Stochastic Loss of Alpha Particles in a Helias Reactor

Ya.I. Kolesnichenko 1), C.D. Beidler 2), V.S. Marchenko 1), I.N. Sidorenko 2), H. Wobig 2)

1) Scientific Centre "Institute for Nuclear Research", Kyiv, 03680, Ukraine
2) Max-Planck-Institut für Plasmaphysik, EUROATOM Association
D-85740 Garching bei München, Germany

e-mail contact of main author: ftd@nucresi.freenet.kiev.ua

Abstract. It is shown that collisionless orbit transformation of the locally trapped particles to the locally passing ones and vice versa in the Wendelstein-line optimized stellarators results in stochastic diffusion of energetic ions. This diffusion can lead to the loss of an essential fraction of energetic ion population from the region where the characteristic diffusion time is small compared to the slowing down time. The loss region and the magnitude of the loss can be minimized by shaping the plasma temperature and density profiles so that they satisfy certain requirements. The predictions of the developed theory are in agreement with the results of numerical modelling of confinement of α-particles in a Helias reactor, which was carried out in this work with the use of the orbit following code. The considered diffusion seems to represent the dominant mechanism of classical losses of α-particles in a Helias reactor.

1. Introduction

The lack of the axial symmetry is known to lead to the loss of all locally trapped α -particles in conventional stellarators. Therefore, a reactor based on such systems is not possible. Several ways are suggested to make acceptable the confinement of alphas. One of them is to optimize the magnetic configuration. This way is implemented in the construction of stellarator Wendelstein 7-X and the development of a Helias reactor [1,2]. In the optimized stellarators (which can also be refered to as Helias configurations) the effects of plasma diamagnetism are sufficiently strong to make closed and weakly deflecting from the magnetic flux surfaces the contours of the longitudinal adiabatic invariant, $J = \oint v_{\parallel} dl$. This implies that superbanana orbits will not arise, and thus, the locally trapped particles, which constitute the main fraction of escaping alphas in conventional stellarators, will be confined in the optimized systems. However, we will show in this work that a considerable fraction of energetic ions can be lost even in the optimized stellarators. The reason for this is the stochastic diffusion of the so-called "transitioning particles", i.e., particles whose orbits are transformed from locally trapped to locally passing ones and vice versa.

2. Diffusion of transitioning particles

We study the diffusion arising due to the following. The adiabaticity of the particle motion in the phase space breaks down near the separatrix between the regions of locally trapped and locally passing orbits (see Fig. 1). Because of this the adiabatic invariant J acquires a phase dependent jump each time when a particle crosses the separatrix [3]-[5]. The phases of the motion do not correlate for successive transitions. Therefore, the multiple crossings of the separatrix are accompanied by the random walk of particles in the J space, resulting in spatial diffusion. Note that stochastic diffusion having the same nature may take place also in tokamaks, where, however, it plays a minor role, being associated with the presence of the ripple wells [6].

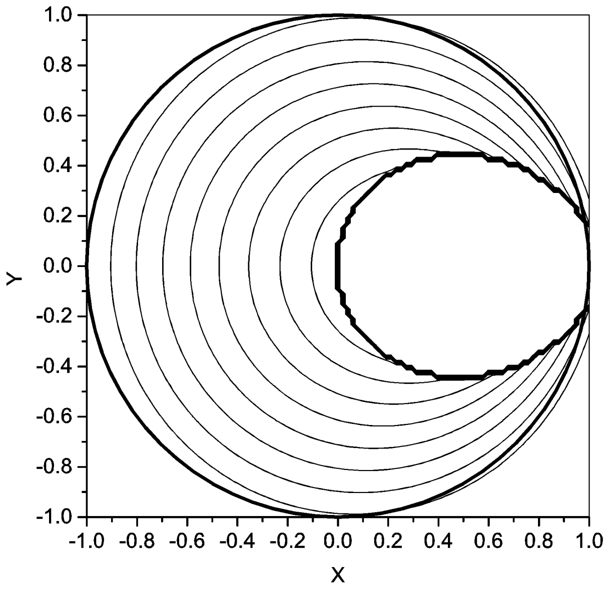

Fig. 1. Sketch of level contours of J for the locally trapped particles (thin lines) and the $\kappa = 1$ contour (bold line inside the plasma) in a Helias. Locally trapped particles moving along the $J = 1$ contours become locally passing ones after crossing the bold line.

