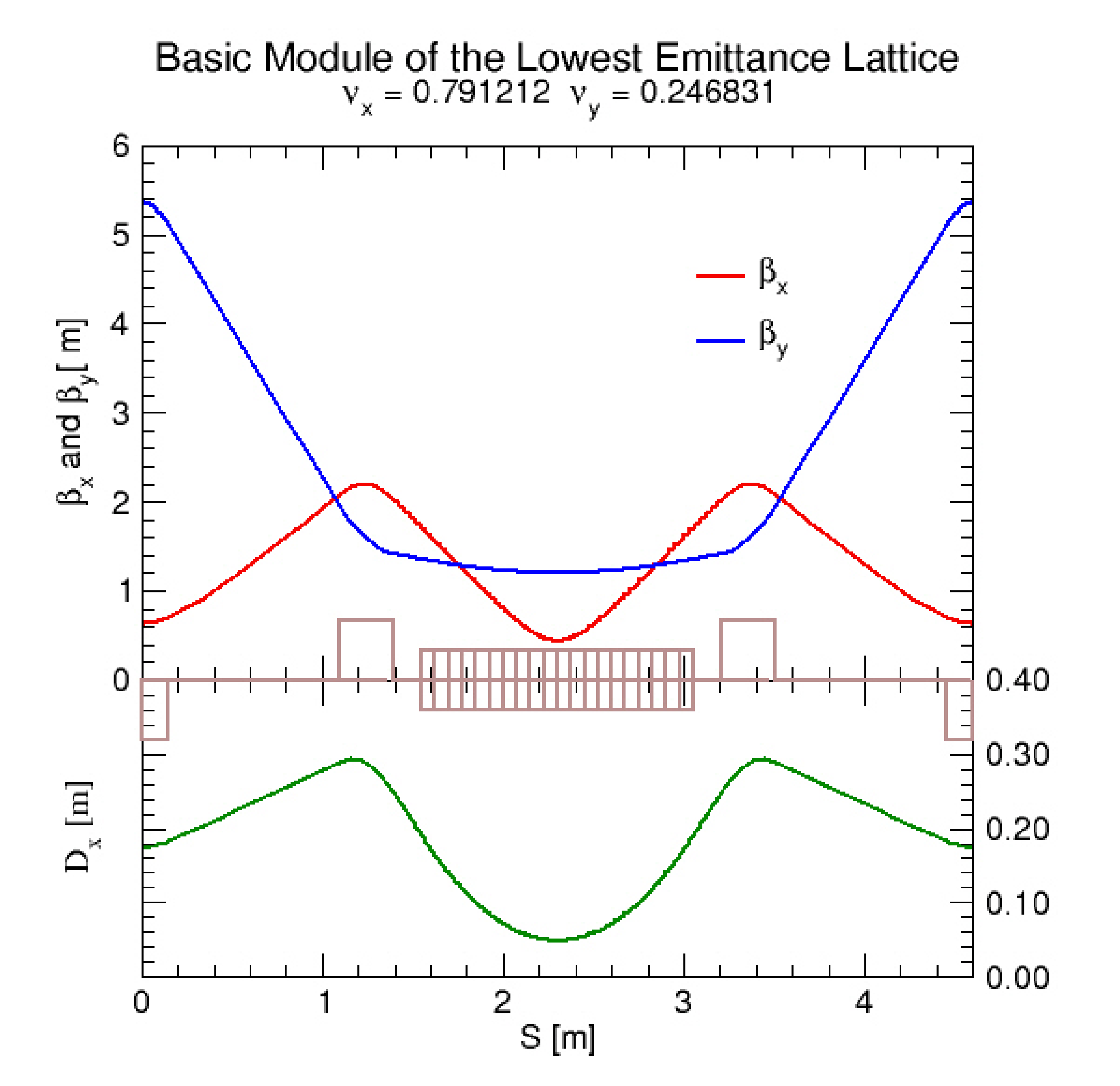
fast proton FFAG @KEK (2001)

What are the basic parameters for the proton and Carbon therapy:

- **Required Range of Energies (or $\delta p/p$)**
  - The “central” energy or momentum p_o is in two examples presented later set are for protons to $\mathbf{p_o}$ **=486 *MeV/c*** or for carbon accelerator **726 *MeV/c*/**nucl. The acceleration would be possible from E_{ko} **=31 *MeV*** up to E_{km} **= 250 *MeV*** or for carbon ions from **68 *MeV/nucl***. up to **500 *MeV/nucl.***
  - Aperture limitation is defined by the maximum value of the DISPERSION function: $\Delta x < D_x * \delta p/p < +/- 35$ mm
    - if the $0.5 < \delta p/p < 1.5$ then:
    - **D_x < 70 mm**
- Why is the Minimum of the < H > function relevant?
  - The normalized dispersion amplitude corresponds to the $<H>^{1/2}$!!!

$$\Delta C = \left[\oint_C \frac{D(s)}{\rho} ds \right] \delta \quad where\ \delta = \frac{\Delta p}{p}.$$

The minimum emittance lattice = to the minimum of the average value of the <H>$_{min}$ function:



β_{min}= Ld/2√15

D_{xmin}=θ∗Ld/24

Scaling or non- scaling FFAG?

Scaling FFAG properties:

- Zero chromaticity.
- Orbits parallel for different energies.
- Large momentum acceptance.
- Relatively large circumference (θ_1/θ_2).
- Relatively large physical aperture.
- RF:large aperture-follows the energy.
- Tunes are fixed for all energies.
- Negative momentum compaction.
- Orbits of the high energy particles are at high field, low energy particles at low field.

Non-Scaling (linear) FFAG properties:

- Chromaticity is changing.
- Orbits are not parallel.
- Large momentum acceptance.
- Relatively small circumference.
- Relatively small physical aperture.
- RF:small aperture-at the crest.
- Tunes move 0.4-0.1 in basic cell.
- Momentum compaction changes.
- Orbits of the high energy particles are at high field, low energy particles at low field.

Recent non-scaling FFAG Phys. Rev. Spec. Topics article

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **8**, 050101 (2005)

Design of a nonscaling fixed field alternating gradient accelerator

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BNL, Upton, New York 11973, USA
(Received 7 February 2005; published 19 May 2005)

We present a design of nonscaling fixed field alternating gradient accelerators (FFAG) minimizing the dispersion action function H. The design is considered both analytically and via computer modeling. We present the basic principles of a nonscaling FFAG lattice and discuss optimization strategies so that one can accelerate over a broad range of momentum with reasonable apertures. Acceleration schemes for muons are discussed.

DOI: 10.1103/PhysRevSTAB.8.050101 PACS numbers: 29.20.−c, 41.75.Lx

I. INTRODUCTION

The fixed field alternating gradient (FFAG) configuration, introduced independently by Ohkawa [1], Symon [2], and Kolomensky [3], has received much attention in recent years. A "proof of principle" machine has been built at KEK [4], followed by a 150 MeV proton synchrotron which is being commissioned [5]. In the scaling FFAG design the particle orbits "scale" with momentum, and acceleration over a large range of momentum requires large apertures. In nonscaling FFAGs the aperture requirements can be significantly reduced.[1] Nonscaling FFAGs have been discussed as a part of the general FFAG family [8] and in the context of muon acceleration [9–14], where the short muon lifetime prohibits slow ramping of the magnetic fields. The FFAG acts similar to a recirculating linear accelerator (RLA), but all the orbits go through the same lattice, obviating the need for separated arcs.

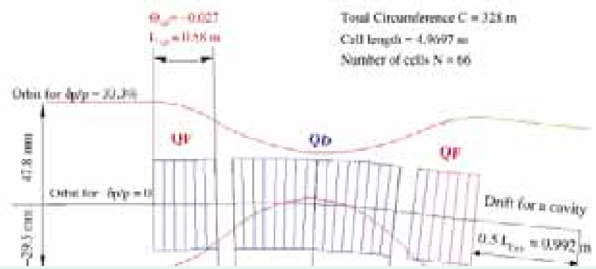
A. The Basic Cell

We end up with the rather simple configuration of Fig. 1. The accelerator is composed of a large number (66 in our case) of identical unit cells. Each cell contains a magnet triplet, with a relatively long gradient bending magnet QD ("combined function") having a strong central field and negative gradient (horizontally defocusing) at the center, flanked by a pair of negative bend magnets QF that are

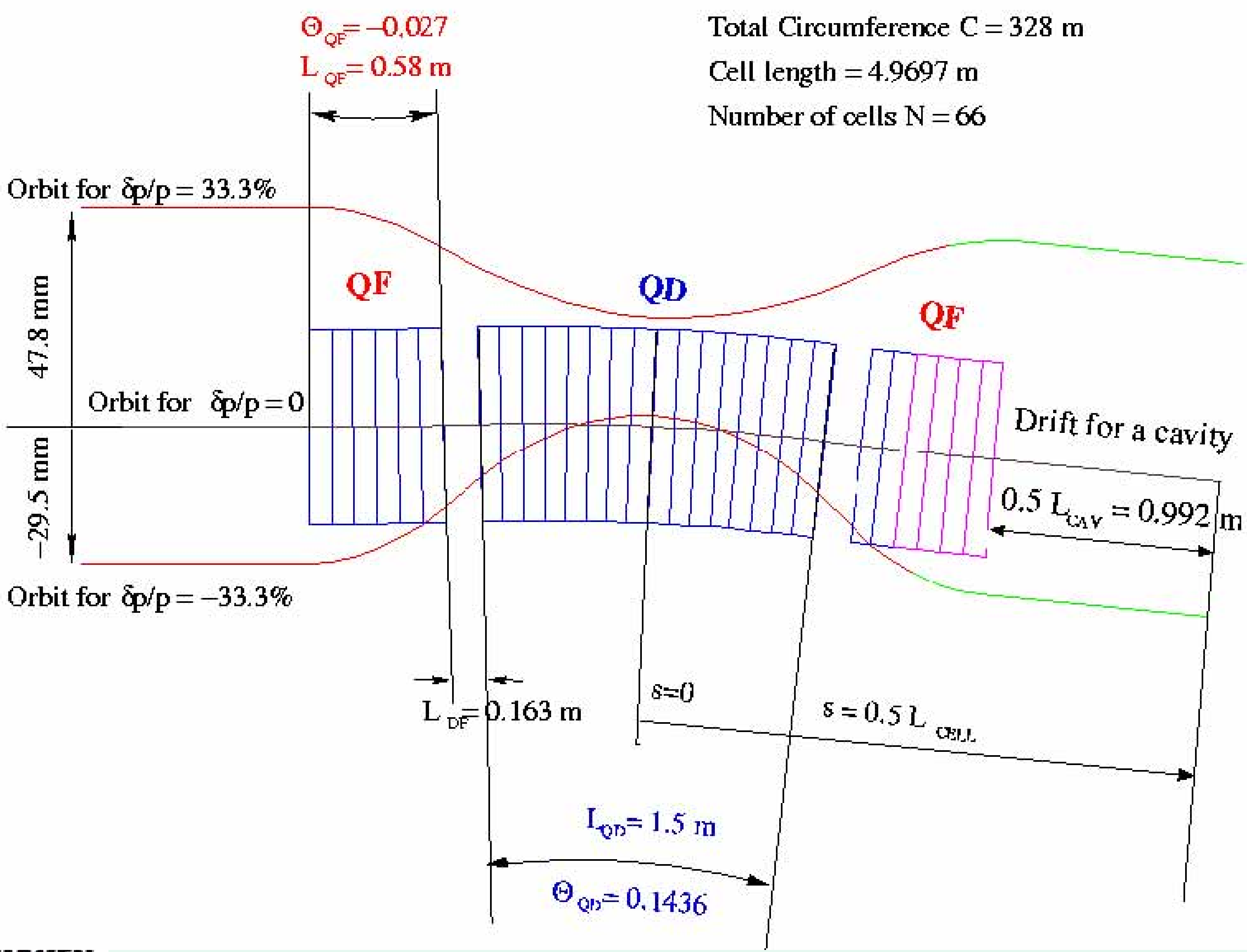
II. PARTICLE MOTION AT VERY LARGE MOMENTUM OFFSET

We consider a particle of momentum p and a reference particle with momentum p_0 and charge q; the momentum offset is $\delta = (p - p_0)/p_0$. The magnetic rigidity of the reference particle is $(B\rho)_0 = p_0/q$; the reference particle is on a reference orbit (assumed planar) with local radius of curvature ρ_0 and vertical field $B_0(s) = (B\rho)_0/\rho_0(s)$. In the cases considered here the field $B_0(s)$ and with it the radius of curvature $\rho_0(s)$ are constant in each magnet, so that the reference orbit consists of circular arcs in the magnets and straight sections between the magnets; we also assume that the magnet edges are straight at right angles to the reference orbit. We assume that the magnetic field in the dipole magnets is linear:

$$B_y = B_0 + Gx, \quad (1)$$



Triplet for the minimum emittance



Proton and Carbon Cancer Therapy

The cancerous tumors are removed most efficiently by the ion radiation as it had been previously (1946) recognized by R. Wilson.
[Radiological use of fast protons. Radiology 47:487-91, 1946].

