Nonlinear Tearing Mode in Turbulent Plasmas

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Abstract. A kind of large eddy simulation model is proposed to incorporate the effect of microturbulence into MHD. This model is applied to the analysis of magnetic island dynamics in the presence of strong pressure gradient. It is found that the growth of magnetic island is accelerated by the anomalous resistivity and the propagation frequency of magnetic island is dictated by turbulent viscosity.

1. Introduction

Neoclassical effect is considered as a driving mechanism of magnetic islands in high temperature plasmas. Theoretically, much work has been done based on neoclassical MHD model[1]. Although the theoretical prediction of saturated island width is not inconsistent with experimental observations[2], the dynamic behavior such as the fast growth of magnetic island has not been explained so far. Since plasma confinement is degraded due to tearing modes, understanding of dynamics is necessary for an achievement of high performance in fusion plasma. In this article, a kind of large eddy simulation (LES) model is proposed to incorporate the effect of microturbulence into MHD. The localized micro perturbations, produced by the magnetic stochasticity generated due to the symmetry-breaking[3] or by the normal cascade from global MHD mode is considered to cause anomalous resistivity near the separatrix. The magnetic island dynamics in the presence of strong pressure gradient is investigated based on this model.

2. Model

We start from high β reduced MHD equations[4]. The nonlinear effect of microturbulence is renormalized as an eddy viscosity based on one-point renormalization approximation and the transport matrix is obtained via the mean field approximation[5]. The effect on global MHD mode is introduced as the turbulent transport coefficients.

$$\frac{\partial}{\partial t}\nabla^2\phi = -[\phi, \nabla^2\phi] + \left(\mu_c + \mu_N\right)\nabla^4\phi - \nabla_{//}\nabla^2 A + \boldsymbol{b_0}\times\kappa\cdot\nabla p \qquad (1)$$

$$\frac{\partial}{\partial t}A = \left(\eta_c + \eta_N\right)\nabla^2 A - \nabla_{//}\phi \qquad (2)$$

$$\frac{\partial}{\partial t}p = -[\phi, p] + \left(\chi_c + \chi_N\right)\nabla^2 p \qquad (3)$$

where $\{\phi, A, p\}$ are the electrostatic potential, the component of the vector potential parallel to the magnetic field and the fluctuating electron pressure, respectively. $\{\mu_c, \eta_c, \chi_c\}$ are the collisional ion viscosity, resistivity and the electron thermal conductivity. $\{\mu_N, \eta_N, \chi_N\}$ are the renormalized eddy viscosities due to the background turbulence. In the strong turbulence limit, these coefficients are proportional to the amplitude of turbulence. The back reaction from MHD mode to the microturbulence is not solved, i.e., we regard the background turbulence as parameters. $[,]$ is the Poisson's bracket denoting the $\boldsymbol{E} \times \boldsymbol{B}$ nonlinearity and $_{//} = \partial / \partial z - [A,]$ where z is the direction of the main magnetic field. Only the modes with low poloidal and toroidal mode numbers contribute these nonlinearities.

3. Tearing mode with eddy viscosities

When the width of magnetic island is smaller than the width of resistive layers, the $\boldsymbol{E} \times \boldsymbol{B}$ nonlinearities in eqs.(1)-(3) are neglected. We consider the case that the background turbulence localizes around the rational surface and has the short wave length. In this limit, the eddy viscosities have a relation;

$$\mu_N : \eta_N : \chi_N = \frac{1}{3} : 1 : 1 \qquad (4)$$

which corresponds to the electrostatic turbulence. Figure.1 shows the dependence of the growth rate of tearing mode on eddy viscosities with the relation eq.(4). Here $m = 2 / n = 1$ mode with typical parameters for large size of tokamak is chosen.

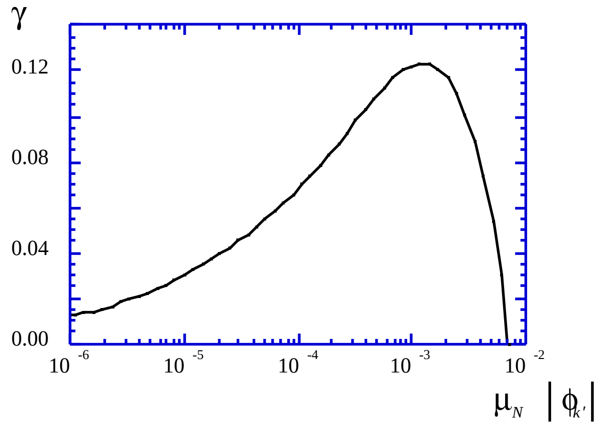

Fig.1 the dependence of the growth rate of tearing mode on eddy viscosities.

