Improved Stability due to Local Pressure Flattening in Stellarators

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Abstract. It is demonstrated that the stability of low n pressure gradient driven modes is improved by introducing local pressure flattening at low order rational surfaces in LHD (Large Helical Device) with the inward magnetic axis shift of 25cm, where n is the toroidal mode number.

1. Introduction

The largest stellarator/heliotron device, called LHD (Large Helical Device), has successfully started physics experiments [1]. The electron and ion temperatures, $T_e \sim 3.8$keV and $T_i \sim 2.8$keV, were obtained in the low density range, $\bar{n}_e \sim 1.5 \times 10^{19}\mathrm{m}^{-3}$ [2]. The obtained energy confinement time was about 50% better than the International Stellarator Scaling of energy confinement [3]. The maximum average beta value, $\bar{\beta} \simeq 2.4\%$, exceeded 2%, which was the highest beta obtained in stellarator/heliotron devices [4]. Since $\bar{\beta} \simeq 2.4\%$ is not limited by MHD instabilities and the target beta value of LHD is 5%, a higher power heating is expected in an optimized magnetic configuration.

In this paper we will discuss effects of local pressure profile flattening on interchange modes which may affect stability and confinement properties of LHD. For studying MHD stability in stellarator/heliotron devices, the Mercier criterion [5] is valuable. For three-dimensional MHD equilibria under the assumption of the existence of flux surfaces, the Mercier criterion is usable for evaluating the beta limit [6]. Another important ingredient for three-dimensional MHD equilibrium and stability is the formation of magnetic islands [7]. This problem is related to the existence of three-dimensional nested flux surfaces [8]. The magnetic islands may be produced by resonant perturbed magnetic fields which are generated by internal resistive MHD instabilities or external error fields. If the magnetic islands appear at low order rational surfaces, it is expected that the pressure profile becomes flat in the island regions. It is shown that the MHD stability based on Mercier criterion changes significantly, although the pressure flattening is highly localized in the neighborhood of rational surfaces [9-11]. For this situation the stability limit of low mode number interchange modes becomes important.

2. Reduced MHD Equations and a Model Pressure Profile with Locally Flat Regions at Rational Surfaces

For analyzing pressure driven instabilities in stellarator/heliotron devices, we use the ideal reduced MHD equations [6] [10] [12], which are written as

$$\frac{\partial \psi}{\partial t} = -(R/R_0)^2 \mathbf{B} \cdot \nabla u \tag{1}$$

$$\rho \frac{d}{dt} \nabla_\perp^2 u = -\mathbf{B} \cdot \nabla(\nabla_\perp^2 \psi) + R_0^2 \nabla\Omega \times \nabla P \cdot \nabla \zeta \tag{2}$$

$$\frac{dP}{dt} = 0 \tag{3}$$

where

$$\mathbf{B}\cdot\nabla = \frac{R_0B_0}{R^2}\frac{\partial}{\partial\zeta} - \nabla\psi\times\nabla\zeta\cdot\nabla \tag{4}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (R/R_0)^2\nabla u\times\nabla\zeta\cdot\nabla \tag{5}$$

$$\Omega = \frac{1}{2\pi}\int_0^{2\pi} d\zeta\left(\frac{R}{R_0}\right)^2\left(1+\frac{|\mathbf{B}-\overline{\mathbf{B}}|^2}{B_0^2}\right) . \tag{6}$$

Here ψ, u and P denote the poloidal flux function, the stream function and the plasma pressure, respectively. The axisymmetric component of the magnetic field is given by $\overline{\mathbf{B}}$ and R and ζ denote the major radius and the toroidal angle, respectively. The magnetic axis is $R = R_0$, and the toroidal field at $R = R_0$ is B_0. Since the free-boundary effect is not significant for the stability of currentless plasmas, a perfectly conducting wall is usually placed at the plasma boundary [10]. It is noted that the equilibrium state of ψ is consistent with the rotational transform profile due to stellarator fields.

