MIXED SOC-DIFFUSIVE DYNAMICS AS A PARADIGM FOR TRANSPORT IN FUSION DEVICES

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ABSTRACT: A recently proposed paradigm for plasma turbulent transport dynamics, based on the concept of self-organized criticality (SOC), is extended to include other transport mechanisms existent in real plasmas. This extension might clarify the experimentally observed violation at fluctuation scales of the scale-invariance essential to the SOC model. It might also provide with new experimental tests that could help to validate its relevance. Finally, it might give some hints to understand the central role played by the plasma edge in transport dynamics.

1 INTRODUCTION

Self-organized criticality [1] (SOC) has been recently proposed as a paradigm for plasma turbulent transport dynamics in fusion devices [2]. SOC systems organize themselves, in the presence of a drive, to fluctuate around a state marginal to major disruption, while still being able to drive significant transport out of the system. This state lacks any characteristic length or time, exhibiting self-similar spatial and temporal spectra leading to power laws. Transport is driven by avalanches of all sizes and durations, whose PDF are also well described by power laws. Key ingredients of SOC systems are: 1) an instability threshold; 2) two disparate time scales, one associated to the drive, and the other to the instability relaxation. It is easy to find candidates for all these elements in a confined plasma. With this in mind, some modes (such as drift waves or ITG at the core, and resistive interchanges or ballooning at the edge) have been proposed as candidates for driving significant avalanche transport while staying subcritical on average, with the plasma behaving as a SOC system. Such a paradigm could help to understand many experimental results not properly accounted for by other models [2]: 1) the strong measured transport even when profiles are on average stable to the suspected responsible instability; 2) the scaling of the confinement in L-mode discharges with the system size; 3) the decorrelation of avalanches by a sheared flow and 4) the universal indexes found in measured fluctuation spectra.

Some experimental evidence consistent with this paradigm has been reported [3]. Results of the analysis of edge electrostatic fluctuation data from several experiments showed self-similarity down to time scales of the order of ten times the turbulence decorrelation time and strong evidence of radial correlations over distances longer than the turbulence correlation length. Experiments supporting the radial propagation of avalanche-like events have also been reported in the DIII-D tokamak. But at the same time, some groups have reported on small deviations from self-similarity when going down to fluctuation scales, which seem to compromise the validity of the SOC assumption [4]. In this paper we will show that these small scale-invariance violations are however to be expected, since a number of other active transport mechanisms exist in these plasmas that drive a continuous outflux that might break the SOC scale-invariance [5]. These mechanisms can interact with the dominant SOC dynamics, efficiently erasing certain ranges of temporal and spatial scales. To study this interaction, we split the outgoing flux in a subdominant continuous flux, referred to as the **diffusive channel**, to which all collision-driven mechanismsand the supercritical turbulence contribute, and a dominant discontinuous one, referred to as the avalanche or **SOC channel**, to which the sub-critical turbulence contributes.

2 DIFFUSIVE-SOC SANDPILE RESULTS

2.1 Sandpile model

To identify the characteristic traces of the interplay between these two transport channels we turn to a simpler model, that of a standard running sandpile [6]. We have modified it to include a diffusive transport component which may interact with varying relative intensity with the dominant avalanche-like one. The SOC character of the sandpile dynamics comes from the existence of a critical slope, $|Z_c|$ that when locally overcome, gives rise to the removal of the excess sand to adjacent positions. The sandpile domain is divided into L cells. In each of them, labeled by the index n with $0 < n \leq L$, the amount of sand is h_n. The sandpile state is evolved as follows. First, a grain of sand is added randomly to the cells with probability P_0. Next, all cells are checked for stability by comparing the local gradient, Z_n, to the critical gradient; finally, the cells are time advanced, with the unstable cells overturning and moving a prescribed amount of sand, N_f, to the next cell: $h_n = h_n - N_f$ and $h_{n+1} = h_{n+1} + N_f$. Typical values in the simulations are $L = 400$, $Z_c = 200$ and $N_f = 30$. The boundary condition is open at $x = L$ and closed at $x = 0$. To this standard automata model we have added a continuous diffusive component in the following way: a diffusive flux at the n-th cell is computed as $\Gamma_n = D_0 \left(Z_{n+1} - Z_n\right)$, with the flux going to the $(n+1)$-th cell when positive and to the $(n-1)$-th cell otherwise.