This result shows that a finite amplitude of fluctuations enhances the growth rate of tearing mode. For $\chi \approx 1\ m^2 / \sec$, the growth rate is estimated as $50 - 100\ (\mu\mathrm{sec})^{-1}$ which is order of experimental observation[6]. As an example, the growth rate of tearing mode enhanced by the resistive ballooning mode turbulence is estimated. The growth rate is given by

$$\gamma \quad \alpha^{3/5} s^{-1/5} \eta_c^{3/5} \qquad (5)$$

where $s = r\, d \ln q / dr$ is the shear parameter, q, the safety factor, $\alpha = q^2 \beta R / L_p$ is the ballooning parameter with $L_p^{-1} = - d \ln p / dr$. It is found that the tearing mode is stabilized by the magnetic shear and is destabilized by the pressure gradient through the resistive ballooning mode turbulence. This example shows that the linear picture which gives $\gamma \quad \eta_c^{3/5} s^{2/5}$ is drastically changed under the effect of microturbulence.

4. Equations for island width and propagation frequency

When the width of magnetic island exceeds the width of resistive layers, the inertia effect dominates the island evolution[7]. We consider the perturbation of the phase $\exp(im\theta - inz/R - i\omega t)$ with island width w. The nonlinear dynamics of $(w, \hat{\omega})$ is analyzed. Including the effect of turbulence, we develop a new equation of the magnetic island as

$$G_1 \frac{dw}{dt} = \eta \left(\frac{\Delta'}{4} + \sqrt{\varepsilon}\, \frac{q}{\varepsilon} \frac{L_s}{L_p} \beta \frac{G_2}{w} + \frac{\hat{\omega}_g \hat{\omega}_*}{\delta^2 \beta} \frac{q^2}{s^2} \frac{G_2}{w} - \hat{\omega}(\hat{\omega} + \hat{\omega}_{*i}) \frac{q^2}{s^2} \frac{G_3}{w^3} \right) \tag{6}$$

where the normalization: $r/a \to r$, $\varepsilon v_A t/a \to t$ is used. w is the normalized width of magnetic island, $\eta = \eta_c + \eta_N$, $\varepsilon = r/R$, the inverse aspect ratio, $L_s^{-1} = s/(qR)$, β, the plasma beta, $\hat{\omega}_g = \delta\beta\kappa$, $\delta = c/(a\omega_{pi})$, the normalized ion skip depth, $\kappa = -2(r/R_0)(1 - 1/q^2)$, $\hat{\omega}_* = \delta\beta R/L_p$, $\hat{\omega} = \omega/k_y$, the normalized propagation frequency of magnetic island, $\hat{\omega}_{*i} = -\hat{\omega}_*/2$, the normalized ion diamagnetic frequency, k_y, the wave number, $G_1 = 0.41$, $G_2 = 0.78$, $G_3 = 1.06$. The first term in RHS represents the driving term of tearing mode. The second term represents the bootstrap current term[1]. In the helical systems, this term acts as stabilization effect since $L_s < 0$. The third term represents a stabilizing effect due to the magnetic well for tokamak and a destabilizing effect due to the magnetic hill for helical systems ($\hat{\omega}_g \hat{\omega}_* / (\delta^2 \beta) = \kappa\beta R/L_p$). The fourth term is the ion polarization term[8]. The equation for propagation frequency is given by

