Transport of Energetic Ions in MHD-Active High-Beta Plasmas of Spherical Tokamaks

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Abstract. It is shown that high β (β is the ratio of plasma pressure to the magnetic field pressure) may deteriorate the confinement of trapped energetic ions in spherical tokamaks (ST) during MHD events, such as sawtooth oscillations and internal reconnection events (IRE). This result indicates that moderate rather than very high β may be preferable in STs.

1. Introduction

Experiments on Spherical Tokamaks (ST) have demonstrated a possibility of achieving high values of the parameter β, which have never been reached in conventional tokamaks (CT) [1]. Calculations show that the plasma diamagnetism in such discharges is so strong that it leads to formation of a minimum of the magnetic field strength, B, in the plasma core ("magnetic valley") [2]. This affects particle motion, in particular, essentially changes features of the toroidal precession. Therefore, it may influence plasma processes associated with energetic ions for which precession is especially important. This was shown recently in Ref. [3], where it was found that trapped-particle-induced fishbone mode becomes stable in the presence of the magnetic valley because of the reversal of the direction of the toroidal precession of energetic ions. On the other hand, one can expect that high-β-induced changes in the interaction of energetic ions and MHD perturbations will affect not only plasma stability but also transport of the energetic ions. In the present work we show that this is indeed the case. We are interested here in the MHD events that manifest itself as large-scale helical perturbations terminated by the reconnection of magnetic field lines (this is presumably the case during, e.g., sawtooth oscillations, internal reconnection events etc.). Such events are typical for tokamaks, including spherical ones [1,4]. The presence of MHD activity can significantly deteriorate confinement of fast ions, which has been shown in experiments on CTs. A theoretical study of the effect of sawtooth oscillations on fast ions resulted in the prediction of the critical energy, $\mathcal{E}_{crit}$, such that the trapped ions with energy higher than $\mathcal{E}_{crit}$ are not sensitive to the sawtooth crash (the crash is the relaxation phase of the oscillations), whereas the ions with $\mathcal{E} < \mathcal{E}_{crit}$ are strongly redistributed [5]. This prediction was confirmed experimentally [6]. The existence of the critical energy is a consequence of the fact that the toroidal precession destroys the interaction of ions with magnetic helical perturbations when the ion energy is sufficiently high. Therefore, $\mathcal{E}_{crit}$ exists for all the MHD events mentioned above (rather than for sawteeth only). Its magnitude can be evaluated from the condition that the characteristic time of the precession is equal to the duration of the crash (τ_{cr}) as follows:

$$\mathcal{E}_{crit} = \left. \frac{2\pi M k r R_0 \omega_B}{n\tau_{cr}} \right|_{r=r_s} \propto \frac{A_s B S}{n\tau_{cr}} \ , \tag{1}$$

where n is the toroidal mode number (which was taken equal to unity in Ref. [5]), k is the elongation of the plasma cross section, r_s is the radius of the $q = 1$ flux surface, M is the ion mass, ω_B is the cyclotron frequency, R_0 is the major radius of the torus, S is the square of the $q = 1$ flux-surface cross section, $A_s = R_0/r_s$, and B is the magnetic field strength. When $n = 1$, $\tau_{cr} \sim 10^{-4}$ s, $\mathcal{E}_{crit}$ varies in the range $300 \div 700$ keV for α-particles in TFTR and JET, depending on plasma parameters. Because the product A_sBS in STs is relatively small, $\mathcal{E}_{crit}$ in STs is much less than that in CTs. However, existing theory is relevant for plasmas with low β, in which the effects of plasma diamagnetism are negligible. In high-β plasmas, as we will show below, trapped ions are attached to evolving flux surfaces and, thus, are expelled from the plasma core by crashes of MHD events even when their energy exceeds $\mathcal{E}_{crit}$.

2. Effect of plasma diamagnetism on the particle transport

We assume that the flux surfaces exist throughout the crash and can be described by a flux function, ψ, varying in time. In addition, we assume that the crash duration exceeds the bounce period of the particle under consideration and the particle orbit weakly deviates from a magnetic field line for one bounce. Then the longitudinal adiabatic invariant, $J = M/(2\pi) \oint ds\, v_\parallel$ ($v_\parallel$ is the particle longitudinal velocity, s is the distance along the field line), is approximately conserved and can be used to describe the bounce-averaged particle motion. We use flux coordinates (ψ, θ, ϕ), in which the equation of the field line is $\theta = q^{-1}(\psi)\phi + \eta$, where q is the safety factor, η is the field line label. Then $J = J(\psi, \eta)$, and equations of the bounce-averaged particle motion can be written as follows:

$$\dot{\psi} = C\frac{\partial J}{\partial \eta}, \quad \dot{\eta} = -C\frac{\partial J}{\partial \psi}, \tag{2}$$

where C is a certain function of the metric properties of the magnetic field and the particle pitch angle. For the deeply trapped particles, the derivatives of J are proportional to the corresponding derivatives of B_*, where $B_*(\psi, \eta) = B[\psi, \theta(\psi, \eta, \phi_*), \phi_*]$ is the minimum of the magnetic field on a field line, $\phi_* = \phi_*(\psi, \eta)$ is the location of the minimum. We conclude from Eq. (2) that B_* is conserved, i.e., the banana center moves along a trajectory on which the minimum of the magnetic field on the field line does not change. Now we have to specify $B(\psi, \theta, \phi)$. Assuming that the reconnection process is sufficiently slow for the plasma to be in approximate equilibrium at each moment of time, we can use the following Taylor series expansion of the magnetic field strength in the plasma core [7]:

$$B = B_0(\phi)\{1 + \psi^{1/2}\mathcal{A}_1(\phi)\cos[\theta - \theta_1(\phi)] + \psi\mathcal{A}_2(\phi) + \psi\mathcal{A}_3(\phi)\cos[2\theta - 2\theta_2(\phi)]\}. \tag{3}$$

The $\mathcal{A}_2(\phi)$ term in Eq. (3) characterizes the growth of the flux-surface-averaged magnetic field with the radius (i.e., the average magnetic well). It is known that the increase of the plasma pressure gradient results in the increase of $\mathcal{A}_2$ in both axisymmetric tokamak equilibria and stellarator equilibria. If we assume that β is high, and the $\mathcal{A}_2(\phi)$ term in Eq. (3) dominates, the contours $B_* = \text{const}$ approximately coincide with the contours $\psi = \text{const}$ in the (ψ, η) plane. This implies that the particle moves approximately along a flux surface. The presence of the other terms results only in weak oscillations around this line. We can conclude from this that when the pressure gradient is sufficiently high, the precession tends to attach particles to evolving flux surfaces. Therefore, in contrast to the case of a low-β plasma, the precession in a high-β plasma is a factor promoting the redistribution of particles by the crash.

This conclusion is confirmed by the following analysis, which does not involve the near-axis expansion. We introduce the flux coordinates $(x^1, x^2, x^3) = (r, \alpha \equiv m\vartheta - n\varphi, \varphi)$

associated with the pre-crash equilibrium, where r, ϑ, and φ are the radial, poloidal, and toroidal coordinates, respectively; m is the poloidal mode number. We present the particle velocity as $\vec{v} = \vec{v}_\parallel + \vec{v}_E + \vec{v}_D$ with $\vec{v}_D = c/(e|B|^2)\vec{B}\times(\mu\nabla B + Mv_\parallel^2\vec{\mathcal{K}})$, $\vec{v}_E$ the velocity of the $\vec{E}\times\vec{B}$ drift, e and μ the particle charge and magnetic moment, $\vec{E}$ the electric field of the perturbation, and $\vec{\mathcal{K}}$ the field line curvature. Assuming that $\vec{\mathcal{K}}$ is not affected by the crash and using the expression $\vec{\mathcal{K}} = |B|^{-2}\left[\nabla\left(|B|^2/2 + 4\pi p\right) - \nabla_\parallel |B|^2/2\right]$, where $\nabla_\parallel = \vec{b}(\vec{b}\cdot\nabla)$, $\vec{b} = \vec{B}/B$, p is the plasma pressure, we express ∇B in terms of ∇p and $\vec{\mathcal{K}}$. In addition, we assume that $1 - nq/m \ll 1$ and that approximate helical symmetry is conserved in the considered region so that $\psi = \psi(r,\alpha)$ and $p = p(\psi) = p(r,\alpha)$. Then, performing bounce averaging, we obtain:

$$\dot{r} = v_E^1 + \frac{4\pi c\mu}{e}\left\langle \frac{B_3}{|B|^3\sqrt{g}} \right\rangle \frac{\partial p}{\partial \alpha}\,, \tag{4}$$

$$\dot{\alpha} = v_E^2 - \frac{4\pi c\mu}{e}\left\langle \frac{B_3}{|B|^3\sqrt{g}} \right\rangle \frac{\partial p}{\partial r} + \omega_{pr}^{(0)}\,, \tag{5}$$

