3D Fokker-Planck model for high and moderate energy ions in weakly rippled tokamaks

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Abstract. Fokker-Planck coefficients specifying diffusion and convection transport processes for charged fusion products and NBI ions in tokamaks with weak TF ripples are derived in the COM space describing the collisional ripple transport processes of fast ions, both in the moderate and in the high energy range. The collisional diffusion coefficient of toroidally trapped ions which are in resonance with the TF ripple perturbations (so-called superbananas) is shown to exhibit only a weak dependence on the ripple magnitude and, further, to reach a maximum in the medium collisionality regime where the bounce frequency of superbananas is close to the effective collision frequency. This maximum appears in the energy range from a few tens of *keV* to a few hundreds of *keV* and may exceed the well-known Boozer - ripple plateau diffusion in the case of weak ripples. The radial convection of superbanans induced by slowing down is shown also to exceed the corresponding convection of bananas. The collisional superbanana transport is supposed to be responsible for the enhanced NBI ion loss observed in the intermediate energy range and, hence, should be essentially embodied into any modeling of charged fusion products at energies$E < 1$ *MeV*.

1. Introduction

TF (toroidal field) ripples are known to strongly enhance the radial transport processes of fast ions. In the periphery of a tokamak plasma, where the ripple magnitude δ exceeds the Goldston-White-Boozer stochasticity threshold δ_{cr}, they result in stochastic ripple transport of toroidally trapped high-energetic ions with a diffusion coefficient $D \propto$ 10-100 m^2s^{-1} and hence in fast loss (as compared with the slowing down processes) of the latter. Furthermore, at the outboard region of the plasma, the ripples are responsible for prompt collisionless loss of fast ions localized in the ripple wells. Due to the low density of high-energy ions at the plasma periphery the modelling of this collisionless ripple transport yields usually only a small amount of lost charged fusion products and NBI ions [1]. Generally, the corresponding theoretical predictions agree with losses of charged fusion products with energies E close to the birth energy E_0 and of NBI ions with energies close to the injection energy E_b, measured in present-day tokamaks [1,2]. However, such modelling of fast ion confinement, which considers collisionless ripple transport only, can explain neither the relatively high loss of injected ions in the energy range $T<E<E_b$ observed in JET [2] and in TFTR [1,3], nor the substantial loss of partially thermalized DD and DT charged products ($E/E_0 \approx 0.5-0.7$) measured in TFTR [1,4] in plasmas with weak MHD activity.

In the case of moderate energy ions (or high-energy charged fusion products in the plasma core where usually $\delta < \delta_{cr}$), TF ripples may result in enhanced collisional radial transport of toroidally trapped fast ions that resonate with ripple perturbations [5,6]. The importance of collisional transport has been demonstrated in Monte-Carlo modeling of ripple induced *MeV* alpha loss ($E \geq 0.5E_0$) in TFTR including collisions [7], where pitch angle scattering was shown to lead to a strong synergistic enhancement of the collisionless stochastic losses. Further our 3D Fokker-Planck confinement modeling of *MeV* charged fusion products has shown that collisional ripple transport of resonant bananas (so-called superbananas [8], satisfying

$$l\omega_b - N\omega_d = 0, \quad l = 0, \pm 1, \pm 2, \ldots \tag{1}$$

where ω_b and ω_d are the particle bounce and precession frequencies and N is the TF coil number) may be responsible for the loss of partially thermalised charged fusion products observed in TFTR [1,4]. The recent TF ripple reduction experiments with ferritic inserts in JFT-2M [9] have also proved the NBI ion loss dependence on the TF coil number N and the safety factor q, as was theoretically predicted for the ripple collisional diffusion rate of superbanas [6]. We note however, that the previously used 3D Fokker-Planck model [8] is applicable only for the *MeV* range of energetic ions, in which the regime of weak collisionality for superbananas is valid, i.e. where the superbanana frequency ω_{sb} exceeds the effective collisional frequency ν_{eff}. To extend the applicability of the above model to a lower energy range ($E \leq 0.5E_0$) of ions confined in a slightly rippled tokamak, we derive the Fokker-Planck transport coefficients in the constants-of-motion (COM) space for the case of medium ($\omega_{sb} \approx \nu_{eff}$) and of relatively high collisionality ($\omega_{sb} < \nu_{eff}$). The main purpose of the present report is the consistent derivation of the Fokker-Planck transport coefficients which describe the collisional transport processes of fast ions, both in the moderate and in the high energy range (from a few tens of *keV* to a few *MeV*), in toroidal plasmas with weak TF ripples.