The corresponding diffusion coefficient can be written as follows:

$$D = \frac{\langle(\Delta r)^2\rangle}{\tau} \ , \qquad (1)$$

where Δr is the change of the particle radial coordinate caused by the orbit transformation, $\langle\ \rangle$ means ensemble averaging, r is the effective flux surface radius defined by the equation $\psi = \overline{B}r^2/2$, $\overline{B}$ is the average vacuum magnetic field at the magnetic axis; τ is the characteristic time,

$$\tau = \frac{1}{2}(\tau^l + \tau^p) \ , \qquad (2)$$

τ^l and τ^p are the characteristic times of the particle motion in the locally trapped and passing states. The factor 1/2 takes into account that a particle crosses the separatrix twice per full period of a hybrid passing-localized orbit. The time τ^l is essentially the precession time of a localized particle, whereas $\tau^p = (2/P)\int_{-\theta(\kappa^2=1)}^{\theta(\kappa^2=1)} d\theta/\dot{\theta}$ where $\dot{\theta}$ is the frequency of the poloidal motion of a passing particle, P is the probability of the orbit transformation resulting in the trap of a passing particle. The probability P and the radial jump caused by the separatrix crossing can be expressed in terms of the longitudinal invariant [3]-[7]. All ingredients in Eq. (1) were calculated with using the bounce-averaged equations of the particle motion in the magnetic field

$$B = \overline{B}[1 + \varepsilon_m(\psi) + \varepsilon_0(\psi)\cos N\phi - \varepsilon_h(\psi)\cos(\theta - N\phi) - \varepsilon_t(\psi)\cos\theta\,] \ , \qquad (3)$$

where ψ, θ, ϕ are the magnetic flux coordinates with ψ the toroidal magnetic flux; ε_0 describes the change of the vacuum magnetic field due to finite β; ε_m, ε_h, and ε_t are the amplitudes of the mirror, helical, and toroidal harmonics, respectively, ε_m being dominant in the plasma core; $N >> 1$ is the number of the field periods along the large azimuth of the torus. As a result, a diffusion coefficient was obtained. Its magnitude for $\varepsilon_t << \varepsilon_h << \varepsilon_m$, which is the case in the plasma core of a Helias, can be evaluated as

$$D \approx \frac{4}{\pi}\frac{R^2\omega_B\rho_B^4}{N^2 a r^3}\frac{\varepsilon_h^2}{\varepsilon_m}\varepsilon_0' \ , \qquad (4)$$

where ω_B is the energetic ion gyrofrequency, ρ_B is the gyroradius; *a* and *R* are the minor and major radii of the torus, respectively; $\varepsilon_0' \equiv d\varepsilon_0 / dx$ with *x=r/a*. Equation (4) is relevant to particles with the pitch-angle parameter $\alpha \equiv \mathrm{E}/\mu B - 1 \sim \varepsilon_m + \varepsilon_0$.

The condition that an energetic ion will be lost because of diffusion (rather than displaced within the plasma) is $\tau_d << \tau_s$, where τ_s is the characteristic slowing down time, and τ_d is the diffusion time defined by

$$\tau_d(r) \sim \frac{(a-r)^2}{D} \quad . \tag{5}$$

It is of importance to know the fraction of transitioning particles, which is essentially the stochastic-diffusion-induced loss fraction of alpha particles when the condition $\tau_d << \tau_s$ is satisfied. This quantity relevant to a flux surface is given by

$$\nu(r) = \sqrt{\frac{\alpha_{\max}}{1+\alpha_{\max}}} - \sqrt{\frac{\alpha_{\min}}{1+\alpha_{\min}}} \quad , \tag{6}$$

where $\alpha_{\min} = \varepsilon_m + \varepsilon_0 - \varepsilon_h - \varepsilon_t$, $\alpha_{\max} = \varepsilon_m + \varepsilon_0 + \varepsilon_h + \varepsilon_t$.