By assuming a spatially uniform sand rain, the sandpile domain will be split into two distinct subdomains. In the one extending inwards from the boundary at $x = 0$, most of the transport will go through the diffusive channel. In the other subdomain, which extends inwards from the boundary at $x = L$, transport will be mainly driven by avalanches. They will be linked by a crossover region, where both channels have similar intensities. The location of the crossover region can be easily estimated as $x_t^* = x_t/L = (|Z_c|/L)(D_0/P_0)$, which agrees well with the simulations. This subdomain structure is maintained if the condition $x_t^* < 1$ holds. In terms of the free parameters of the simulation this requires $D_0/P_0 < L/Z_c$, which sets an upper limit for the diffusivity ratio, D_0/P_0. At the same time, it is easy to see that x_t^* is approximately equal to the fraction of the total average transport leaving the sandpile through the diffusive channel.

2.2 Break up of scale-invariance in avalanche transport

The change in the properties of the avalanche transport when diffusivity is increased can be seen in the time evolution of $g(t)$, which gives the total number of overturning (i.e., unstable) sites at each iteration. It is thus a measure of the SOC activity taking place in the system. For the pure SOC case, this evolution is strongly bursty, showing scale-invariance up to system scales, with events of all possible sizes and durations. In contrast, for increasing diffusivity, the evolution of $g(t)$ becomes a more intermittent constant size signal. The transition from burstiness to intermittency is caused by the smoothing of the local slope profile inhomogeneities by the subdominant diffusion, which makes it increasingly more difficult for avalanches to take place. For small values of the D_0/P_0 ratio, the diffusive component cannot however completely balance the source at a submarginal level, causing the slope to build up to a point at which a larger avalanche event can occur. On the other hand, the constant size of the signal is associated with the appearance of a characteristic spatial scale of order $L - x_t$. Its nature will be clarified later.

2.3 Power spectrum of the global avalanche transport

One of the more useful tools to characterize the dynamics of SOC systems is the power spectrum of the time series of $g(t)$, $P(w)$. In the pure SOC case, this spectrum can be split in several distinct regions where it follows power laws, $P(w) \propto w^{-\alpha}$, with varying universal exponents α [6]. Each of these regions is related to a different type of events taking place in the system: 1) the region at low frequencies with $\alpha = 0$ corresponds to large catastrophic single events

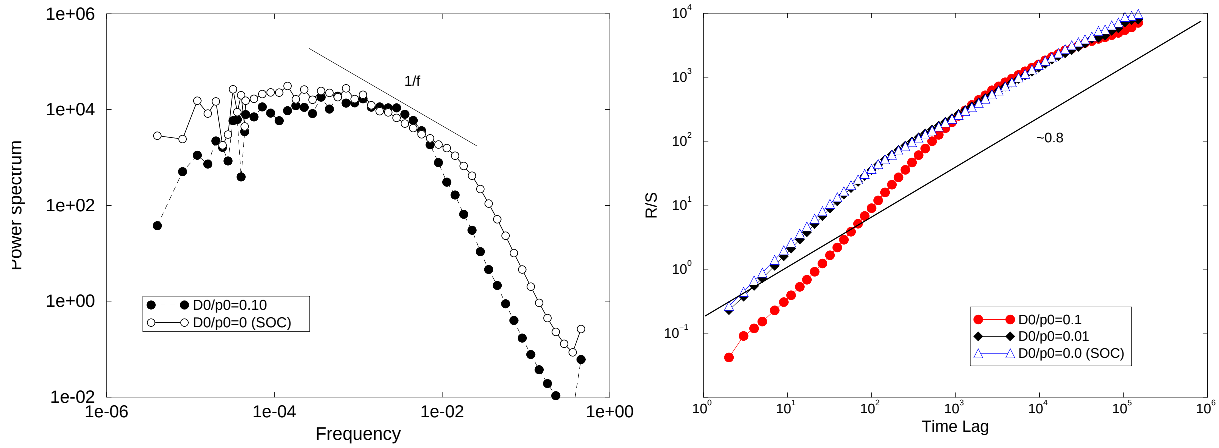

Figure 1: Power spectrum (left) and R/S analysis (right) of the time evolution of the total number of overturning sites for increasing values of the diffusivity ratio, D_0/P_0.