2. Collisional Transport of Resonant Toroidally Trapped Particles

In this section we present the results of a qualitative derivation of the ripple induced transport coefficients which determine the collisional transport processes of fast ions, both in the moderate and in the high energy range.

2.1. Superbanana Orbits

To describe the orbits of resonant toroidally trapped particles in the vicinity of the l-th resonance we express the longitudinal invariant as $J = (m/2\pi)\oint V_{\parallel} ds + (l/N)\, p_{\varphi}$ [10] with the integration performed in a coordinate system moving along the torus with precession velocity and where p_{φ} denotes the toroidal canonical momentum. According to [10] this invariant may be represented by

$$p^2/2 - M\cos(\xi + p)\cos\psi = const \tag{2}$$

where $p = (p_{\varphi} - p_{\varphi}^{l})\, \partial(Nq\vartheta_t)/\partial p_{\varphi}^{l}$ is the banana-guiding-center coordinate, $p_{\varphi}^{l} = p_{\varphi}^{l}(E, \lambda)$ is the value of p_{φ} satisfying the resonant condition, Eq. (1), $\lambda = \mu B_0 / E$ is the normalized magnetic moment, $\xi = \xi(p_{\varphi}^{l}, \lambda)$ is a constant of motion, $\psi = N(\varphi - q\vartheta)$ is the angular variable in the mentioned moving coordinate system, $\vartheta_t = \vartheta(V_{\parallel} = 0)$ measures the poloidal extent of bananas and $M = M\left(p_\varphi^l, \lambda\right) \cong \delta/\delta_1$ with $\delta_1 = \varepsilon/\left(Nq\right)^{3/2}$. Typical superbanana orbits in the $\{p,\psi\}$- plane are shown in Fig.1 and Fig.2 representing cases $\delta > \delta_1$ and $\delta < \delta_1$, respectively.

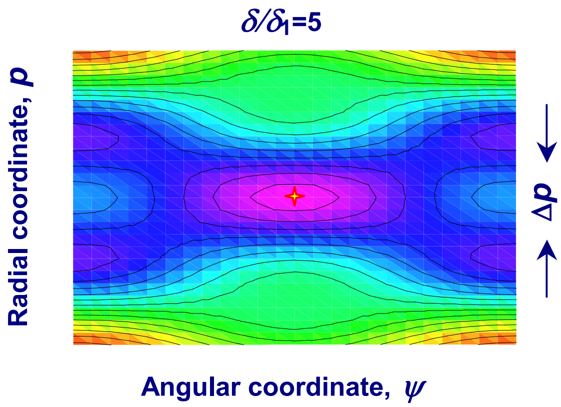

FIG.1. Superbanana orbits for $\delta > \delta_1$ in the $\{p,\psi\}$-plane, ξ = 0. Blue and violet regions correspond to closed orbits with $\Delta p \sim 1$.

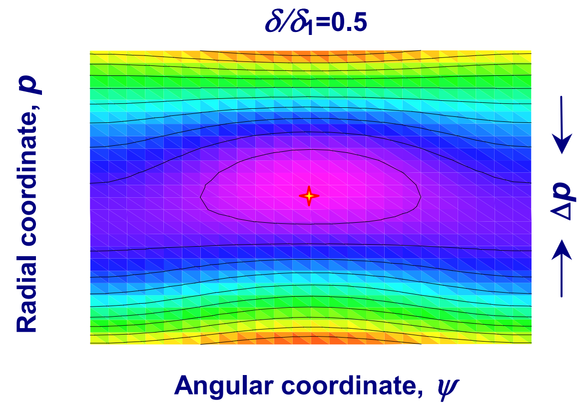

FIG.2. Superbanana orbits for $\delta < \delta_1$ in the $\{p,\psi\}$-plane, ξ = 0.7. Blue and violet regions correspond to closed orbits with $\Delta p \sim M^{1/2}$.

Typical widths Δr_{sb} and frequencies ω_{sb} of superbananas are given in [8,10] as

$$\Delta r_{sb} \cong \left(1 + M^{-1/2}\right)^{-1} r/\left(Nq\right), \qquad \omega_{sb} \cong M\left(1 + M^{-1/2}\right)V_d/r \tag{3}$$

with $V_d \cong V\rho_L/R$ denoting the toroidal drift velocity.