The Relative Biological Efficiency RBE is at least 1.1 better with ions compared to the X-rays.

[A new method of treating leukemia at the Sloan Hospital in New York is by the short lived α**-emitters.** They have to stick to the cancerous cells and energy deposited by radiation destroy DNA].

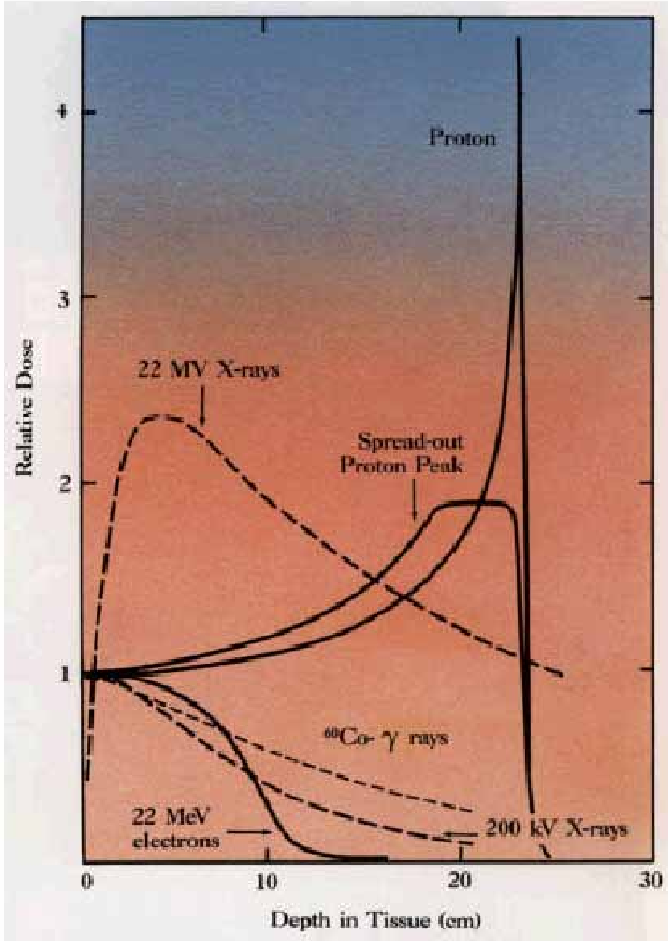


Figure 1: Depth dose curves for an unmodulated proton beam (Bragg peak), a modulated proton beam (spread-out Bragg peak – SOBP), 22 MeV X-rays, 22 MeV electrons, ^{60}Co γ-rays and 200 kV X-rays.

Proton and Carbon Cancer with respect to the photon therapy:

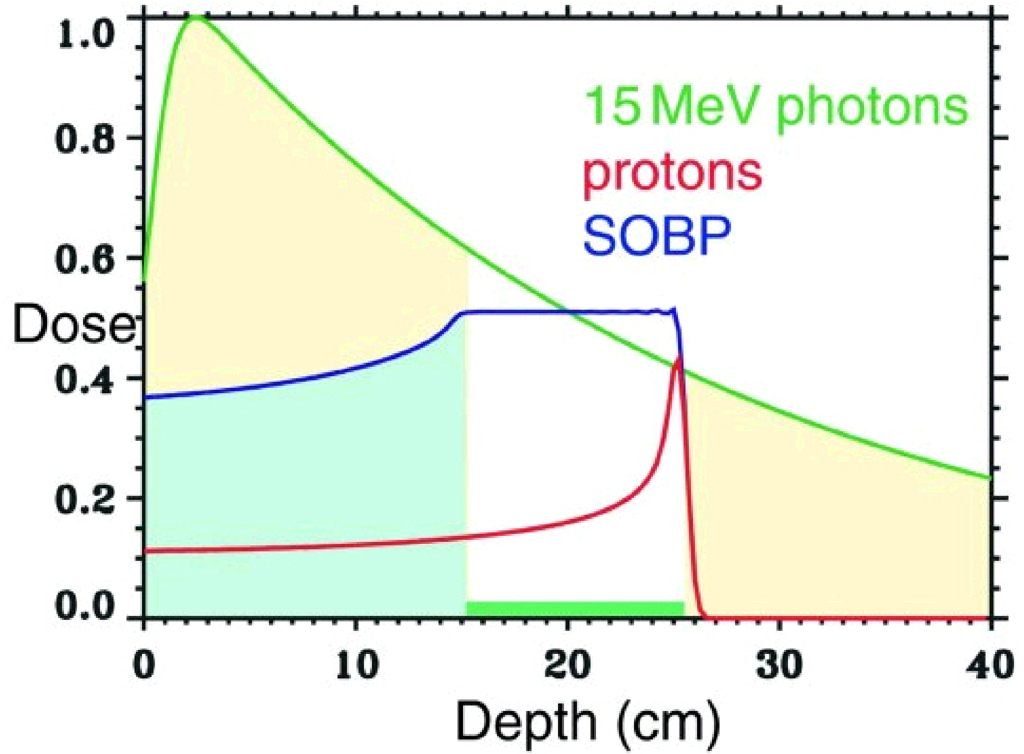


Fig 1 Comparison of the depth dose profiles of proton and photon beams. The photon dose falls off exponentially with depth (green curve). A mono-energetic proton beam is characterized by the presence of the Bragg peak in the region where the protons stop (red profile). Through the superposition (blue curve) of many proton beams of different residual range it is possible to deposit a homogenous dose (SOBP) in the region of the tumor (in this case from 15 to 25 cm depth). One recognizes from the picture the potential of dose sparing of the protons in the entrance and exit region of the beam (the unnecessary dose is painted in pale blue and light yellow).

Simulation of the proton therapy:

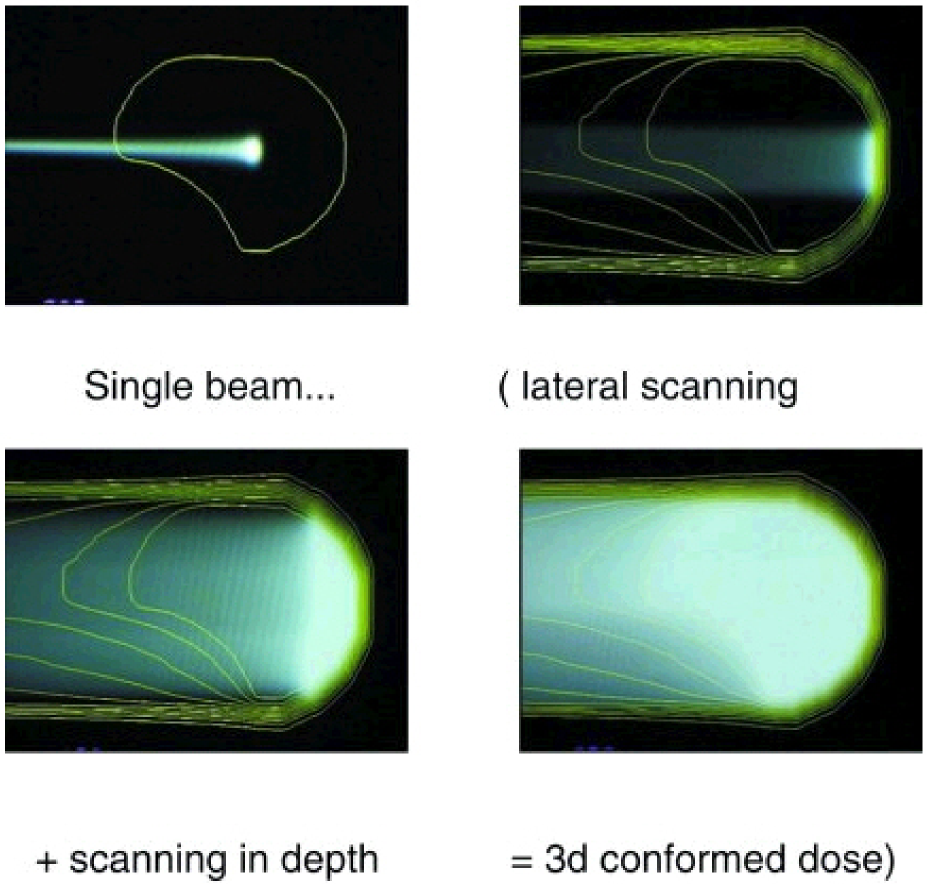


Fig 3 Basic principle used for beam scanning with protons. Through the delivery of individual proton pencil beams one can shape the distribution of the dose in three-dimensions at wish directly under computer control.

Proton and Carbon Cancer Therapy present facilities:

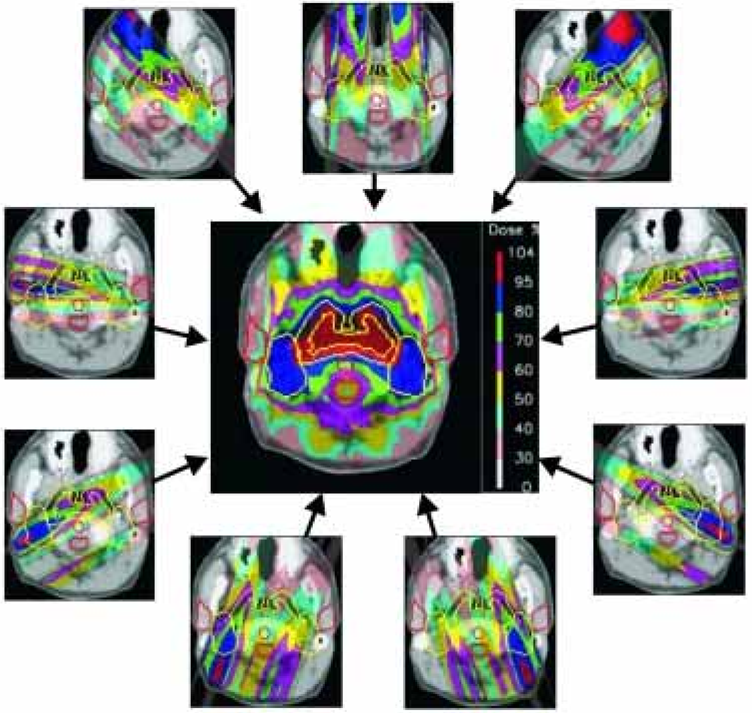


Fig 2 An example of intensity modulated treatment planning with photons. Through the addition of 9 fields it is possible to construct highly conformal dose distribution with good dose sparing in the region of the brain stem (courtesy of T. Lomax, PSI).

