MHD Phenomena and Transport of Energetic Ions in Spherical Tori

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Abstract. Mechanisms of the influence of MHD events on the beam ions in moderate-β plasmas relevant to current experiments on NSTX are studied. Change of the neutron yield caused by particle redistribution is evaluated. Destabilizing effect of the trapped energetic ions on ideal and non-ideal MHD modes in high-β plasmas is predicted.

1. Introduction

It is known that β (the ratio of the plasma pressure to the magnetic field pressure) in Spherical Tori (ST) is considerably larger than that in Conventional Tokamaks (CT). When β is very high, ST plasmas are characterized by a number of specific features. In particular, the fishbone mode associated with trapped particles can be stabilized by the magnetic valley [1]; the circulating-particle-induced fishbone mode tends to be stabilized by the large Shafranov shift [2]; the transport of trapped energetic ions caused by sawtooth oscillations can be enhanced [3]. When β is moderate, the "bounce-frequency" fishbones may appear [4]. On the other hand, a variety of MHD events, such as sawtooth oscillations, internal reconnection events etc., were observed in STs. Experiments on the NSTX spherical torus show that these MHD events often result in strong drops (by a factor of two) of the neutron yield and signals on the Neutral Particle Analyzer (NPA), which implies that the energetic (beam) ions are strongly affected by MHD perturbations [5]. Therefore, the study of the interplay of MHD modes and energetic ions in STs is of large practical importance. Such a study is carried out in this work, where both the influence of energetic ions on plasma MHD activity is investigated and the transport of energetic ions induced by MHD reconnection events is studied.

2. The Influence of Trapped Energetic Ions on the Ideal Kink Instability and Non-ideal MHD Modes

It was predicted theoretically [6] and then confirmed experimentally on TFTR [7] that trapped ions with the energy above a certain critical magnitude, $\mathcal{E}_{crit}$, are not sensitive to the sawtooth crash in CTs. The existence of $\mathcal{E}_{crit}$ is a consequence of the fact that toroidal precession tends to move particles at $r =$ const, where r is the radial coordinate characterizing the flux surfaces before the crash. On the other hand, such a motion is not compatible with the motion of the bulk plasma across the $r =$ const surfaces during the crash. Therefore, the energetic ions stabilize the crash-causing instability when their number is not too small. This was observed experimentally on JET, where it was demonstrated that the neutral beam injection may considerably increase the period of sawtooth oscillations [8]; later a theoretical explanation was suggested [9]. The situation changes in high-β plasmas of STs. As shown in Ref. [3], plasma diamagnetism results in the "diamagnetic precession", which is directed along the evolving flux surfaces (in contrast to toroidal precession). The diamagnetic precession overrides the toroidal precession when β is high ($\beta(0) \gtrsim \epsilon_s$ with $\epsilon_s = r_s/R_0$, r_s the radius of the q=1 surface, R_0 the radius of the magnetic axis), in which case the ions with $\mathcal{E} > \mathcal{E}_{crit}$ are attached to evolving flux surfaces like the bulk plasma particles. Because of this, as we will show, high-energy ions lose their ability to stabilize MHD modes and, furthermore, can play a destabilizing role.

To demonstrate this effect, we consider first the simplest case of the ideal kink instability. Then we can write the instability growth rate as

$$\frac{\gamma}{\omega_A} \approx \lambda_c + \lambda_h, \quad (1)$$

where λ_c and λ_h are the negatives of the normalized MHD kinetic energy and energetic ion response, respectively. It follows from Eq. (1) that, depending on the sign of λ_h, the effect of the energetic ions is either stabilizing or destabilizing. When the population of the energetic ions consists of the trapped particles, $\lambda_h < 0$ for the magnetic field $B \propto 1/R$ with R the distance from the major axis of the torus, which means that the energetic ions have a stabilizing influence on the ideal kink instability in CTs [9]. However, in high-β plasmas of STs $B(R)$ has a minimum ("magnetic valley"); therefore, the sign of the response of the energetic ions may change. This will occur when the particles located at $\partial B/\partial R > 0$ mainly contribute. Note that finite β may have a destabilizing influence on MHD modes even for $\partial B/\partial R < 0$ when marginally trapped particles dominate in the energetic ion population [10].