where $\omega_{pr}^{(0)}$ is the conventional (toroidal) precession; subscripts and superscripts denote co- and contravariant vector components, respectively; $\langle\ldots\rangle$ denotes bounce averaging; $\sqrt{g}$ is the determinant of the metric tensor. Let us analyze Eqs. (4), (5). The first terms in these equations describe the $\vec{E}\times\vec{B}$ drift. They are the terms driving the particle redistribution by freezing the particles to the evolving flux surfaces [5]. The conventional (toroidal) precession is described by the last term in Eq. (5). The competition of the conventional precession and the $\vec{E}\times\vec{B}$ drift motion results in $\mathcal{E}_{crit}$. The other terms in Eqs. (4), (5) are associated with the plasma diamagnetism. Taking into account that $p = p(\psi)$, we conclude from Eqs. (4), (5) that the precession associated with the plasma diamagnetism conserves ψ, i.e., leads to the particle motion along the flux surfaces rather than along the $r = \text{const}$ surfaces. The diamagnetic precession overrides the conventional precession and, thus, attaches the particles to the moving flux surfaces when $\beta(r_s) > \epsilon(r_s)$, where $\beta(r_s) \equiv 8\pi p(r_s)/B_0^2$. The latter is essentially the condition of the existence of the "magnetic valley" in the equilibrium.

The different effect of precession in high-β and low-β plasmas can be explained as follows. The instabilities causing magnetic reconnection in low-β plasmas are essentially Alfvénic in the sense that they weakly perturb the magnetic field strength. Therefore, the particle precession is weakly affected by the perturbations and tends to move particles along pre-crash flux surfaces, thus preventing their redistribution. On the other hand, when β is so high that the plasma diamagnetism results in a magnetic valley, a helical displacement of the plasma core results in a similar displacement of the valley, forming several local magnetic wells in the equatorial plane outside the magnetic axis, where the deeply trapped particles are localized.

3. Numerical simulation

We apply the obtained equations to simulate the transport of ions with $\mathcal{E} > \mathcal{E}_{crit}$ during crashes of sawtooth oscillations in an NSTX-like tokamak. With this purpose we use Eqs. (4), (5), where we have added terms associated the longitudinal particle motion (which are essential when a trapped particle is transformed to a circulating one) and carried out bounce averaging, assuming $r_s/R_0 \ll 1$. We take $v_E^1 = -c(B_0rk)^{-1}\partial\Phi/\partial\alpha$, $v_E^2 = c(B_0rk)^{-1}\partial\Phi/\partial r$, use the conservation of J to follow the evolution of the particle energy, the equation $\vec{E}\cdot\vec{B} = 0$ to relate the perturbed electrostatic potential Φ to the helical magnetic flux ψ, and the equations for $\psi(r,\alpha,\phi,t)$ from the crash model of Ref. [5] approximating the Kadomtsev type of the sawtooth crash. The results of calculations for a well-trapped particle are shown in Fig. 1. We observe that the particle, which was not sensitive to the crash at low β, is expelled from the plasma core in the high-β plasma.