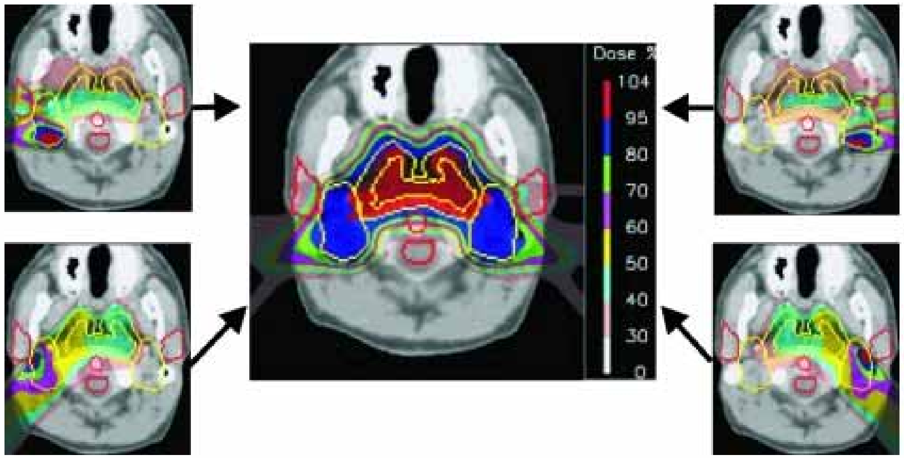
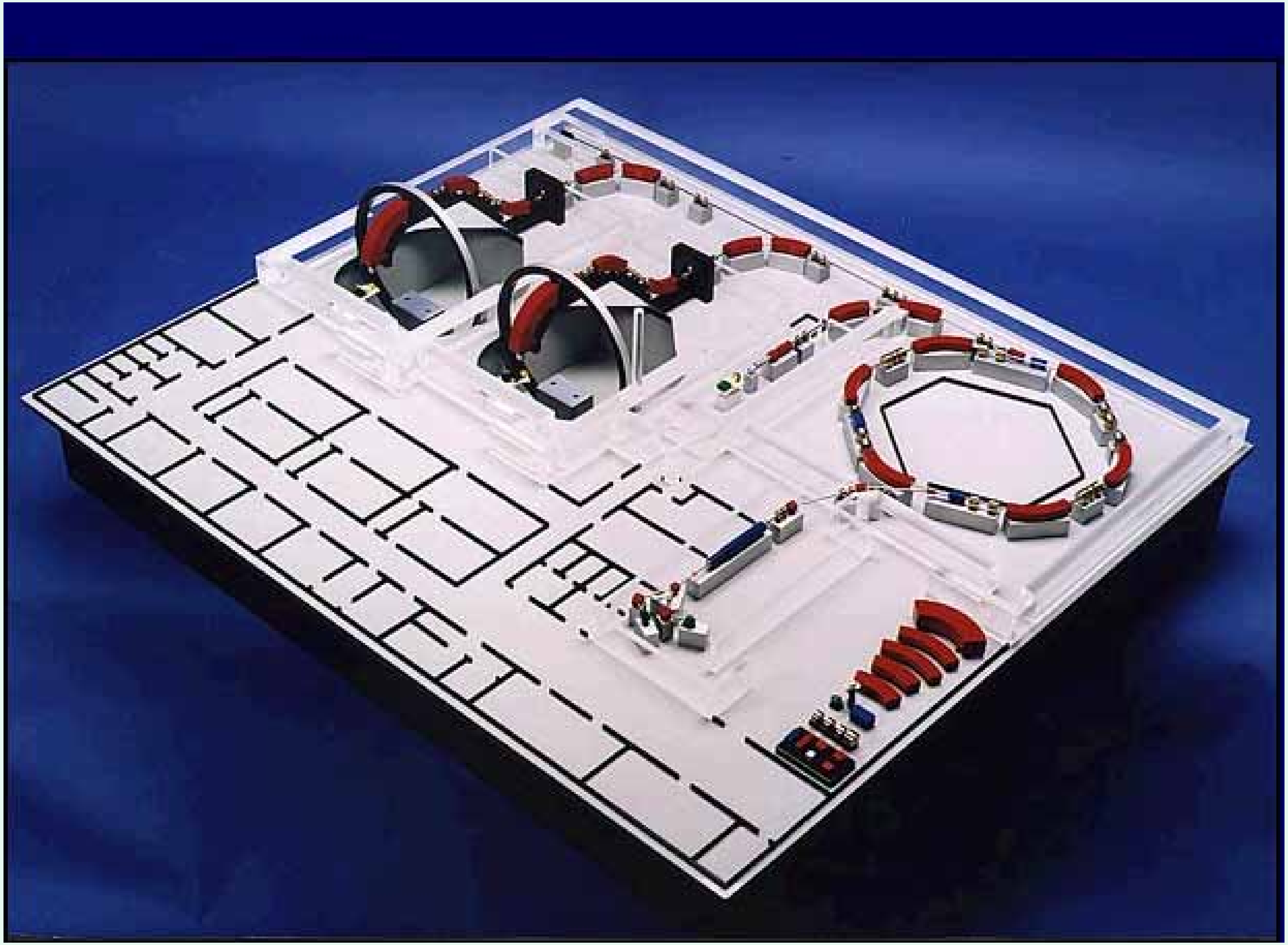


Fig 4 Example of intensity modulated therapy with protons. A high degree of conformity is achieved using a low number of dose fields. The advantage compared with photons is the general reduction of dose burden outside of the target volume (courtesy of T.Lomax, PSI)

The proton therapy facility components:



Proton and Carbon Cancer Therapy

Protons have comparable biological effects in tissue relative to high energy x-rays used in conventional radiotherapy; in fact the RBE value of protons is approximately 1.1. Measured RBE values can be found in Table 2.

Table 2: RBE values of modulated proton beams at the Bragg peak compared to $^{60}Co^{iii}$.

Reference	Tissue	Endpoint	Proton energy (MeV)	No. fractions	RBE
Tepper (1977)	Crypt cell	Survival	160	1	1.19
	Crypt cell	Survival	160	20	1.23
	Skin	Acute reaction	160	20	1.13
Urano (1980)	Fibrosarcoma	Survival	160	1-10	1.16
Urano (1984)	Mammary ca.	$TCD_{50/120}$	160	1	1.11
	Lens	Cataract	160	1	1.09
	Lung	$LD_{50/100}$	160	1	1.02
	Testis	Weight loss	160	1	1.23
	Tail vertebrae	Growth	160	1	1.32
Anso(1985)	Fibrosarcoma	Survival	70	1	1.06(1.01-1.12)
	Fibrosarcoma	Survival	250	1	1.06(1.03-1.09)
Tatsuzaki(1993)	Mouse	$LD_{50/30}$	250	1	1.09
	Skin	Contraction	250	10	1.03

Proton and Carbon Cancer Therapy present facilities:

WHO	WHERE	WHAT	DATE FIRST RX	DATE LAST RX	RECENT PATIENT TOTAL	DATE OF TOTAL
Berkeley 184	CA. USA	p	1954	1957	30	
Berkeley	CA. USA	He	1957	1992	2054	
Uppsala (1)	Sweden	p	1957	1976	73	
Harvard	MA. USA	p	1961	2002	9116	
Dubna (1)	Russia	p	1967	1996	124	
ITEP, Moscow	Russia	p	1969		37858	Dec-04
Los Alamos	NM. USA	p	1974	1982	230	
St. Petersburg	Russia	p	1975		1145	Apr-04
Berkeley	CA. USA	ion	1975	1992	433	
Chiba	Japan	p	1979		145	Apr-02
TRIUMF	Canada	p	1979	1994	367	
PSI (SIN)	Switzerland	p	1980	1993	503	
PMRC (1), Tsukuba	Japan	p	1983	2000	700	July-00
PSI (72 MeV)	Switzerland	p	1984		4182	Dec-04
Uppsala (2)	Sweden	p	1989		418	Jan-04
Clatterbridge	England	p	1989		1372	Dec-04
Loma Linda	CA. USA	p	1990		9585	Nov-04
Louvain-la-Neuve	Belgium	p	1991	1993	21	
Nice	France	p	1991		2555	Apr-04
Orsay	France	p	1991		2805	Dec-03
iThemba LABS	South Africa	p	1993		468	Nov-04
MPRI (1)	IN USA	p	1993	1999	34	Dec-99
UCSF - CNL	CA USA	p	1994		632	June-04
HIMAC, Chiba	Japan	C ion	1994		1796	Feb-04
TRIUMF	Canada	p	1995		89	Dec-03
PSI (200 MeV)	Switzerland	p	1996		209	Dec-04
G.S.I Darmstadt	Germany	C ion	1997		198	Dec-03
H.M.I. Berlin	Germany	p	1998		546	Dec-04
NCC, Kashiwa	Japan	p	1998		300	Oct-04
Dubna (2)	Russia	p	1999		296	Dec-04
HIBMC, Hyogo	Japan	p	2001		483	Dec-04
PMRC (2), Tsukuba	Japan	p	2001		492	Jul-04
NPTC, MGH	MA USA	p	2001		973	Dec-04
HIBMC, Hyogo	Japan	C ion	2002		30	Dec-02
INFN-LNS, Catania	Italy	p	2002		82	Oct-04
WERC	Japan	p	2002		19	Oct-04
Shizuoka	Japan	p	2003		100	Dec-04
MPRI (2)	IN, USA	p	2004		21	Jul-04
Wanjie, Zibo	China	p	2004		1	Dec-04
					1100 pions	
					4511 ions	
					40801 protons	
				TOTAL	46412 all particles	

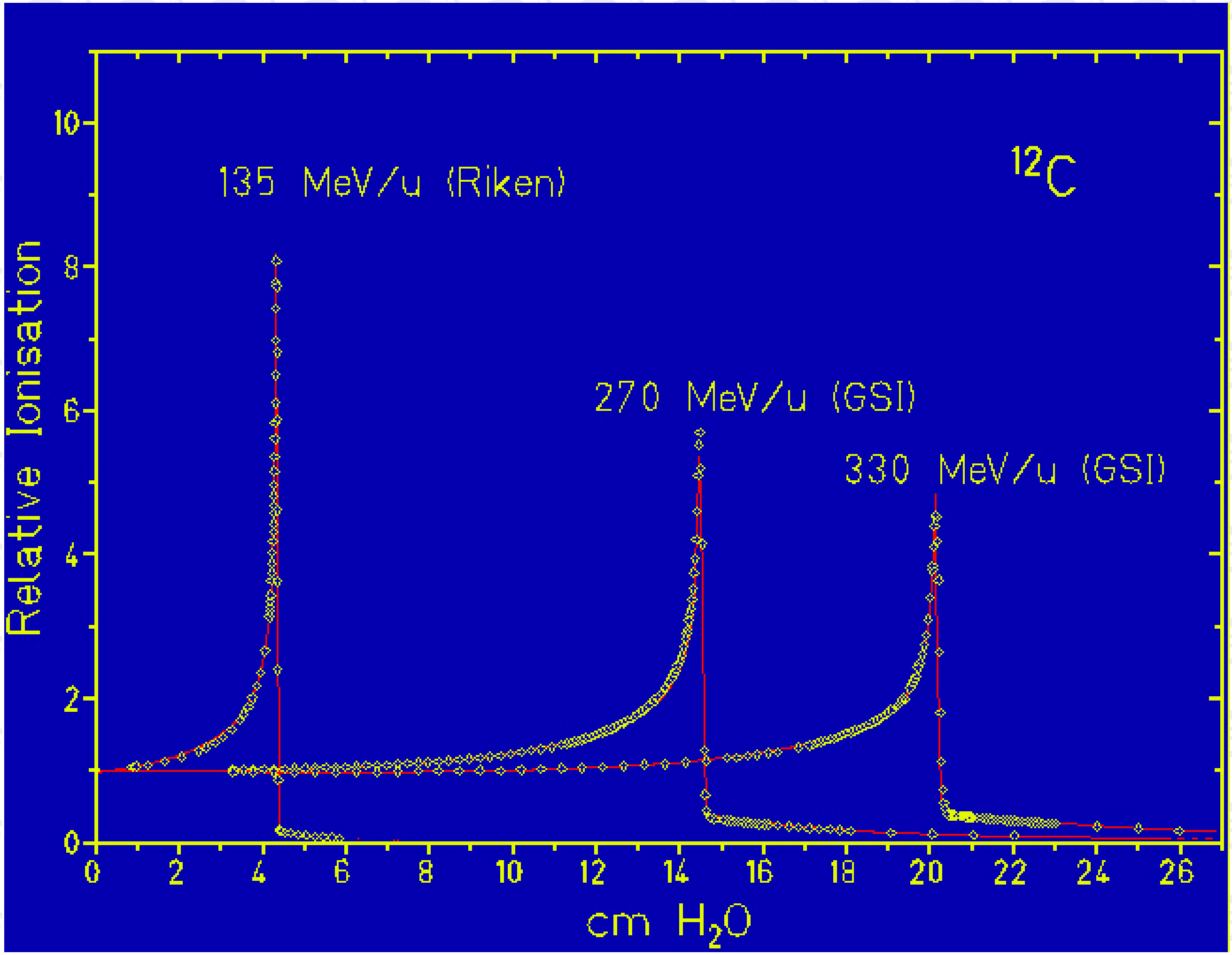
Future proton therapy facilities:

Table 4: Proposed New Facilities for Proton &Ion Beam Therapy

INSTITUTION	PLACE	TYPE	1ST RX?	COMMENTS
Bratislava	Slovakia	p, ion	2003	72 MeV cyclotron; p; ions; +BNCT, isot prod.
IMP, Lanzhou	PR China	C-Ar ion	2003	C-ion from 100MeV/u at HIRFL expand to 900 MeV/u
Shizuoka Cancer Center	Japan	p	2003	synchrotron 235 MeV; 2 gantries; 1 horiz; funded.
Rinecker, Munich	Germany	p	2003	4 gantries, 1 fixed beam, 230 MeV, scanning beams.
Wanjie, Zibo	China	p	2003	Under construction. 230 MeV synchrotron, 2 treat rooms.
PSI	Switzerland	p	2004	Addition of a 250MeV cyclotron, 2nd gantry, new 1 fixed
NCC, Seoul	Korea	p	2005	235 MeV cyclotron, 2 gantries, 1 horiz.
IThemba LABS, Somerset West	South Africa	p	2006	230MeV,1 gantry,1 horiz.+30o beams,1 horiz.+15o.beams
CGMH, Northern Taiwan	Taiwan	p	2001?	250MeV synchrotron/230MeV cyclotron;3 gantry,1 fixed
Erlangen	Germany	p	?	4 treatment rooms, some with gantries.
CNAO, Milan & Pavia	Italy	p, ion	2004?	synchrotron; 2 gantry;1 fixed beam rooms;1 exp. room
M. D. Anderson Cancer Center	TX, USA	p	2004?	Accelerator, 3 gantries; 1 fix + 1 exp beam rooms
University of Florida	FL, USA	p	2004?	235MeV cyclotron; 2 gantries; 1 fix + 1 exp beam rooms
Heidelberg	Germany	p, ion	2005?	
Med-AUSTRON	Austria	p, ion	2007?	2p gantry;1 ion gantry;1 fixed p;1 fixed ion;1 exp room
Xi an, Shanxi Province	China	p	?	Contract signed with IBA.
Central Italy	Italy	p	?	cyclotron; 1 gantry; 1 fixed
Clatterbridge	England	p	?	230 MeV cyclotron; part of the CASIM project
TOP project ISS Rome	Italy	p	?	70 MeV linac; expand to 200 MeV?
3 projects in Moscow	Russia	p	?	including 320 MeV; compact, probably no gantry
Krakow	Poland	p	?	60 MeV proton beam.
Proton Development N.A. Inc.	IL USA	p	?	300 MeV protons; therapy & lithography

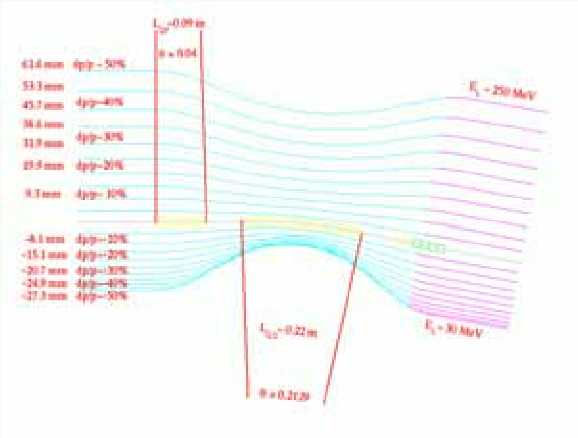
Advantages of Carbon with respect to proton:

Physical beam model for carbon ion radiotherapy

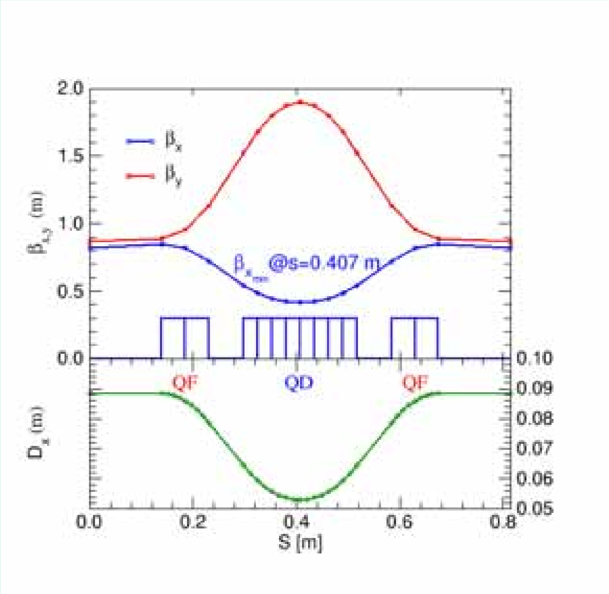


The non-scaling FFAG

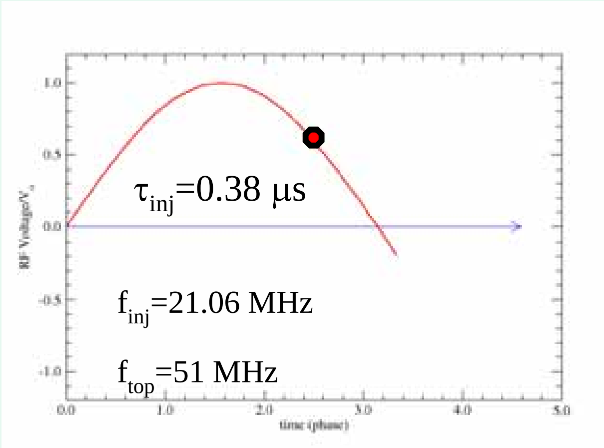
A . Particle orbits



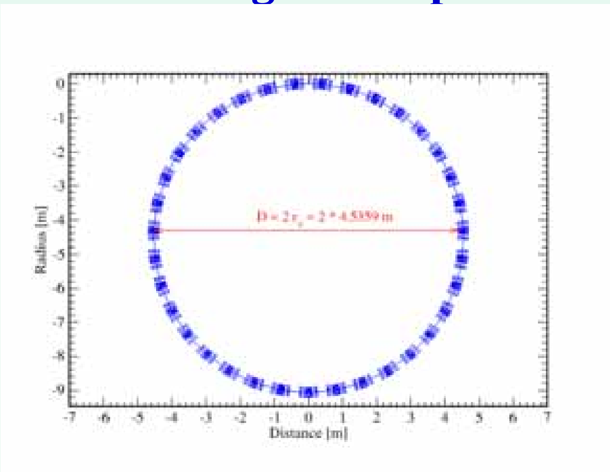
B. Lattice functions

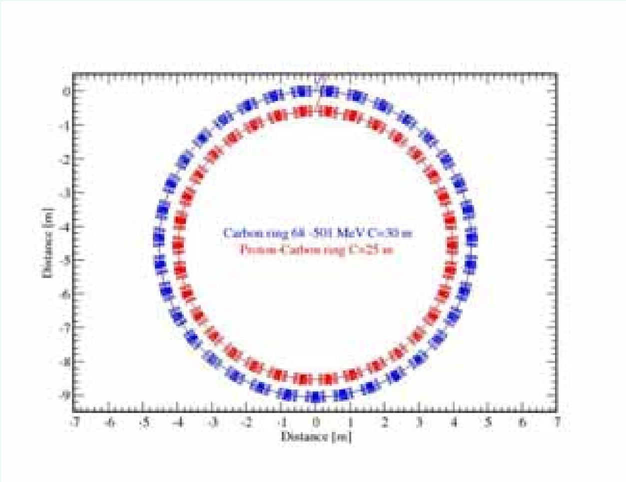


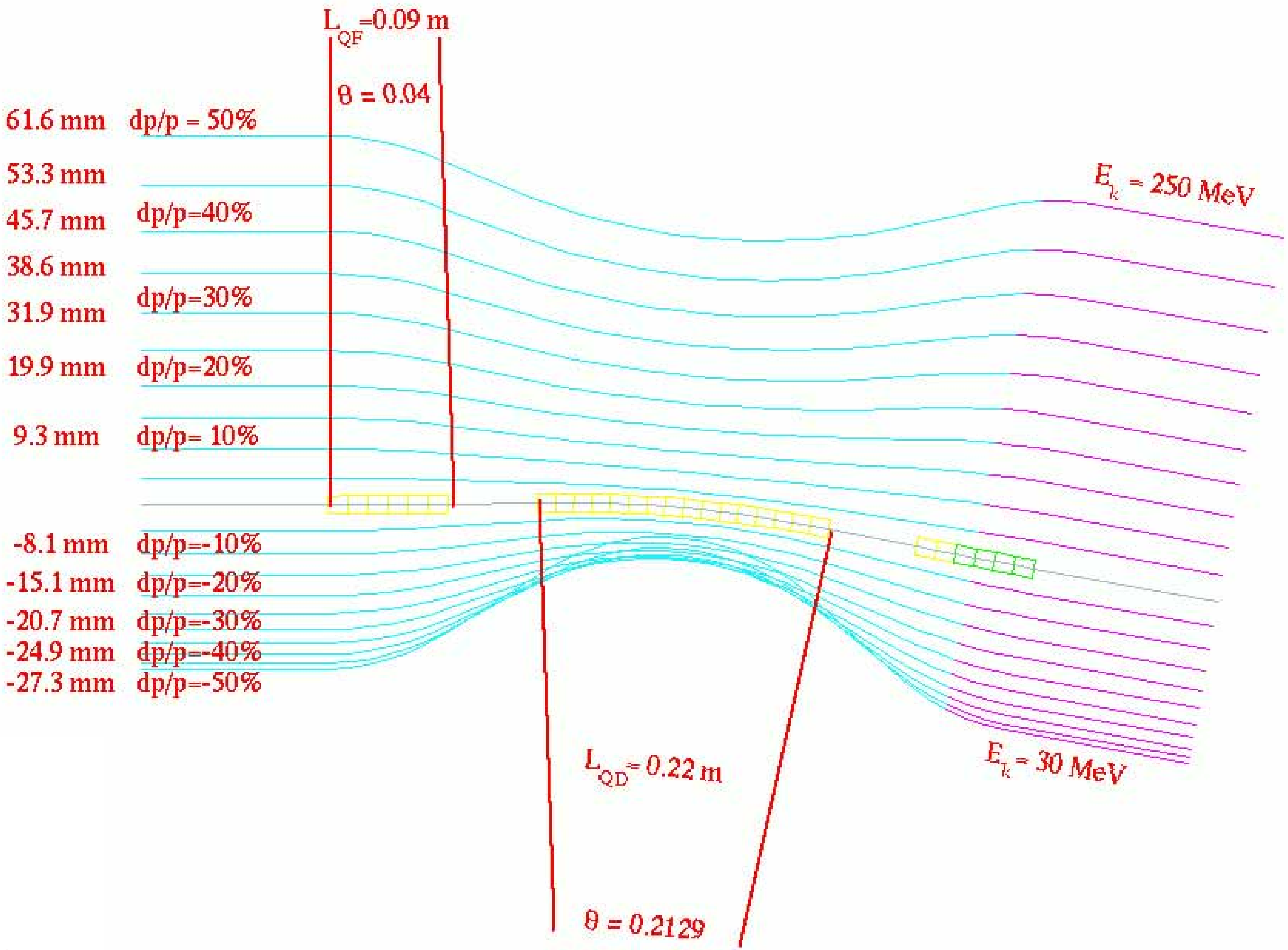
C. Acceleration



D. Lattice design and update

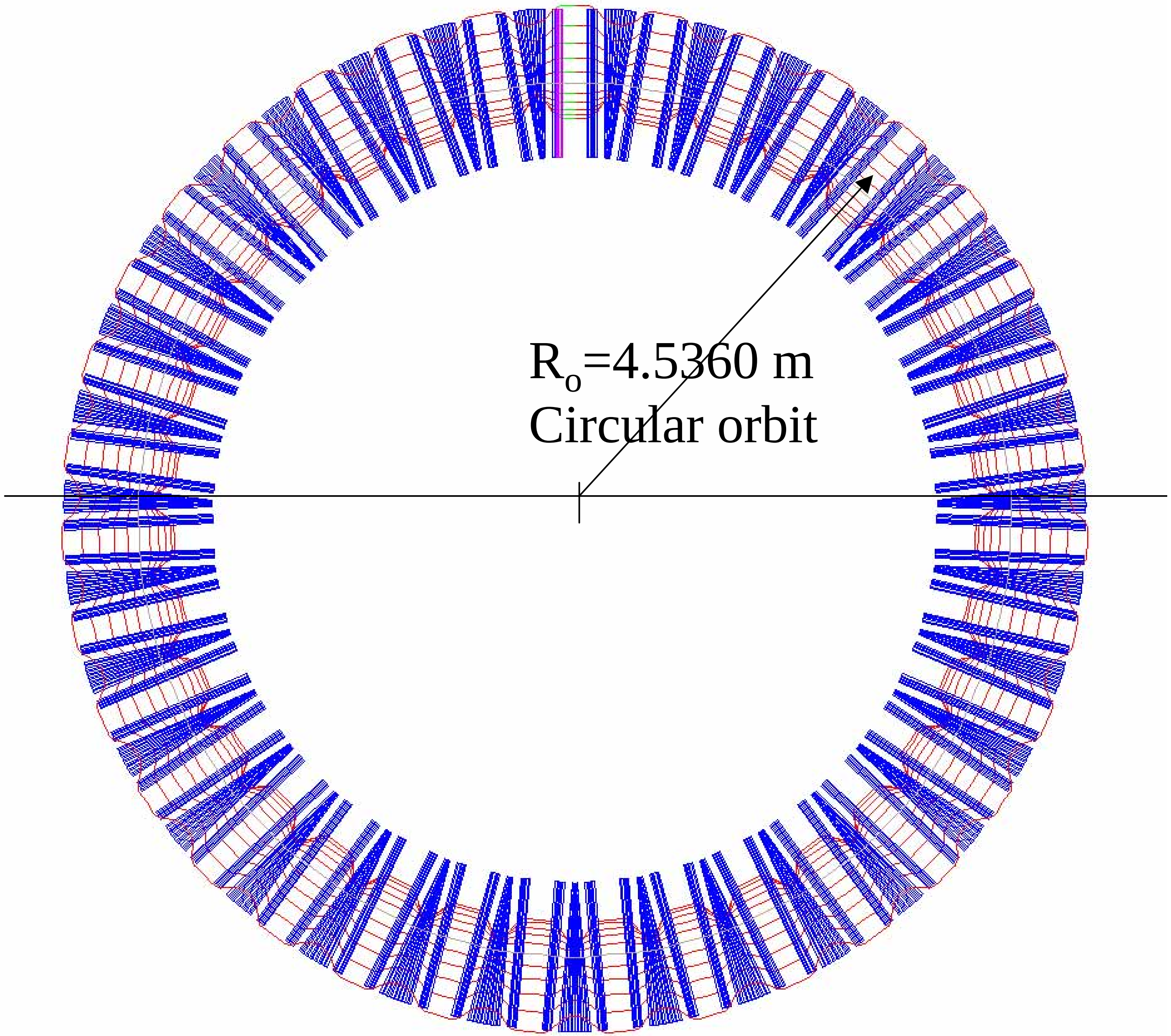




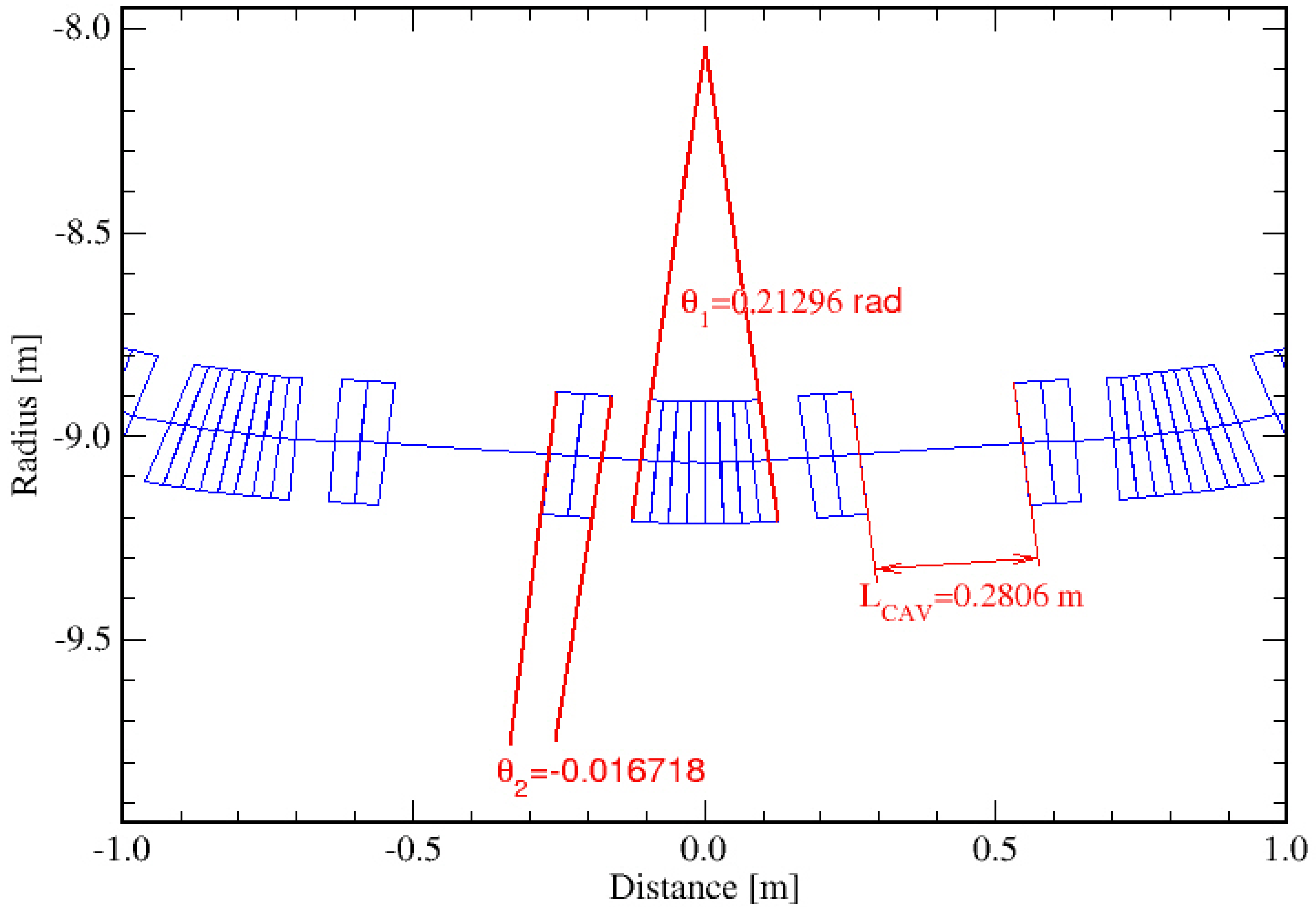


Particle paths during acceleration:

Orbits in the whole ring



The basic Cell – Magnets layout



BROOKHAVEN NATIONAL LABORATORY

TABLE 1. PROTON THERAPY REQUIREMENTS:

	Injection ($\delta p/p$=–50%)	Cent. momentum ($\delta p/p$=0)	Top energy $\delta p/p$=50%
p (GeV/c)	0.243044580	0.486089161687	0.72913374
E_k (MeV)	30.9673989490	117.545851	250.000000
E (GeV)	0.969239427944	1.05671049655	1.18827203
γ	1.03300471291	1.12623041494	1.26644725
$\beta\gamma$	0.25903423884	0.51806847766	0.77710271
$B\rho$ (Tm)		1.621418914	

TABLE 2. MAGNET PROPERTIES FOR THE FFAG LATTICE FOR PROTON ACCELERATION FROM 30 TO 250 GeV KINETIC ENERGY

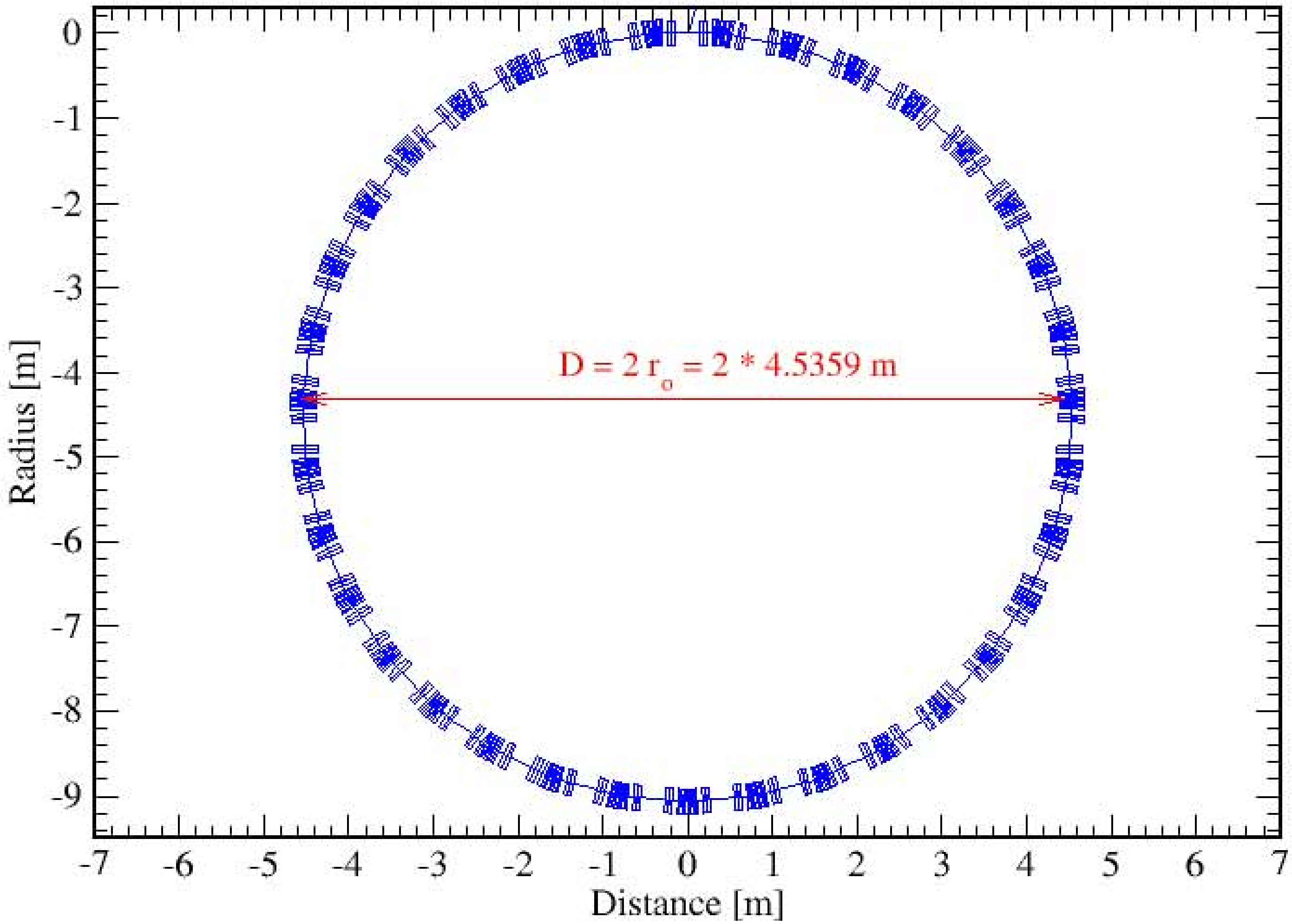
Combined function magnet	QF focusing gradient	QD defocusing gradient
Length	0.09	0.22
Bending Field (T)	0.30	1.563239217
Bending angle (rad)	0.016718929	0.212957438
Gradient (T/m)	41.227598	28.954765
Length of cavity drift (cm)	28.077650	
Drift between magnets (cm)	6.6675464	

TABLE 3. ACCELERATION OF FULLY STRIPPED CARBON IONS WITH THE 250 MeV PROTON NON-SCALING FFAG. THE MAXIMUM ACHIEVABLE KINETIC ENERGY PER NUCLEON IS EQUAL TO E_{kmax} = 68.271 MeV/n.

	Injection ($\delta p/p$=–50%)	Cent. momentum ($\delta p/p$=0)	Top energy $\delta p/p$=50%
p (GeV/c)	0.1210364084	0.24207281687	0.363109225
E(GeV/n)	0.9393250132	0.96243488967	0.999764961
E_k(MeV/u)	7.8306931643	30.940569673	68.27064122
γ	1.0084065925	1.0332160583	1.073291527
$\beta\gamma$	0.1299378921	0.2598757842	0.389813676
$B\rho$ (Tm)		1.1614936003	