We write the magnetic field strength as $B = B_0[1 - \epsilon\cos\theta + \epsilon^2(\alpha + \sigma\cos^2\theta)]$, where α and σ are parameters [1]. In low-β plasmas relevant to CTs these parameters are small, but $\alpha \gg 1$ in STs with very high β. The magnetic valley at $r < r_s$ arises when $\alpha + \sigma > 0.5A_s$, where $A_s = \epsilon_s^{-1}$. Using this magnetic field, we calculated $\Lambda_h \equiv \lambda_h\hat{s}/(\epsilon_s\beta_{ph})$ ($\hat{s}$ is the magnetic shear, $\beta_{ph} = 8\pi p_h/B_p^2$, B_p is the poloidal magnetic field, p_h is the fast ion pressure) for an NSTX-like torus with $\epsilon_s = 1/3$. The results are shown in Fig. 1. We observe that $\lambda_h > 0$ when α exceeds a certain magnitude, $\alpha_{crit}(r_s/r_h)$, where r_h is a characteristic width of the radial profile of the energetic ions, α_{crit} growing with r_s/r_h. To explain the obtained dependence of α_{crit} on r_s/r_h, we have to take into account that the region with $\partial B/\partial R > 0$ grows with α and that the number of the energetic ions in the region $r \lesssim r_s$ grows with r_h^{-1}.

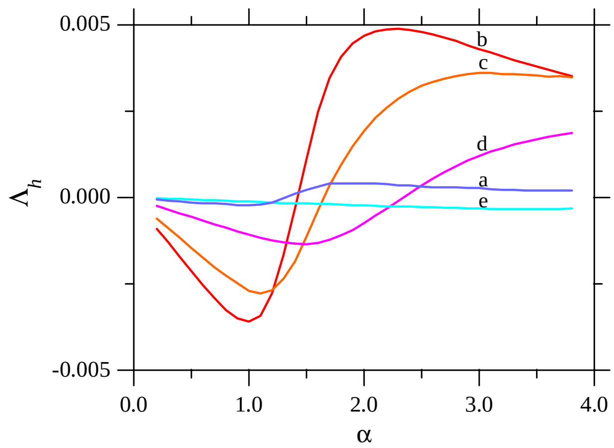

FIG. 1. $\Lambda_h \equiv \lambda_h(0)\hat{s}/(\epsilon_s\beta_{ph})$ vs α in a spherical torus with $A = 1.3$, $\epsilon_s = 1/3$, $\sigma = 0.1$ for various r_s/r_h: a – 0.2; b – 1.2; c – 2; d – 3; e – 7. Here r_h characterizes the radial profile of the energetic ions taken in the form: $n_h(r) = n_h(0)\exp(-r^2/r_h^2)$. The region of $\alpha \geq 1.4$ corresponds to an equilibrium configuration with the minimum of the magnetic field at $r \leq r_s$.

In STs the ion Larmor radius (ρ_i) typically exceeds both the width of the resistive reconnection layer, δ_{res} [$\delta_{res} \sim r_s S_M^{-1/3}\hat{s}^{-1/3}$, where $\hat{s}$ is the magnetic shear, S_M is the magnetic Reynolds (Lundquist) number] and the electron skin depth, d_e ($d_e \equiv c/\omega_{pe}$, ω_{pe} is the electron plasma frequency). Therefore, non-ideal instabilities, which are presumably responsible for the sawtooth crashes in tokamaks, may arise. For this reason, in addition to the ideal kink instability, we studied the semi-collisional tearing mode and the collisionless tearing mode. We found that in both cases the increase of β has a stabilizing influence on the modes.

Let us consider a specific example relevant to NSTX. We take $A = 1.3$, $B = 0.3\,\mathrm{T}$, the plasma radius $a = 68\,\mathrm{cm}$, the plasma density $n_i = 3\times10^{13}\,\mathrm{cm}^{-3}$, $T = 1\,\mathrm{keV}$, the cross-section elongation $k = 2$, $r_s = a/2$, $\hat{s} = 0.5$, $\mathcal{E}_b = 80\,\mathrm{keV}$. Then $\rho_i = 2.15\,\mathrm{cm}$, $\delta_{res} = 0.33\,\mathrm{cm}$, and $d_e = 0.1\,\mathrm{cm}$, which implies that semi-collisional tearing instability may occur. However, the ratio δ_{res}/d_e depends on plasma parameters in a way that the regime of the collisionless tearing mode can take place in a plasma with the same β as in the example considered above. Indeed, $\delta_{res}/d_e \propto n^{2/3}T^{-1/2}B^{-1}\hat{s}^{-1/3}(R_0 r_s)^{1/3}$; therefore, increasing the temperature by, e.g., a factor of 4 and keeping $nT = \text{const}$, we obtain the regime with $\delta_{res} < d_e$.

3. Redistribution of Energetic Ions and Change of Neutron Emission During MHD Activity

An MHD event affects the neutron yield of the DT reaction in two ways. First, it redistributes the beam and plasma particles and, second, it leads to particle loss. Typically the core-localized MHD activity (such as sawtooth oscillations) results in negligible loss of the energetic ions in CTs. The situation in STs is not clear. Although NPA signals in experiments on NSTX strongly drop during the IRE and sawteeth [5], this information is not sufficient to conclude that the fast ions were lost. Therefore, we evaluate the maximum change of the neutron yield in the absence of the particle loss in order to see whether the redistribution itself can explain the observed drop of the neutron yield.