$$\frac{d\hat{\omega}}{dt} \left[\frac{K_1}{w} - \frac{w}{\hat{\omega}} \frac{\hat{\omega}_g^2}{\delta^2 \beta} \frac{\hat{\omega}_* K_2}{(\hat{\omega} - \hat{\omega}_{*e})^2} + \frac{1}{\hat{\omega} w} \frac{\hat{\omega}_g^3 K_3}{(\hat{\omega} - \hat{\omega}_{*e})^2} + \frac{\hat{\omega}_g}{\hat{\omega} w} \left(1 + \frac{2\hat{\omega}_g \hat{\omega}_{*e}}{(\hat{\omega} - \hat{\omega}_{*e})^2} \right) K_1 \right]$$
$$= -\frac{dw}{dt} \left[(\hat{\omega} + \hat{\omega}_{*i}) \frac{K_4}{w^2} + \frac{\hat{\omega}_g \hat{\omega}_*}{\delta^2 \beta} \frac{K_5}{\hat{\omega}} \right] + \chi \frac{\hat{\omega}_g \hat{\omega}_*}{\delta^2 \beta} \frac{1}{\hat{\omega} w} K_1 + \mu \frac{\hat{\omega} + \hat{\omega}_{*i}}{w^3} K_6 \tag{7}$$

where $\hat{\omega}_{*e} = \hat{\omega}_*/2$, the normalized electron diamagnetic frequency, $\chi = \chi_c + \chi_N$, $\mu = \mu_c + \mu_N$, $K_1 = 1.83$, $K_2 = 5.45$, $K_3 = 1.03$, $K_4 = 1.14$, $K_5 = 2.44$, $K_6 = 11.24$. Influence of turbulent transport on propagation is seen in the right hand side. The detailed derivation will discussed in [9].

The case of pressure-driven island ($\Delta' < 0$) is studied in the following. Setting $\dot{w} = \dot{\omega} = 0$, the stationary solution is obtained as

$$w_s = -G_2 \frac{4}{\Delta'} \left[\sqrt{\varepsilon}\, \frac{q}{\varepsilon} \beta \frac{L_s}{L_p} + \kappa \frac{L_s^2}{R L_p} \beta \left(1 + \frac{K_7}{P_r} \right) \right] \tag{8}$$

and

$$\hat{\omega}_s^{\pm} = \frac{1}{2} \left(-\hat{\omega}_{*i} \pm \sqrt{\hat{\omega}_{*i}^2 - 4 \frac{\kappa\beta R}{L_p} \frac{K_8}{P_r} w_s^2} \right) \tag{9}$$

where $P_r = \mu/\chi$ is the turbulent Prandtl number, $K_7 = 0.221$ and $K_8 = 0.163$. It is found that the saturated island width is weakly affected by the turbulent Prandtl number. The contribution of anomalous resistivity to the island width is higher order correction. The stationary propagation frequency is dictated by the turbulent Prandtl number. A novel point is that the ion polarization term also acts destabilization for the helical systems.

5. Stability analysis around the stationary solution

The stability analysis of the fixed points $(w_s, \hat{\omega}_s^{\pm})$ is performed. First, the case of tokamak plasmas is studied. The parameters are chosen as $a = 1[m]$, $R = 3[m]$, $B = 2[T]$, $\beta = 0.012$, $\delta = 0.041$, $\varepsilon = 0.167$, $q = 2$, $L_p = 1$, $s = 1$, $L_s = 6$, $\kappa = -0.25$, $\Delta' = -6$, $\eta = \chi = 1\times 10^{-5}$, $P_r = 2$. Then we obtain the stationary solutions, $w_s = 0.164$, $\hat{\omega}_s^{\pm} = -4.09\times 10^{-3}, 4.84\times 10^{-3}$ and the characteristic frequencies $\hat{\omega}_{*e} = 7.52\times 10^{-4}$, $\hat{\omega}_g = -1.25\times 10^{-4}$. Fig.2(a) and 2(b) shows $\dot{w}$ versus w at $\hat{\omega} = \hat{\omega}_s$ and $\dot{\hat{\omega}}$ versus $\hat{\omega}$ at $w = w_s$. It is found that $\hat{\omega}_s^{-} = -4.09\times 10^{-3}$ is an unstable fixed point. On the other hand $\hat{\omega}_s^{+} = 4.84\times 10^{-3}$ is a stable fixed point. This analysis shows that the propagation frequency is in the direction of electron diamagnetic drift ($\hat{\omega}_s^{+} > \hat{\omega}_{*e}$) if the microturbulence drives the rotation of the magnetic island. It is found that $P_r > P_{rc} \approx 1.9$ is the necessary condition for the stable fixed point for these parameters. We also check the case the first term in RHS is only retained in the coefficients of $d\hat{\omega}/dt$ in eq.(7). It is found that both solutions are unstable in this case, therefore coefficients of $d\hat{\omega}/dt$ is sensitive to the stability.