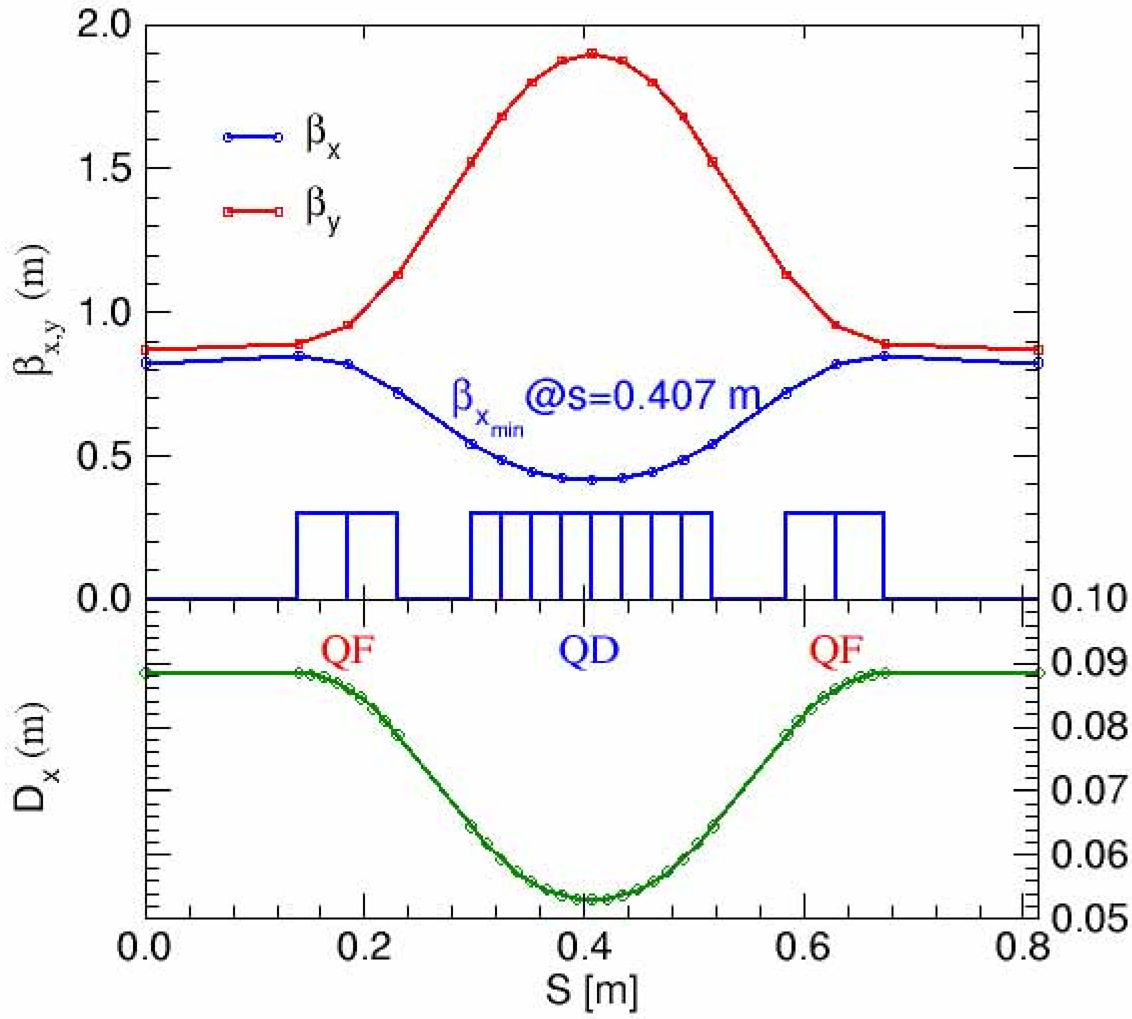
TABLE 5. MAGNETIC PROPERTIES FOR THE *NON-SCALING* FFAG LATTICE FOR CARBON ACCELERATION FROM 68.27 MeV/nuc. TO 501.79 MeV/nuc. KINETIC ENERGY

Combined function magnet	QF focusing gradient	QD defocusing gradient
Length	0.09	0.22
Bending Field (T)	4.689711518	0.900000000
Bending angle (rad)	0.212957490	0.016718955
Gradient (T/m)	123.6833	-86.864685
Length of cavity drift (cm)	28.077920	
Drift between magnets (cm)	6.6753299	

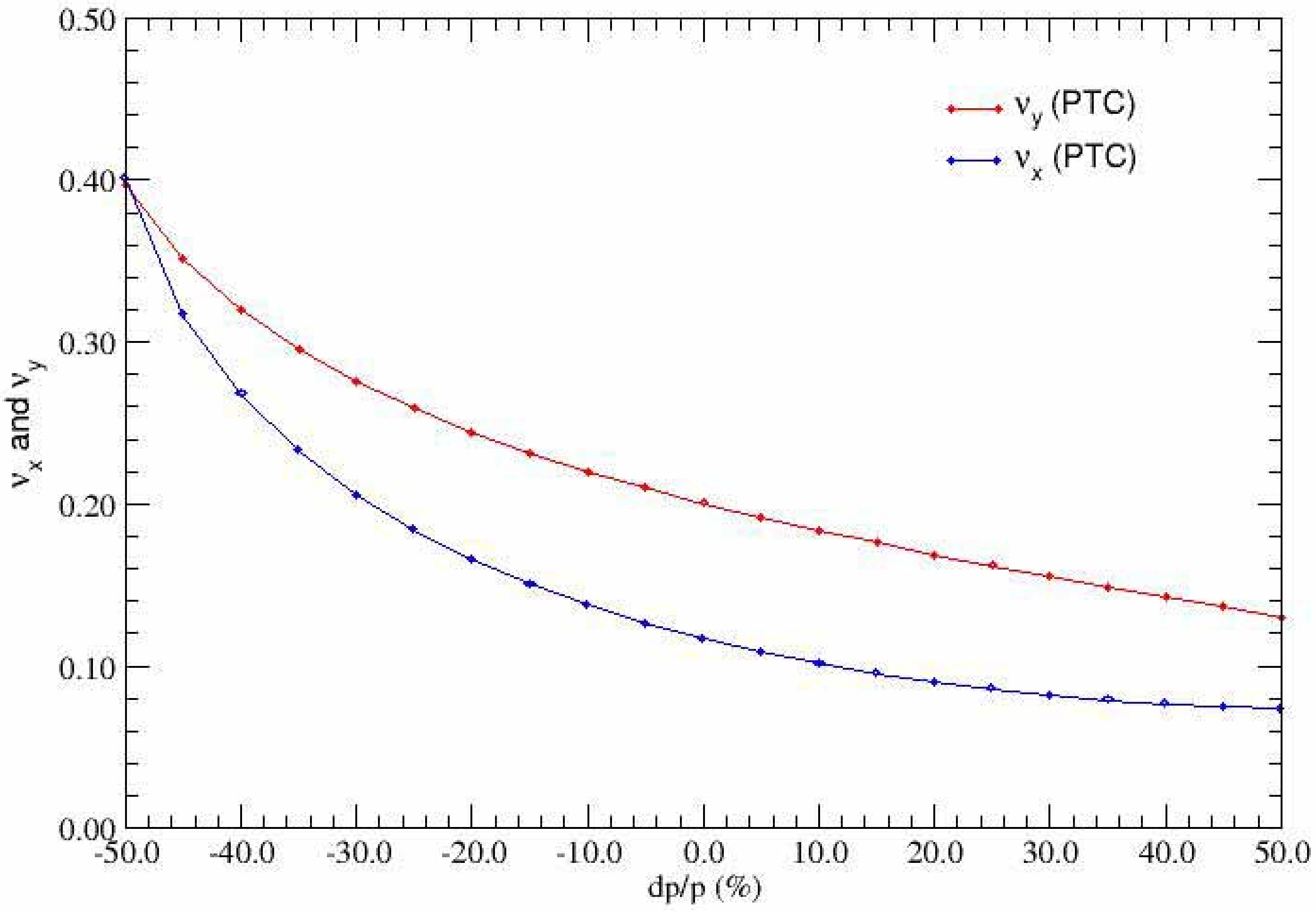


Radius [m]
Distance [m]
D = 2 r$_o$ = 2 * 4.5359 m
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7

Amplitude and dispersion functions in the basic cell



Tunes vs. momentum



Horizontal Amplitude function dependence on momentum

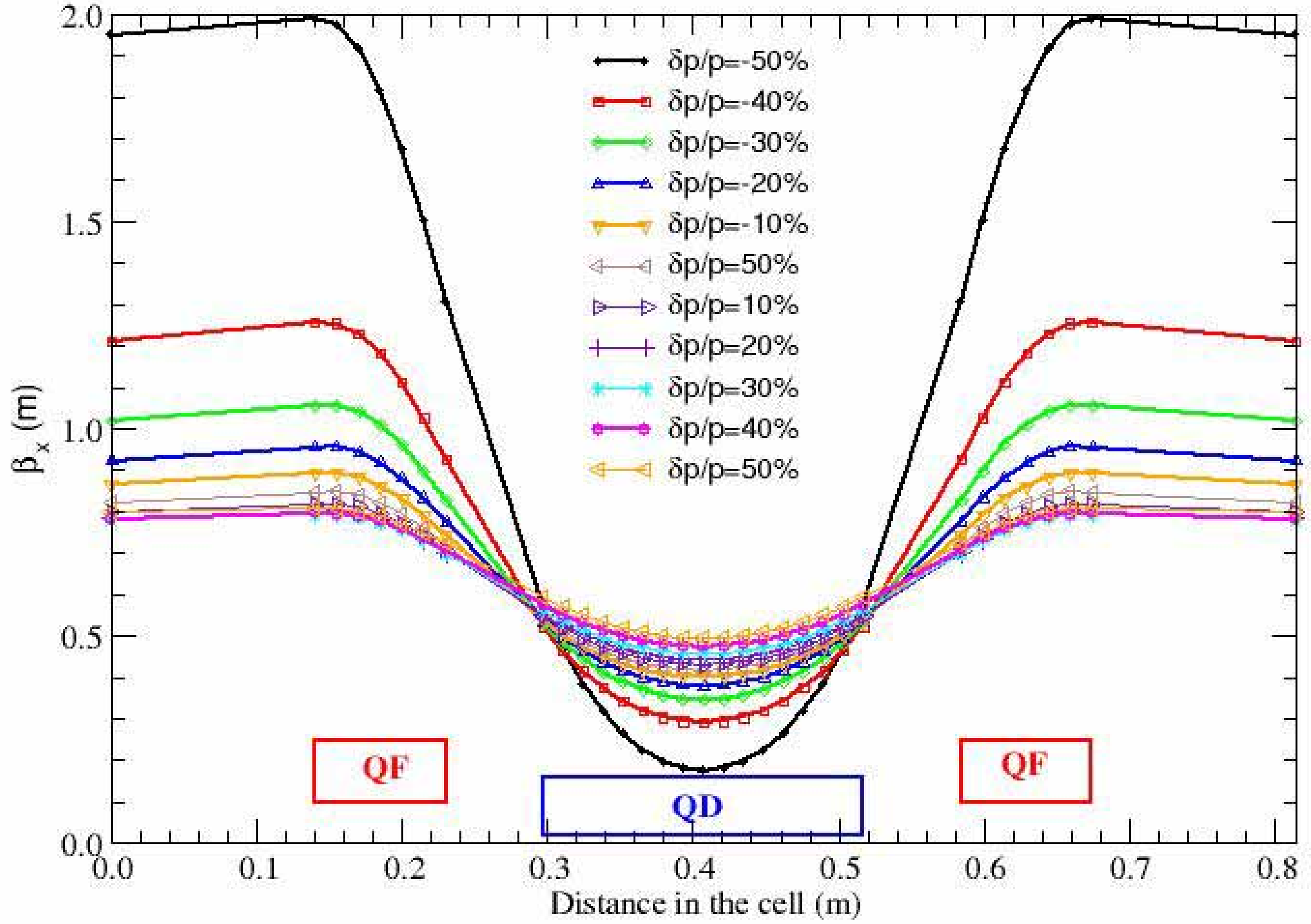


TABLE 4. THE PARAMETERS OF THE SUPER-CONDUCTING NON-SCALING FFAG FOR ACCELERATION OF THE FULLY STRIPPED CARBON IONS. THE CENTRAL MOMENTUM OF p_c=0.72621845 MeV/nucl. DEFINES THE REQUIRED MAGNETIC RIGIDITY OF:

$ze\, B\rho = A \quad ATMU\, \beta\gamma / c = 4.8448$ Tm

	Injection ($\delta p/p$=–50%)	Cent. momentum ($\delta p/p$=0)	Top energy $\delta p/p$=50%
p (GeV/c)$_n$	0.363109225	0.726218450	1.089327675
E(GeV/n)	0.999764961	1.181132891	1.433288684
E_k(MeV/u)	68.27064122	249.6385706	501.7943640
γ	1.073291527	1.267997953	1.538698254
$\beta\gamma$	0.389813676	0.779627352	1.1694410278
$B\rho$ (Tm)		4.844808004	

SUPERCONDUCTING COMBINED FUNCTION MAGNETS

TABLE 5. MAGNETIC PROPERTIES FOR THE *NON-SCALING* FFAG LATTICE FOR CARBON ACCELERATION FROM 68.27 MeV/nuc. TO 501.79 MeV/nuc. KINETIC ENERGY