For describing the locally flat pressure profile,

$$P(\rho) = C[P_0(\rho) + P_{ax}(\rho) + P_{res}(\rho) - A] \tag{7}$$

is assumed, where $P_0(\rho)$ denotes a smooth and standard pressure profile, $P_{ax}(\rho)$ corresponds to a pressure profile flattening near the magnetic axis given by

$$P_{ax}(\rho) = [P_0(0) - P_0(\rho)]\exp\left[-\frac{1}{2}\left(\frac{\rho}{w_a}\right)^4\right] , \tag{8}$$

and $P_{res}(\rho)$ acts to flatten the pressure at rational surfaces

$$\begin{aligned} P_{res}(\rho) \;=\; & \sum_m \{[P_0(\rho_m) + P_{ax}(\rho_m)] - [P_0(\rho) + P_{ax}(\rho)]\} \\ & \times \exp\left[-\frac{1}{2}\left(\frac{\rho-\rho_m}{w_m}\right)^4\right] . \end{aligned} \tag{9}$$

Here ρ denotes the square root of the normalized toroidal flux. In expression (7), A and C are numerical factors to fix pressures at both the magnetic axis and the plasma surface. In expression (8), w_a denotes the width of a region to make the pressure profile flat near the magnetic axis. Also, in expression (9), ρ_m denotes the position of the m-th rational surface and w_m denotes the width of a region to make the pressure profile flat at the rational surface $\rho = \rho_m$.

In order to calculate fixed boundary MHD equilibria for the LHD configuration with the pressure shown by expression (7), the VMEC code was applied. In the following calculations the LHD configuration with an inward magnetic axis shift of 25cm is assumed. Ideal MHD stability against pressure driven interchange modes was studied with the RESORM code [13], which solves linearlized equations of eqs(1-3) as an initial value problem.

3. Stabilization of Low-*n* Interchange Modes with Flat Pressure Regions at Rational Surfaces in Toroidal Plasmas

For the pressure profile given by Eq.(7) in the LHD model configuration, global pressure driven modes with $n = 1, 2, 3$ are examined with the RESORM code [13], where n is the toroidal mode number.

(I) $n = 1$ mode
The smooth pressure profile $P_0(\rho)$ and the rotational transform profile $\iota(\rho)$ are plotted in Fig.1. The rational surfaces for the $n = 1$ mode, $\iota = 1/1$, and $\iota = 1/2$, are shown with the dotted lines in Fig.1. Here the central beta value is assumed 2%. The unstable $n = 1$ mode exists with a growth rate of 4.846×10^{-2}, which is destabilized at the $\iota = 1/2$ surface. Here the growth rate is normalized by the poloidal Alfvén time. When the flat pressure region with the width w is increased at the $\iota = 1/2$ surface, the growth rate decreases. The pressure profile marginally stable against the $n = 1$ with $w = 0.045$ is shown in Fig.2.

(II) $n = 2$ mode
Here the same pressure and rotational transform profiles as shown in Fig.1 are used for the stability analysis of the $n = 2$ mode. However, the relevant rational surfaces increase; $\iota = 2/2$, $\iota = 2/3$, $\iota = 2/4$ and $\iota = 2/5$. The RESORM code shows that the $n = 2$ mode is destabilized at the two rational surfaces, $\iota = 2/4$ and $\iota = 2/5$. Here the growth rate is $\gamma = 7.187 \times 10^{-2}$ at $\beta(0) = 2\%$. Thus it is required to introduce two locally flat pressure regions with different widths at $\iota = 2/4$ and $\iota = 2/5$ for stabilizing the $n = 2$ mode. When $w = 0.02$ at $\iota = 2/4$ and $w = 0.04$ at $\iota = 2/5$, the instability is suppressed completely and the obtained pressure profile is shown is Fig.3.