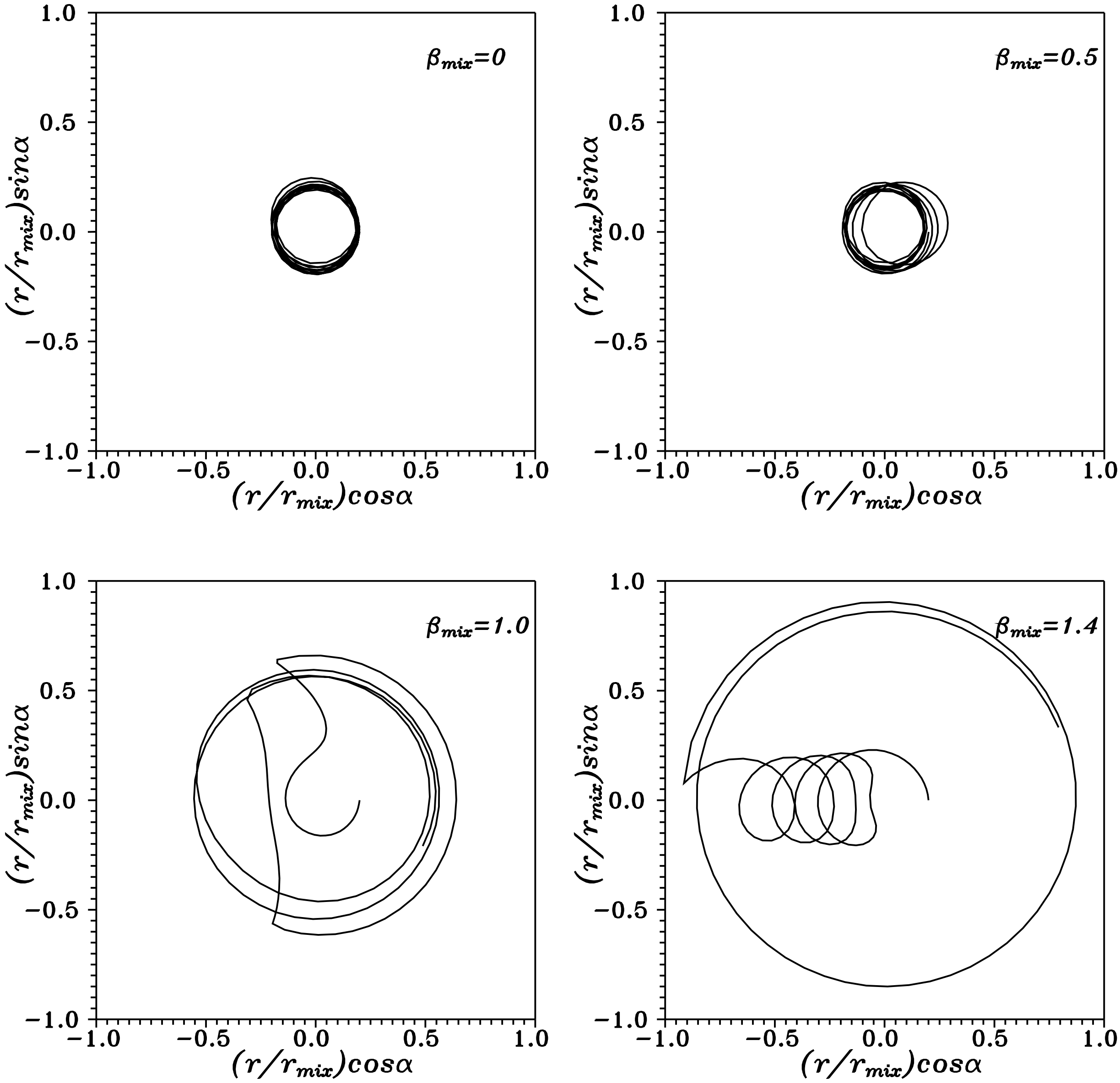

FIG. 1. *Motion of trapped energetic ions with* $\mathcal{E} \gtrsim \mathcal{E}_{crit}$ *in an NSTX-like tokamak with* $r_{mix}/a = 1/3$ *for various* β *during sawtooth crashes. Notations:* r_{mix} *is the sawtooth mixing radius,* $\alpha = \theta - \phi$, $\beta_{mix} = 8\pi[p(0) - p(r_{mix})]/B_0^2$, B_0 *is the magnetic field on the magnetic axis in the presence of the plasma. We observe that the crash has a strong influence on the ions only when* β *is sufficiently high.*

4. Conclusions

It has been shown for the first time that the confinement of trapped energetic ions during MHD events, such as sawtooth oscillations and internal reconnection events (IRE), essentially depends on β. Namely, when β is high, trapped ions with $\mathcal{E} > \mathcal{E}_{crit}$ are expelled from the plasma core, whereas they are only weakly influenced by the MHD activity at relatively low β. These differences are associated with the different character of the particle precessional motion in low-β and high-β plasmas. In spherical tokamaks the critical energy given by Eq. (1) is rather low, and the trapped particles constitute a quite considerable fraction of the population of energetic ions in STs even in the case of tangential Neutral Beam Injection (NBI) [2]. Thus, the predicted β-dependence of transport in MHD-active plasmas may concern a quite considerable fraction of energetic ions and, therefore, the discovered mechanism of the energetic ion transport in MHD-active plasmas may be of importance for the optimization of β in spherical tokamaks. Experimental investigation of the problem, as well as theoretical study of resonance mechanisms of the particle transport [8,9], are required to draw a final conclusion concerning optimal β in STs.

The research described in this work was made possible in part by the International Atomic Energy Agency Research Contract No. 10539 and the Award No. UP2-2114 of the U.S. Civilian Research & Development Foundation. Two of the authors (Ya.K. and Yu.Y.) acknowledge the hospitality of the Princeton Plasma Physics Laboratory.

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