Combined function magnet	QF focusing gradient	QD defocusing gradient
Length	0.09	0.22
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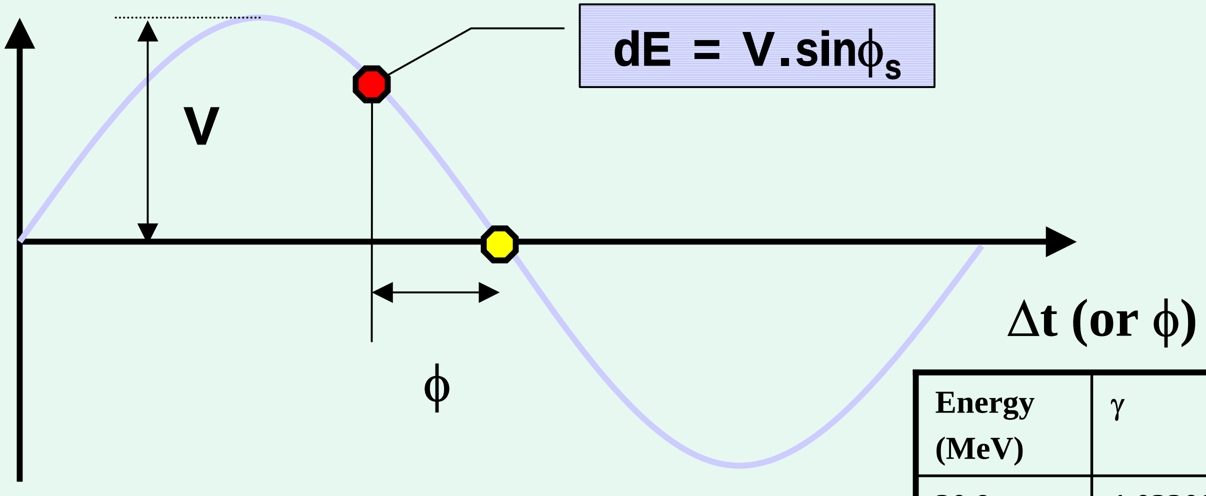
Bending angle (rad)	0.212957490	0.016718955
Gradient (T/m)	123.6833	-86.864685
Length of cavity drift (cm)	28.077920	
Drift between magnets (cm)	6.6753299	

Acceleration:



Always 'above' transition

Energy (MeV)	γ	γ_t	α	η
30.9	1.033005	0.40737 i	-6.0257	6.9628
117.545	1.126230	0.79920 i	-1.56564	2.35403
250.00	1.266447	1.12390 i	-0.79163	1.41511

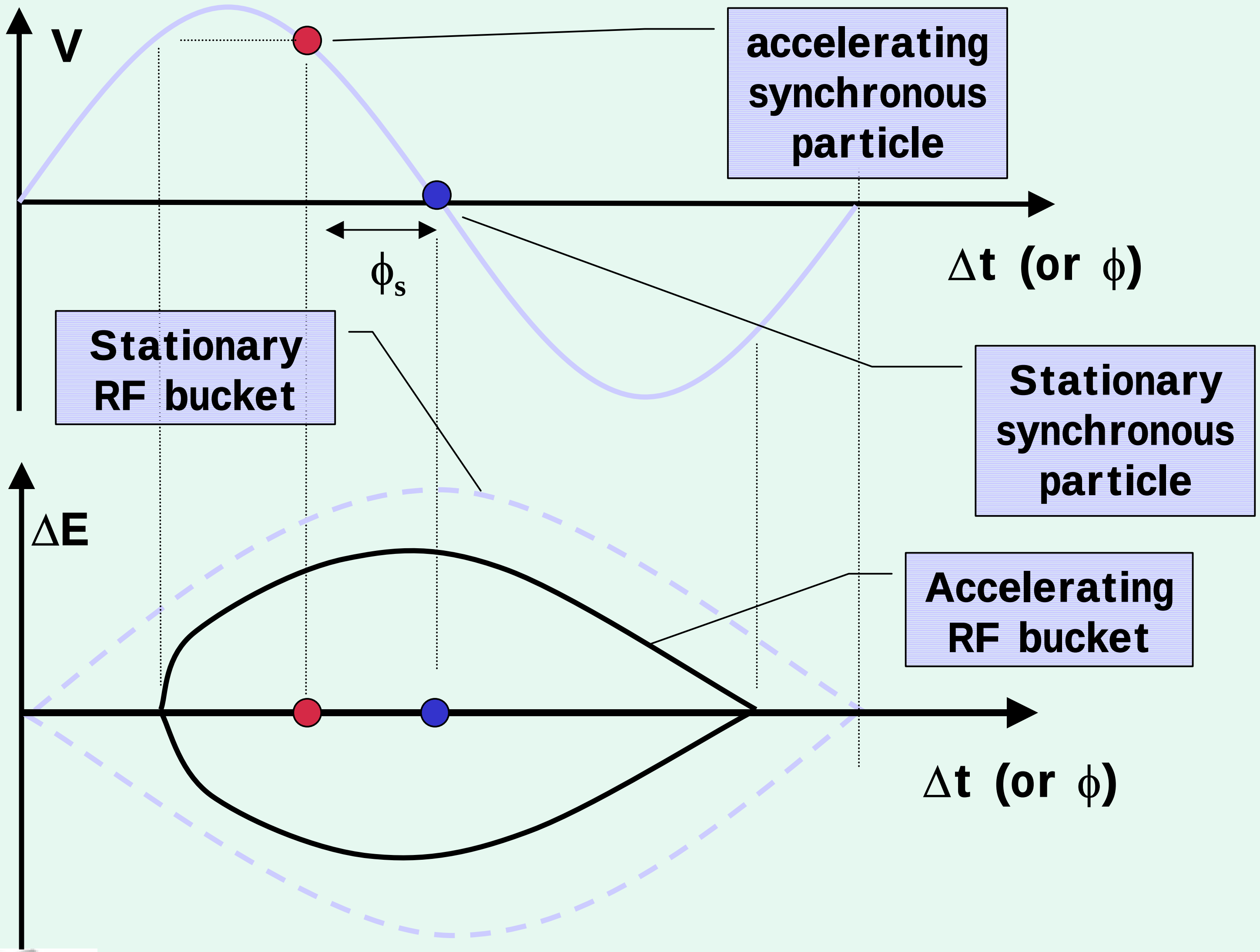
$$\frac{df}{f} = \left(\frac{1}{\gamma^2} - \alpha_p\right)\frac{dp}{p} = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}\right)\frac{dp}{p}$$

The phase of the cavity is set for the synchronous particle at ϕ_s and it sees always an accelerating voltage

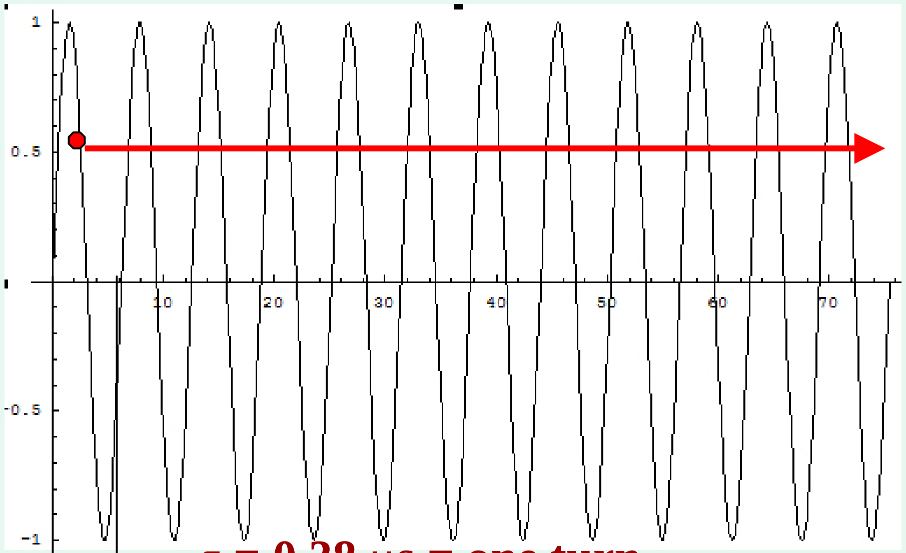
$$\mathbf{V_s = V\sin\phi_s} = \mathbf{V}\Gamma = \text{energy gain/turn} = \mathbf{dE}$$

$$\therefore \frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

Acceleration & RF bucket shape



Harmonic number (h=12, h=8)