3. Alpha particle loss in a Helias reactor: predictions of a theory and numerical simulations

Using Eqs. (4)-(6) let us evaluate the diffusion time and the fraction of transitioning α-particles in a Helias reactor with *N=5*, *R/a =10*, and β ~ 5% [8]. At first, we make a simple estimate by approximating Fourier harmonics of the magnetic field in Eq. (1) as follows:

$$\varepsilon_m = 0.1 \,, \; \varepsilon_0 = 0.08x^2 \,, \; \varepsilon_t = 0.05x \,, \; \varepsilon_h = 0.08x \,. \tag{7}$$

Then at *r=a/2* we obtain:

$$\tau_d(a/2) \approx \frac{5}{\omega_B}\left(\frac{a}{\rho_B}\right)^4, \qquad \nu(a/2) \approx 15\% \,. \tag{8}$$

When *B=5 T,* Eq. (8) yields $\tau_d \approx 0.02s$ for $a/\rho_B = 30$ and $\tau_d = 0.06s$ for $a/\rho_B = 40$. On the other hand, the slowing down time of a *3.5 MeV* α-particle in a plasma with the electron density $n_e = 2-3\times10^{20} m^{-3}$ and the temperature $T = 10-15 keV$ is $\tau_s \sim 0.1\sec$. This means that an essential fraction of alphas will be lost to the wall from a flux surface of the radius *r/a=0.5* in a system with $a/\rho_B = 30$, but particles can hardly escape from the *r/a=0.5* surface when $a/\rho_B = 40$. Furthermore, when profile shapes of the plasma density and temperature are such that τ_s decreases with the radius, the condition $\tau_d << \tau_s$ may violate near the plasma edge. Then energetic ions will diffuse to the periphery and thermalize in that region. In this case the main effect of the stochastic diffusion will be the broadening of the radial profile of the power deposition of energetic ions rather than their loss. As $\tau_s \propto T^{3/2}/n_e$, this will be the case when the temperature profile is peaked, whereas the $n_e(r)$ profile is flat.

Now we calculate numerically the diffusion coefficient, the fraction of transitioning particles and the diffusion time, using corresponding equilibrium data. The obtained dependence of τ_d on α at *r/a =0.5* is presented in Fig. 2. We observe that the dependence of τ_d on α has a rather flat minimum around $\alpha = \varepsilon_m + \varepsilon_0$ where *D* was expected to be maximum; the magnitude of τ_d is in qualitative agreement with the estimates above.

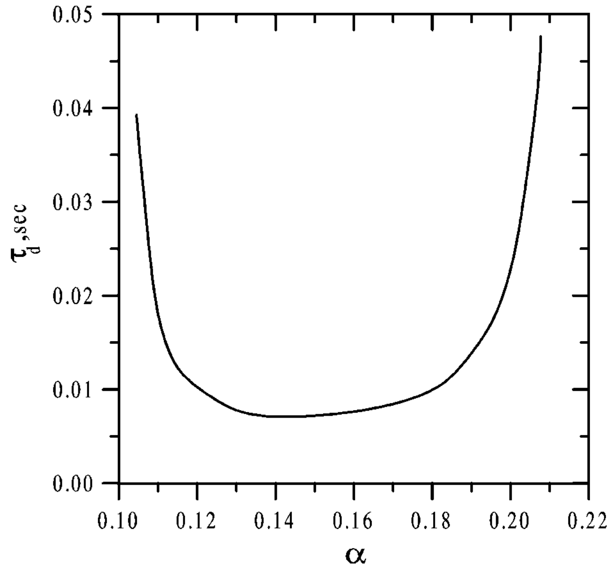

Fig. 2. The diffusion time versus the pitch parameter $\alpha \equiv E/\mu B - 1$ *at* $r = 0.5a$ *in a Helias reactor with* $\beta = 4.7\%$.