The largest particle redistribution takes place when the particles follow evolving flux surfaces, which may occur for $\Delta r_b \ll r_{mix}$, where Δr_b is the particle orbit width, r_{mix} is the mixing radius of the MHD activity. In STs r_{mix} can be rather large because in many cases $r_s \sim a/2$ and $r_{mix} \sim 1.3 r_s$. Therefore, we can assume that the condition $\Delta r_b \ll r_{mix}$ is satisfied. Then the relative change of the neutron yield is completely determined by the particle density before and after the reconnection event. Using the approach of Ref. [11], we obtain that the largest drop of the global neutron yield (I_G) occurs when the sawtooth crash is described by the Kadomtsev model and the beam density is very peaked. Then $|\delta I_G| = 10 - 17\%$ for $n_i \sim (1 - r^2/a^2)^\nu$ with $\nu = 0.5 - 1$, which is much less than the drop of the neutron yield observed experimentally on NSTX in, e.g., the shot #104505. We infer from this that MHD events lead to considerable loss of the energetic ions in the mentioned experiments.

We proceed to studying the mechanisms responsible for the redistribution of the various groups of the energetic ions during MHD activity. We assume that a plasma is characterized by $\beta(0) < \epsilon_s$, which is the case in current experiments on NSTX. Then the magnetic valley is either absent or very shallow and located far from the magnetic axis.

We begin with the consideration of the trapped particles. Their critical energy [6], $\mathcal{E}_{crit} \sim A_s BS/(n\tau_{crash})$ (with S the plasma cross-section square, τ_{crash} the crash duration, n the toroidal mode number), is rather low because STs are characterized by small A_s, B, and S. On the other hand, the diamagnetic precession [3] at $\beta(0) \sim 15\%$ is weak. Therefore, trapped energetic ions are not sensitive to the crash unless they are in resonance with the MHD perturbation. The resonance condition $n\langle\dot{\varphi}\rangle = s\omega_b$ (angular brackets mean bounce averaging) yields:

$$\omega_b(1 - pq_1) + p\nu\omega_D = 0, \qquad (2)$$

where $\nu = 1$, $q_1 = \omega_b^{-1}\langle\dot{\vartheta}q\rangle = (2\pi)^{-1}\oint d\vartheta q(r(\vartheta)) \sim \hat{s}q_t\vartheta_t\Delta r_b/(2\pi r_t)$, Δr_b is the banana width, "t" refers to the particle turning point, $\omega_D = \langle q v_D^2\rangle$ is the precession frequency, v_D^2 is the second contra-variant component of the drift velocity (we use Boozer coordinates), $p = n/s$, s is an integer. We conclude that Eq. (2) can be satisfied only for very high n or for marginally trapped particles with $\langle\dot{\varphi}\rangle = 0$. This implies that low-$n$ perturbations have a weak influence on most trapped energetic ions, as is the case in CTs.

Equation (2) is valid also for the "semi-trapped" particles, for which $\langle\dot{\vartheta}\rangle = 0$ but $v_\|$ does not change the sign. Note that such particles may constitute a considerable fraction of the fast ion population in an ST.

In contrast to trapped particles, the circulating particles "feel" the perturbation due to their motion along the magnetic field lines. Because of this, they are attached to evolving flux surfaces in CTs, which provides their transport [6]. Below we show that the situation may change in STs. One can show that a general resonance condition for the circulating particles in the presence of a perturbation with the mode numbers (m, n), $n\langle\dot{\varphi}\rangle - m\langle\dot{\vartheta}\rangle = s\omega_b$, leads again to Eq. (2) but with $\nu = \mathrm{sgn}(v_{\|})$ and $p = n/[m + \mathrm{sgn}(v_{\|})s]$ and $q_1 \approx q(\bar{r})$, where $\bar{r}$ is the average radial particle location. The first and second terms in Eq. (2) describe the longitudinal motion and the precession motion, respectively. The precession frequency can be written as $\omega_D = \xi v^2/(R_0^2\omega_B)$ with ω_B the gyrofrequency. Here $\xi < 1$ for both CTs and STs with $\beta(0) < \epsilon_s$. However, the ratio ρ/R_0 in STs considerably exceeds that in CTs, and ξ grows with β. Because of this, the precessional motion may become considerable, which tends to prevent attaching the particles to flux surfaces. When Eq. (2) is satisfied, the particles move along the resonance island, which implies that an additional transport mechanism does exist for them. On the other hand, taking the characteristic magnitudes of the terms in Eq. (2), we can introduce a critical energy for the circulating particles defined by $|p|\omega_b(r_{mn})\Delta q = |\omega_D(r_{mn})|$, where Δq is the variation of q on the magnetic island half-width, r_{mn} is the $q = m/n$ radius, cf. Ref. [6]. The particles with the energy well below the critical energy are attached to flux surfaces. Note that the considered mechanism provides particle redistribution not only during sawtooth crash but also during quasi-steady-state MHD events, e.g., when magnetic islands arise due to tearing instability, which seems to be observed in the shot #104162, where strong but slow drop of the neutron emission was observed.