2.2. Diffusion Induced by Pitch Angle Scattering

Here we present qualitative expressions for the diffusion coefficient of resonant toroidally trapped particles in the case of intermediate and weak collisions. Taking into account that – according to [8,10] – the effective collisional frequency of superbananas induced by pitch-angle scattering of the rate $\nu_\perp$ is given by

$$\nu_{eff}^{sb} \cong \left(1 + M^{-1/2}\right)^2 \nu_{eff}^{nr}, \quad \nu_{eff}^{nr} = \nu_\perp N^2 q^2/\varepsilon \tag{4}$$

we derive the local diffusion coefficient for superbananas as

$$D = (\Delta r_{sb})^2 \nu_{eff} \Big/ \left[1 + \left(\nu_{eff}^{sb}/\omega_{sb}\right)^2\right] \equiv \nu_\perp r R \Big/ \left[1 + \left(\nu_{eff}^{sb}/\omega_{sb}\right)^2\right]. \tag{5}$$

In view of the small size of resonant regions (blue and violet areas in Figs. 1,2) the effective superbanana diffusion coefficient D^{SB}, averaged over the toroidally trapped particle domain, may be represented as

$$D^{SB} = f_R D, \; f_R \cong M^{1/2}\left(1 + M^{1/2}\right)\Delta r_b/r \tag{6}$$

where f_R is the fraction of resonant domains and $\Delta r_b \cong q\rho_L/\sqrt{\varepsilon}$. From Eq. (6) it follows that D^{SB} scales with $\delta^{0.5}N^{0.75}q^{1.75}$ in the plasma core ($\delta < \delta_1$), whereas $D^{SB} \propto \delta N^{1.5}q^{2.5}$ at the periphery ($\delta > \delta_1$). Note that for NBI loss measurements in JFT-2M [9] a heat load proportionality $\propto N^{2.25}q^{3.25}$ (see also [6]) was obtained that is close to $D^{SB}(N,q)$ given by Eq. (6) for $\delta > \delta_1$. In Figs. 3-5 the energy dependence of D^{SB} is displayed for different values of δ/δ_1. It becomes evident that

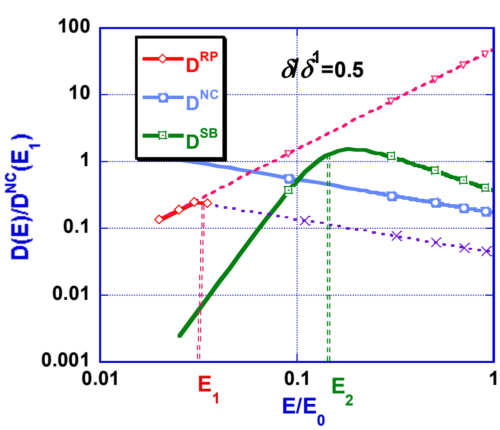

FIG.3. Ripple-induced diffusion coefficients normalized to the value of the neoclassical banana diffusion $D^{NC}(E_1)$ versus energy for $\delta/\delta_1 = 0.5$. D^{RP} is the Boozer ripple-plateau diffusion [11]. E_1 and E_2 are determined by the conditions $v_{eff}^{nr}(E_1) = \omega_b(E_1)$, $v_{eff}^{sb}(E_2) = \omega_{sb}(E_2)$.

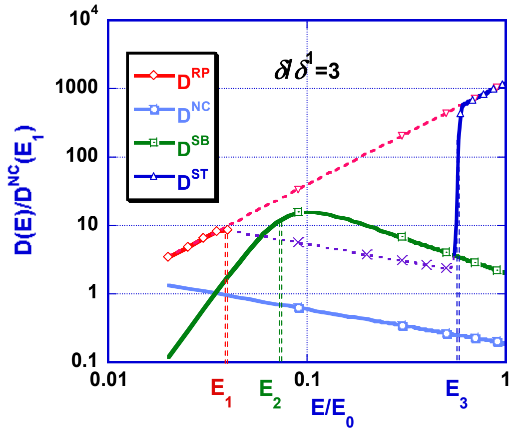

FIG.4. Ripple-induced diffusion coefficients normalized to the value of the neoclassical banana diffusion $D^{NC}(E_1)$ vs. energy for $\delta/\delta_1 = 3$. E_3 is determined by condition $\delta_{st}^{GWB}(E_3) = \delta$. δ_{st}^{GWB} is the ripple stochasticity threshold of Goldston–White-Boozer [12].