(III) $n = 3$ mode
For the LHD configuration with the pressure and rotational transform profiles shown in Fig.1, there are six rational surfaces; $\iota = 3/3,\ 3/4,\ 3/5,\ 3/6,\ 3/7,\ 3/8$. For the case of Fig.1 the RESORM code gives the growth rate $\gamma = 8.233 \times 10^{-2}$ at $\beta(0) = 2\%$, and the unstable mode is localized at the central region with a ballooning structure. These are typical characteristics of the toroidal non-resonant pressure-driven mode [14]. In order to suppress this non-resonant mode, the central pressure profile is flattened first with $w_a = 0.6$ in Eq.(8). Then the growth rate decreases to $\gamma = 6.975 \times 10^{-2}$, and the unstable mode has a typical interchange mode structure destabilized at $\iota = 3/4,\ 3/5,\ 3/6$ and $3/7$. For suppressing the pressure driven interchange mode with $n = 3$ completely, flat pressure regions are generated at the four rational surfaces with $w = 0.03$ at $\iota = 3/7$, $w = 0.03$ at $\iota = 3/6$, $w = 0.02$ at $\iota = 3/5$ and $w = 0.02$ at $\iota = 3/4$. The obtained pressure profile with $\beta(0) = 2\%$ is shown in Fig.4. The average beta value is changed from $\bar{\beta} = 0.632\%$ (see Fig.1) to $\bar{\beta} = 1\%$ (see Fig.4).

It was demonstrated that the pressure driven modes with $n = 1$, $n = 2$ and $n = 3$ can be stabilized by generating the locally flat pressure regions at the relevant rational surfaces separately. Furthermore it is confirmed that the $n = 1,\ 2,\ 3$ modes become stable simultaneously when the pressure profile is described with $w_a = 0.6$ and locally flat regions with $w = 0.025,\ 0.03,\ 0.065,\ 0.03,\ 0.025,\ 0.02$ at $\iota = 0.4,\ 3/7,\ 0.5,\ 0.6,\ 2/3,\ 0.75$, respectively.

4. Concluding Remarks

It is expected that locally flat pressure regions are produced by the non-linear evolution of resistive interchange modes which become unstable for beta values less than the Mercier limit. The other possibility to produce the locally flat pressure regions is external application of resonant helical magnetic fields. It has been demonstrated that pressure driven instabilities with low toroidal mode numbers are stabilized by modifying the pressure profile to make locally flat pressure regions in the LHD model configuration. It is noted that large pressure gradients near the magnetic axis destabilize the non-resonant modes with medium toroidal mode numbers such as $n = 3$ or 4. Therefore, in order to increase the ideal beta limit, broad pressure profiles with several locally flat pressure regions at dominant rational surfaces may be appropriate in LHD.

There are some indications that the experimental beta values exceed the Mercier limit when smooth and monotonic pressure profiles are assumed in CHS [15,16] and Heliotron E [17]. Formation of the above mentioned locally flattened pressure profiles at low order rational surfaces may explain the discrepancy. The important assumption is that the resistive interchange instabilities unstable in the magnetic hill region are responsible for generating such profiles. The other possibility to explain the discrepancy is that high-n pressure driven modes do not play a role due to the finite Larmor radius stabilization [6]. It is noted that the LHD high beta plasma has already obtained $\bar{\beta} \simeq 2.4\%$, which seems to exceed the Mercier limit. Future experiments on LHD high beta plasmas are expected for checking our conjecture.

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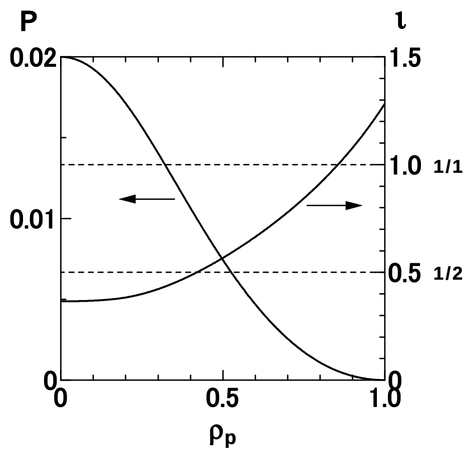

FIG.1. An assumed pressure profile and a rotational transform profile in the LHD configuration with the inward magnetic axis shift of 25cm obtained by the VMEC code[18]. The dotted lines show $\iota = 1/1$ and $\iota = 1/2$. The central beta value is $\beta(0) = 2\%$ and the average beta value is $\bar{\beta} = 0.632\%$. The radius ρ_p denotes the square root of the normalized poloidal flux, which is also used in Figs 2-4.