These results are in agreement with the confinement time and the fraction of lost particles calculated numerically using a guiding center code in Ref. [9]. In the present work, similar calculations were carried out for a Helias reactor [8]. The simulations began with an ensemble of 250 α-particles, all of which have the same initial energy $E_0 = 3.52\ MeV$ but are assigned to different starting radii according to the appropriate birth profile; the remaining spatial and velocity coordinates were chosen randomly. The plasma parameters and profiles were taken from 1-D numerical simulations for two scenarios: first, a high-density (central electron density $n_e(0) = 3\times10^{20}\,m^{-3}$), low-temperature (central temperature $T(0) = 15\ keV$) case similar to that considered previously [9]; second, low-density ($n_e(0) = 1.5\times10^{20}\,m^{-3}$), high-temperature ($T(0)=25keV$) case. These scenarios lead to nearly identical birth profiles for the α-particles but slowing-down times which differ by a factor of four. Given the arguments above, one must therefore expect greater losses of fast α-particles for the low-density, high-temperature scenario as even those particles born near the plasma center (with $\tau_s \approx 0.46\,s$) can diffuse to the plasma edge before slowing down. The carried out calculations confirmed this. It was found that (i) for the high-*n*, low-*T* scenario 19 particles are lost during a simulation time of *0.12 s* leading to a lost-energy fraction of 0.02, 60% of which is due to 5 particles with $E_{loss}>1\ MeV$, where E_{loss} is the energy of escaping ions; (ii) in the low-*n*, high-*T* scenario losses increase to 54 particles during a simulation time of *0.44 s*. The lost-energy fraction is 0.09, of which 85% can be attributed to 32 particles with $E_{loss}>1MeV$.

4. Conclusions

We have shown that transitioning energetic particles in advanced stellarators of Wendelstein line undergo the stochastic diffusion associated with the orbit transformation of localized and passing particles. This diffusion may lead to the loss of α-particles and other energetic ions from the plasma core of a Helias reactor and Wendelstein 7-X for the time ~ *0.01 s*. A key parameter affecting the magnitude of the diffusion coefficient is the ratio a/ρ_B ($D \propto \omega_B(\rho_B/a)^4$). The fraction of escaping α-particles can be of the order of *10%* for α-particles produced at $r \sim a/2$. It can be even more for the injected ions when their pitch angles correspond to transitioning particles, $v_{\parallel}/v \sim \sqrt{\varepsilon_m} \sim 0.3$. However, the diffusion process is relatively slow. Therefore, the diffusion not necessarily leads to the loss of ions of high energy to the wall. When the electron temperature is characterized by strongly peaking radial distribution, whereas the electron density profile is flat, the energetic ions may be thermalized near the edge before being lost.

A general conclusion which follows from our work is that the stochastic diffusion of transitioning particles may represent the dominant mechanism of the loss of energetic ions in optimized stellarators. The dependence of the obtained diffusion coefficient on plasma parameters and the relatively large diffusion time indicate that the loss region and the loss fraction of energetic ions in Helias configurations can be minimized by shaping the plasma temperature and density profiles so that they satisfy certain requirements.

Acknowledgements

The work is carried out within the Partner Project Agreement No. P-034 between the Science and Technology Center in Ukraine, the Scientific Centre “Institute for Nuclear Research”, and Max-Planck-Institut für Plasmaphysik. The authors thank V. Lutsenko for the help with graphical tools. Two of the authors (Ya.K. and V.M.) would like to acknowledge the hospitality of the Max-Planck-Institut für Plasmaphysik.

References

[1] HERRNEGGER, F., RAU, F., WOBIG, H., (Ed.), Contributions to Wendelstein 7-X and Helias Reactor 1991-1998, Report IPP 2/343 (1999).
[2] WAGNER, F., “Stellarators and optimised stellarators”, Transactions of Fusion Technology **33** (1998) 67.
[3] TIMOFEEV, A.V., “On conservation of an adiabatic invariant with the change of the character of the motion”, Sov. Phys.-JETP **48** (1978) 656.
[4] CARY, J.R., ESCANDE, D.F., TENNYSON, J.L., “Adiabatic invariant change due to separatrix crossing”, Phys. Rev. A **34** (1986) 4256.
[5] NEISHTADT, A.I., “On the change of adiabatic invariant due to separatrix crossing”, Sov. J. Plasma Phys. **12** (1986) 992.
[6] MARCHENKO, V.S., “Collisionless diffusive fluxes of locally trapped ions in tokamaks with rippled magnetic field”, Nucl. Fusion **35** (1995) 69.
[7] YUSHMANOV, P.N., “Ripple-induced diffusion transport processes in tokamaks”, Reviews of Plasma Physics, vol.16, Consultants Bureau, New York (1990) 55.
[8] WOBIG, et al., “Power balance in stellarator reactors”, Fusion Energy 1998 (Proc. 17th IAEA Conference, Yokohama) IAEA 4, (1999) 1235.
[9] LOTZ, W., et al., “Collisionless α-particle confinement in stellarators”, Plasma Phys. Control. Fusion **34** (1992) 1037.