τ = 0.38 μs = one turn

τ = 0.38/12 μs = 0.03167 μs

$f_{rf} = 1/\tau_{rf}$ =31.57 MHz

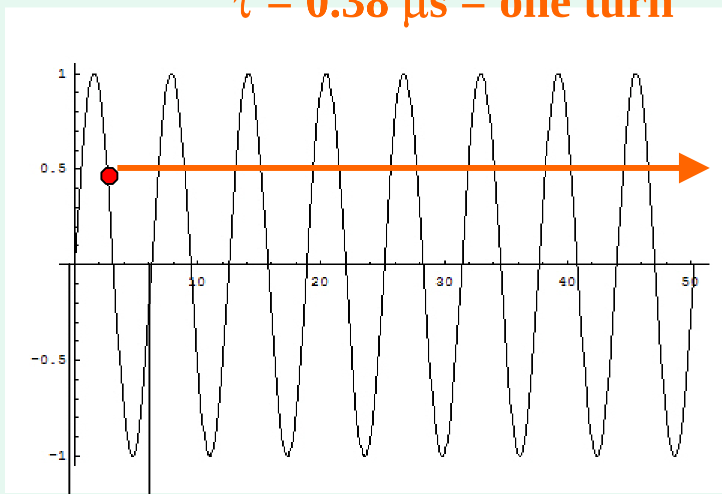
$$f_{rf} = h \times f_{rev}$$

Variable for $\beta < 1$

Frequency of cavity voltage

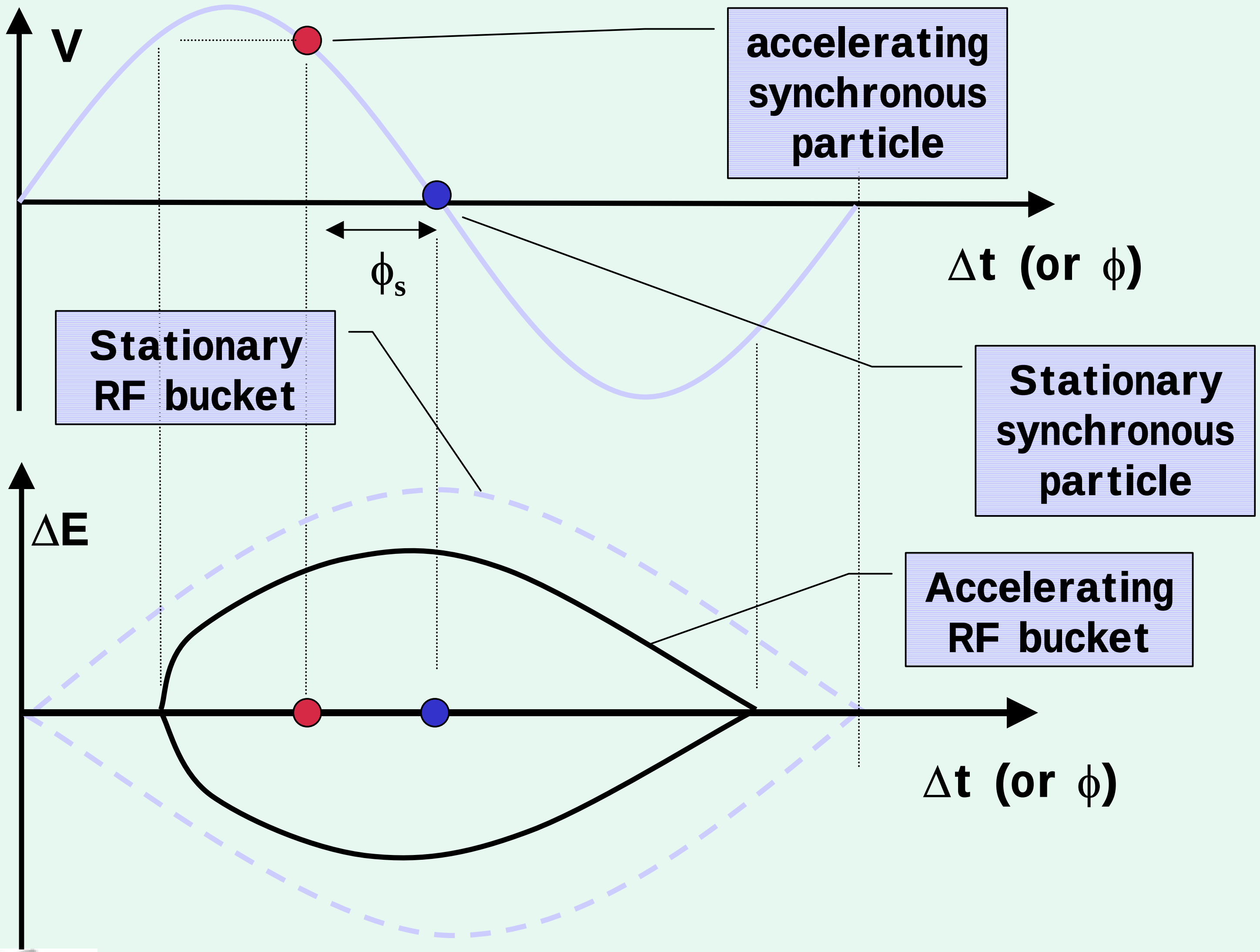
Harmonic number

τ = 0.38 μs = one turn

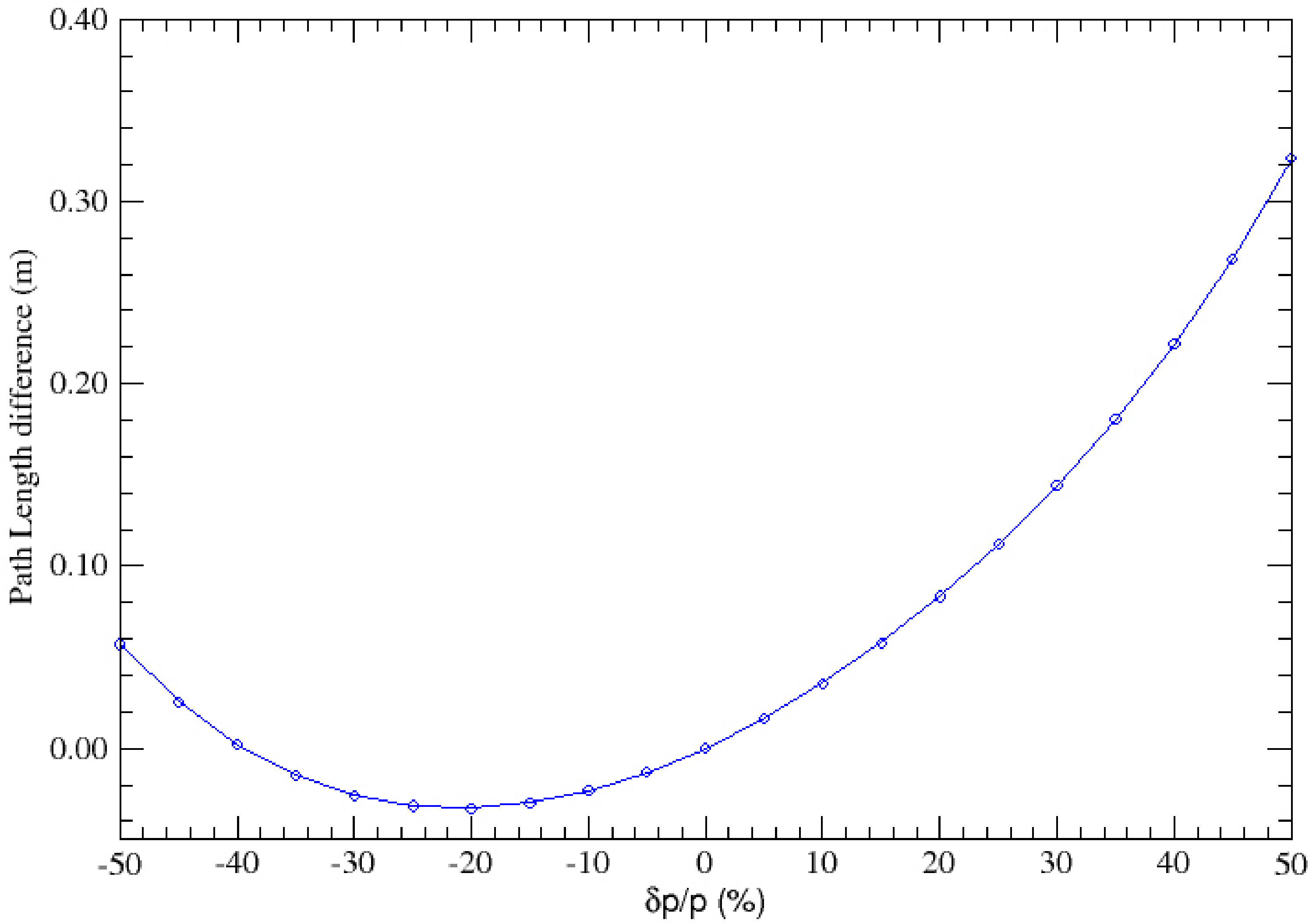


$f_{rf} = 1/\tau_{rf}$ =21 MHz

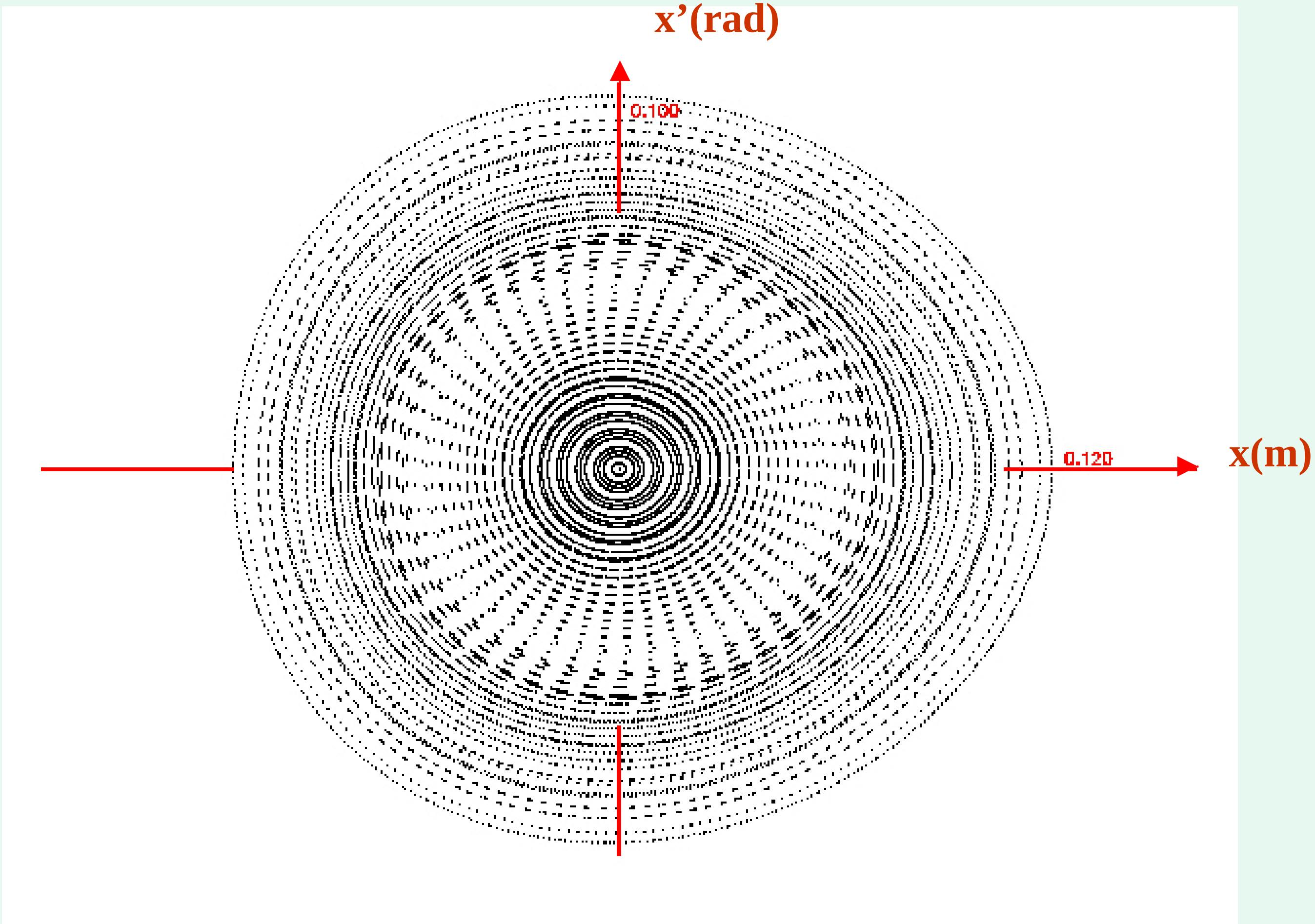
Acceleration & RF bucket shape (1)



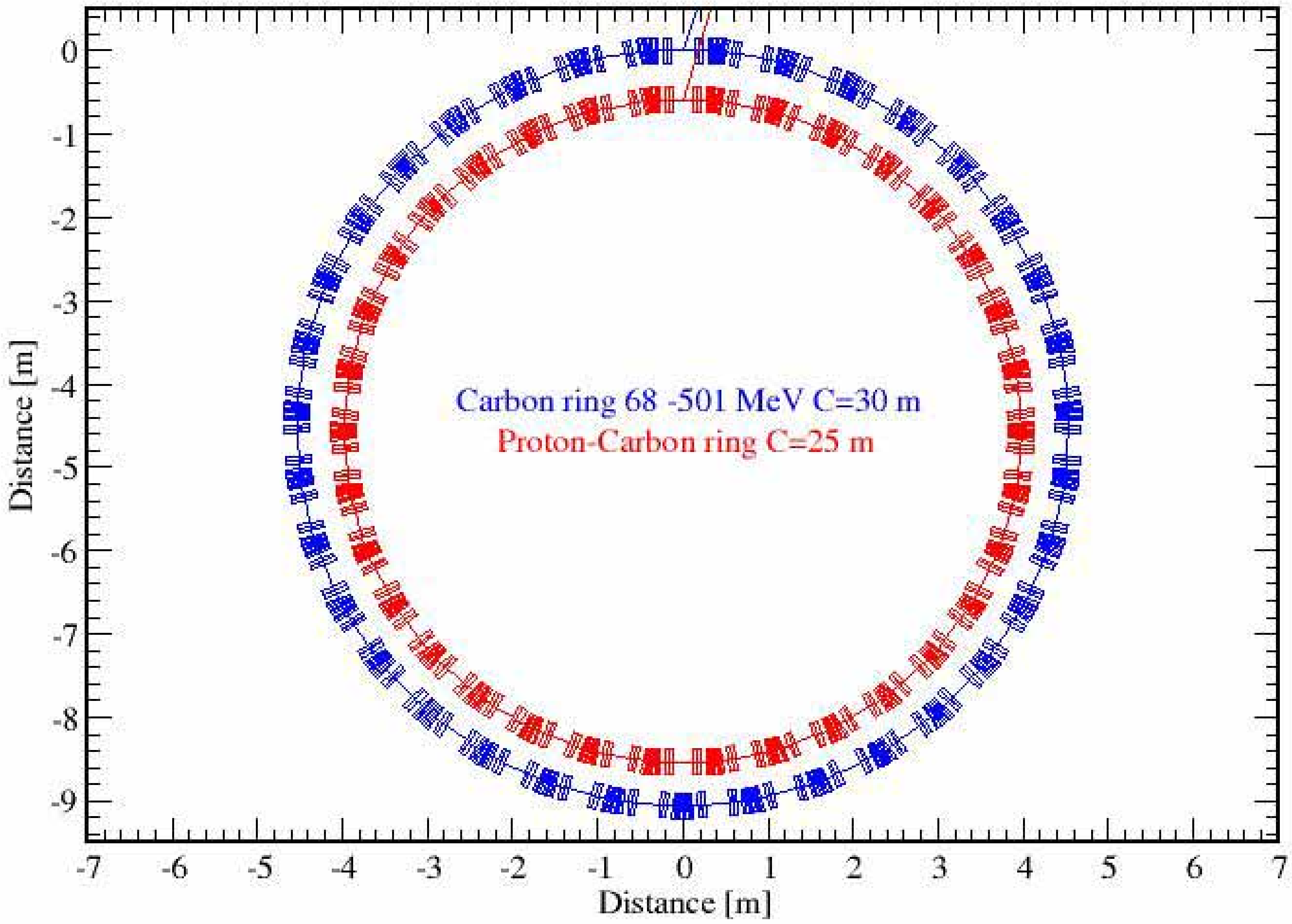
Path Length variation



Proton tracking at central energy



Update on the proton-carbon rings design:



CONCLUSIONS:

- **Examples of the *non-scaling* Fixed Field Alternating Gradient proton and carbon accelerators for cancer therapy show few important advantages with respect to the other CYCLOTRON or SYNCHTROTRON comparative solutions:**
  - **Very small magnets – tight focusing lattice**
  - **Small orbit offsets during acceleration**
  - **Possibility for fast acceleration**
  - **Fast "spot scanning" possibility**
- **Future work requires:**
  - **Electron demonstration ring [this is in progress in England and USA there are already submitted proposals].**
  - **Magnet design and detail six dimensional tracking with errors.**
- **There is already interest by the commercial facilities.**