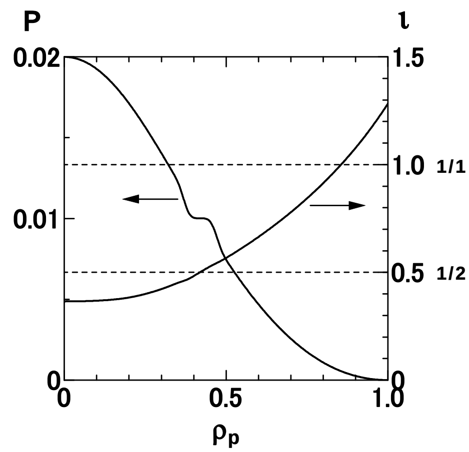

FIG.2. A pressure profile marginally stable against the $n = 1$ pressure driven mode with the width of the flat pressure region $w = 0.045$. The rotational transform profile obtained by the VMEC code, and rational surfaces with $\iota = 1/1$ and $\iota = 1/2$ are also shown.

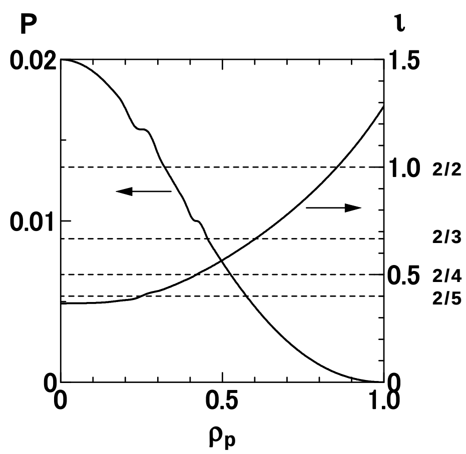

FIG.3. A pressure profile marginally stable against the $n = 2$ pressure driven mode with the flat pressure regions given by $w = 0.04$ at $\iota = 2/5$ and $w = 0.02$ at $\iota = 2/4$. The rotational transform profile obtained by the VMEC code, and rational surfaces with $\iota = 2/2,\ 2/3,\ 2/4,\ 2/5$ are shown.

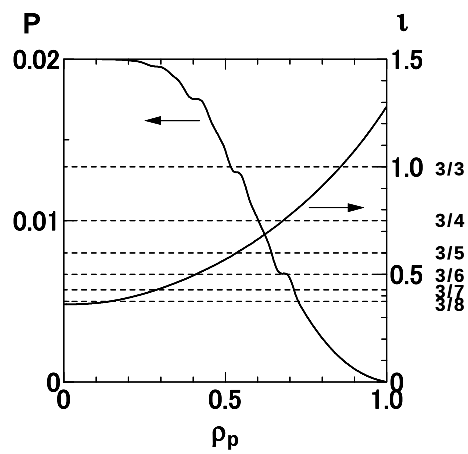

FIG.4. A pressure profile marginally stable against the $n = 3$ pressure driven mode with the flat pressure regions given by $w = 0.03$ at $\iota = 3/7,\ w = 0.03$ at $\iota = 3/6,\ w = 0.02$ at $\iota = 3/5$ and $w = 0.02$ at $\iota = 3/4$. The rotational transform profile obtained by the VMEC code, and rational surfaces with $\iota = 3/3,\ 3/4,\ 3/5,\ 3/6,\ 3/7,\ 3/8$ are shown.