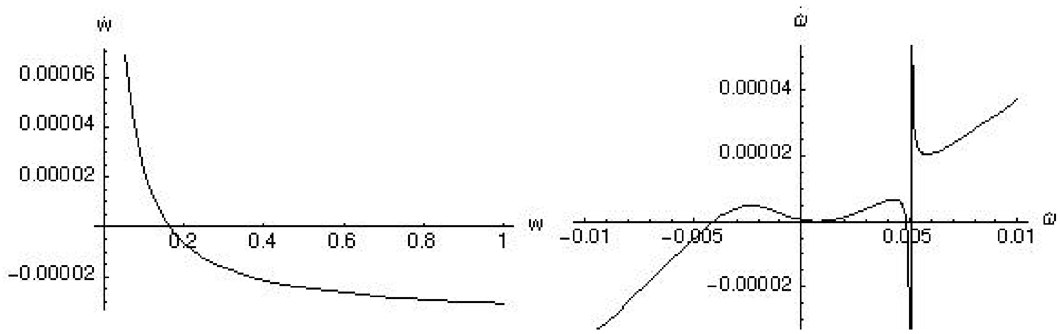

Fig. 2a $\dot{w}$ *versus* w *at* $\hat{\omega} = \hat{\omega}_s$ (a) and $\dot{\hat{\omega}}$ *versus* $\hat{\omega}$ *at* $w = w_s$ (b).

Next we investigate the stability in the helical systems (with magnetic hill and $L_s < 0$). From eqs.(8) and (9), we obtained the following conditions for $w_s > 0$

$$\kappa \frac{|L_s|}{R}\left(1 + \frac{K_7}{P_r}\right) > \frac{q}{\sqrt{\varepsilon}} \tag{10}$$

and the real frequency

$$\hat{\omega}_{*i}^2 > 4\frac{\kappa\beta R}{L_p}\frac{K_8}{P_r} w_s^2 \tag{11}$$

If the shear is very weak, this condition could be satisfied. For the parameters chosen as $a = 1[m]$, $R = 4[m]$, $B = 1.8[T]$, $\beta = 0.015$, $\delta = 0.041$, $\varepsilon = 0.125$, $q = 0.7$, $L_p = 1$, $s = -0.1$, $L_s = -28$, $\kappa = 0.26$, $\Delta' = -6$, $\eta = \chi = 1 \times 10^{-5}$, $P_r = 2$, we obtain $w_s = 0.0094$, $\omega_s^+ = 1.14 \times 10^{-3}$, $\hat{\omega}_{*e} = 1.24 \times 10^{-3}$ and $\hat{\omega}_g = 1.61 \times 10^{-4}$. The threshold value of the Prantdl number is given by $P_r > P_{rc}^h \approx 1.8$ in these parameters. In comparison with tokamak case, the island size is one order smaller.

6. Summary

A kind of large eddy simulation model is proposed to incorporate the effect of microturbulence into MHD. This model is applied to the analysis of magnetic island dynamics in the presence of strong pressure gradient. It is found that (1) the abrupt growth of tearing mode enhanced by microturbulence is possible and the linear picture is drastically changed under the microturbulence, (2) the enhanced growth of magnetic island in the Rutherford regime is expected if η_N is enhanced, (3) the saturation width is enhanced by the sink of rotation of island through the ion polarization current term, (4) the propagation frequency is dictated by the turbulent Prandtl number, (5) the propagation is in the electron diamagnetic drift for typical parameters of large size tokamak and (6) excitation of nonlinear tearing instability in high-β helical system is newly predicted. The break down of stability means that the conventional Rutherford theory is not applicable. In such cases, we must solve (0,0) mode evolution equations simultaneously. Generally, the anomalous viscosity might depend on the transport model inside the island[10]. The wave-particle interactions play a role of the effective parallel thermal conductivity which sets an upper limit on the perturbed pressure gradient in the island region. As a future work, such effect should be incorporated into the anomalous viscosity.

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