superbanana diffusion reaches its maximum at $E \simeq E_2$ (regime of medium collisionality, where ω_{sb} is close to the effective collision frequency) with E_2 found from

$$\left(E_2/E_0\right)^{5/2} = M^{-1}\left(1+M^{-1/2}\right)N^2q^2\,\nu_\perp(E_0)R/V_d(E_0) = v_{eff}^{sb}(E_0)/\omega_{sb}(E_0). \tag{7}$$

The curves in Figs. 3-5 marked by crosses suggest the diffusion coefficient for non-resonant toroidally trapped ions in the weak collionality regime. As illustrated in Fig. 6, E_2 is strongly dependent on the flux surface radius and varies from a few tens of *keV* to a few hundreds of *keV*. Figures 3-6 reveal that superbanana diffusion is important first of all in the plasma core ($r/a <$ 0.8-0.9) where $D^{SB}(E_2) \sim (10\text{-}10^2)D^{NC}(E_2) \sim 10^3\text{-}10^4\ cm^2 s^{-1}$.

2.3 Convection Induced by Slowing Down

Note that the energy dependence of $p_\varphi^l = p_\varphi^l(E,\lambda)$ yields a high radial convection rate of resonant particles, $|dr/dt| \propto |dr/dE|E\,\nu_s \propto \Delta r\,\nu_s \propto 10^2\ cm\ s^{-1}$ with $\Delta r=|dr/dE|E$ being determined by the resonance condition, Eq. (1), and where ν_s the slowing down frequency. This transport mechanism may be consequential for the confinement of charged fusion products in tokamaks as well.

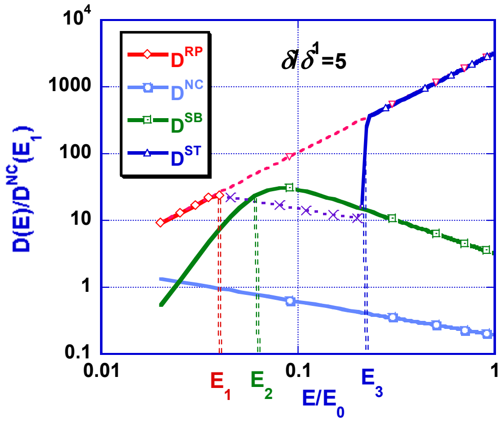

FIG.5. Ripple-induced diffusion coefficients normalized to the value of neoclassical banana diffusion $D^{NC}(E_1)$ *versus energy for* $\delta/\delta_1 = 5$.

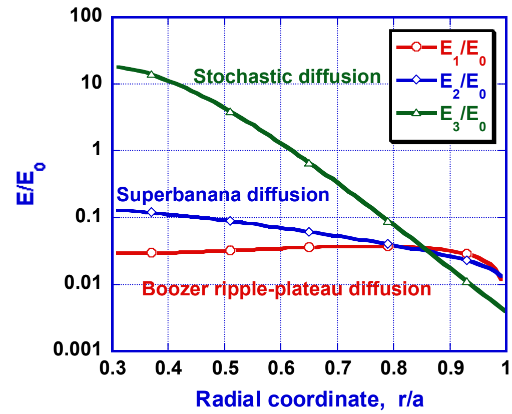

FIG.6. Critical energies E_1, E_2, E_3 *as functions of flux surface radius for typical TFTR-like parameters (N=20, q(a)=5, A=3,* $\delta_{max}\cong 0.01$, $\nu_\perp(E_0)\cong 0.1s^{-1}$*).*

3. SUMMARY

For typical tokamak parameters, toroidally trapped ions with energies from a few tens of *keV* to a few hundreds of *keV* being in the resonance with the TF ripples are shown to diffuse radially due to pitch angle scattering with a coefficient ~10^3-10^4 $cm^2\ s^{-1}$. This collisional ripple diffusion is supposed to be responsible for the enhanced loss of NBI ions observed in the intermediate energy range in JET and TFTR [1,2] as well as recently in JFT-2M [9]; it should be essentially embodied into any modeling of fast ion behavior in tokamaks at energies $E < 1$ *MeV*.

Acknowledgement

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