In order to study the transport of the energetic ions numerically, we used code OFSEF [12] extended to include effects of the plasma pressure. The finite pressure terms may play an important role even at moderate β because the large local pressure gradients appear during the reconnection. We used parameters of the shot #104505 in NSTX. Figure 2 demonstrates the results for 80-keV ions. We observe that the particles are considerably affected. They move along an island, which leads not only to mixing of the energetic ions inside the plasma but also to their loss. In addition, motion of a significant part of the particles is stochastic, which may result in additional loss when the MHD event lasts long enough. The precession of the considered particles was found small (which was confirmed by calculations with the code ORBIT [13]), so that the motion along the perturbed flux surfaces mainly determines the particle transport in the absence of stochasticity.

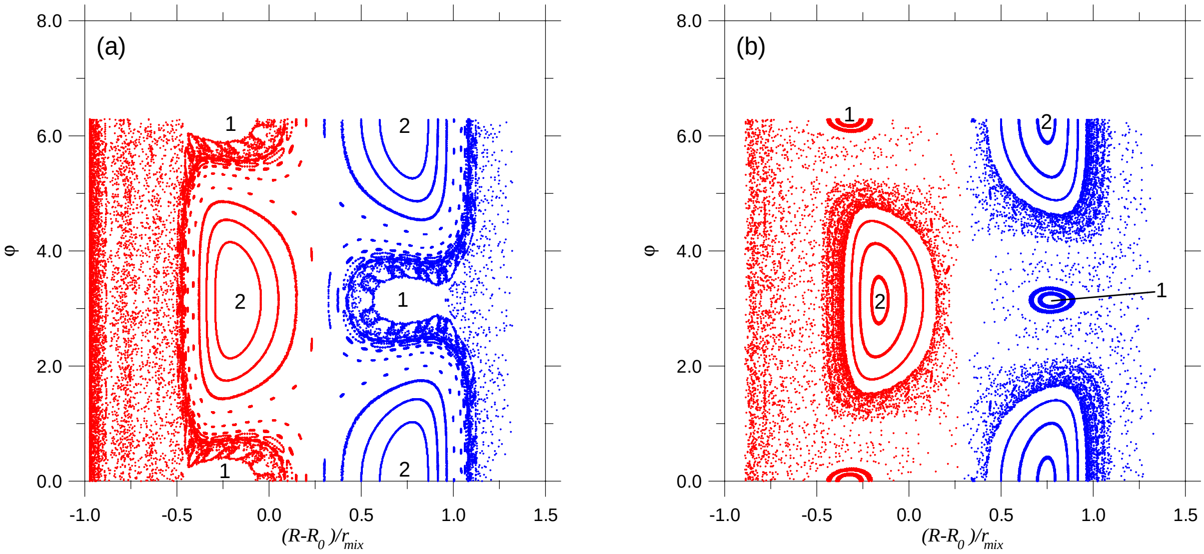

FIG. 2. Poincare map of the motion of 80-keV ions with $\lambda = 0.58$ *in the equatorial plane of an NSTX-like torus at the final stage of a sawtooth crash. (a),* $\beta(0) = 0$*; (b),* $\beta(0) = 16\%$*. 1, remnant of the pre-crash near-axis drift surfaces; 2, magnetic island. In the map (b), an extensive stochastic region appears at the island separatrix. The blue and red colors mark the footprints left when the particle moves up and down, respectively.*

4. Conclusions

We have shown that the increase of β may deprive the trapped energetic ions of their ability to stabilize MHD modes in STs. Moreover, we have found that the trapped energetic ions may trigger the sawtooth crashes in high-β plasmas of STs (rather than to prevent them as is the case in CTs). The destabilization occurs for $\beta(0) \gtrsim \epsilon_s$, i.e., when β is so high that a magnetic valley arises close to the magnetic axis.

The neutron yield change is evaluated. We have found that the particle redistribution itself (i.e., without particle loss) cannot explain strong drops of the neutron yield observed experimentally on NSTX, in particular, in the shot #104505. This implies that MHD events can lead to considerable losses of energetic ions in NSTX experiments. Mechanisms of the particle transport during MHD activity are analyzed. It is shown numerically that the confinement of circulating and semi-trapped 80-keV ions in NSTX can be considerably deteriorated in the presence of MHD perturbations.

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