that involve all the system; 2) the region at intermediate frequencies with $\alpha = 1$ is related to avalanche overlapping; 3) finally, for large frequencies, a region with exponent larger than 2 exists, corresponding to small scale events. When diffusivity increases (see Fig. 1), the $1/f$ region of the spectrum begins to shrink and eventually disappears over a threshold value as low as $D_0/P_0 = 0.1$!!. Notice that transport is then still strongly dominated by avalanches. This surprising result suggests that diffusion can very efficiently reduce avalanche overlapping, even at much smaller values than needed for taking over control on the global transport. The smoothing of the local profile inhomogeneities and the concomitant decorrelation carried out by the diffusivity seems thus to be able to erase the system *memory* contained in the slope profile, preventing thus the triggering of avalanches. And notice that the flat low frequency region extends to higher frequencies suggesting that increased diffusion enhances isolated avalanches.

2.4 Existence of long-term time correlations in avalanche transport

Another characteristic feature of SOC dynamics is the appearance of long-range time and space correlations. These correlations can be somewhat identified by looking at the autocorrelation function of $g(t)$, $\Delta(\tau)$. The ocurrence of algebraically decaying tails in the autocorrelation function has long been used as a proof of the existence of long-term correlations in the system. For the standard SOC case, this algebraic tail is apparent. However, important changes take place in the autocorrelation function behaviour when D_0/P_0 increases since the algebraic tail vanishes very quickly, for values of the diffusive ratio consistent with the other diagnostics.

Another method to look for long-term correlations is the rescaled adjusted range statistics (R/S analysis) proposed by Mandelbrot and Wallis based on a previous work by Hurst [7]. The R/S analysis of a time series is carried out by computing the R/S ratio as a function of the time lag, n. For some range of n, this ratio may possibly scale as n^H, where H is the Hurst exponent. A constant value of the H parameter over a long range of time lag values is a signature of the self-similarity of the signal in that range. The value of H, on the other hand, unveils hidden long-range dependencies: $H > 0.5$ implies long-term correlations, $H < 0.5$ reveals anti-persistence while $H = 0.5$, a totally random signal. In the standard SOC sandpile, a value for the Hurst exponent close to $H = 0.8$ is obtained, that extends for more than two decades (see Fig. 1). When the diffusivity ratio is increased, the situation changes consistently with other diagnostics: the range over which the R/S ratio scales linearly begins to decrease very quickly, eventually ceasing to exist above the same threshold value, $D_0/P_0 \sim 0.1$, that precluded the existence of the $1/f$ region.

2.5 Role of the sandpile edge in the transport dynamics

The central role that the edge comes to play in SOC-diffusive dynamics is revealed by the change in the PDFs of the sandpile locations where avalanches are inititated or terminated. In the pure SOC case, they are rather flat since the uniform drive makes every location as good as any other to become unstable and inititate an avalanche or to stop an oncoming one (see Fig.2). But when diffusivity increases, it smooths out small scales first and therefore the number of small avalanches diminishes much faster than the number of larger avalanches Amazingly, the PDFs of the start-up and terminating locations of the remaining avalanches becomes strongly become respectively peaked at the edge and at the transition point, x_t (see Fig. 2). This suggests that somehow, most avalanches are initiated at the edge and manage to propagate all through the SOC region. The physical reason for this behaviour is related to the higher average slope existent at the edge: as a result of the instantaneous removal of sand from the last cell, some allowed slope reductions are not possible at the edge. For instance, the slope can be reduced by -1 if a grain of sand falls in the next location. But this slope transition is not allowed at the edge. Similarly, the edge only reduces its slope by $-N_f$ when unstalbe, while the other locations reduce it by $-2N_f$. In the standard SOC case, this steeper edge region only involves the last two points, but it expands inwards to accomodate the outgoing flux for increasing diffusivity. It thus becomes the most probable location for initiating avalanches which, thanks to the previous diffusive smoothing of the inner slope profile, will find no obstacle to propagate all across the SOC region.

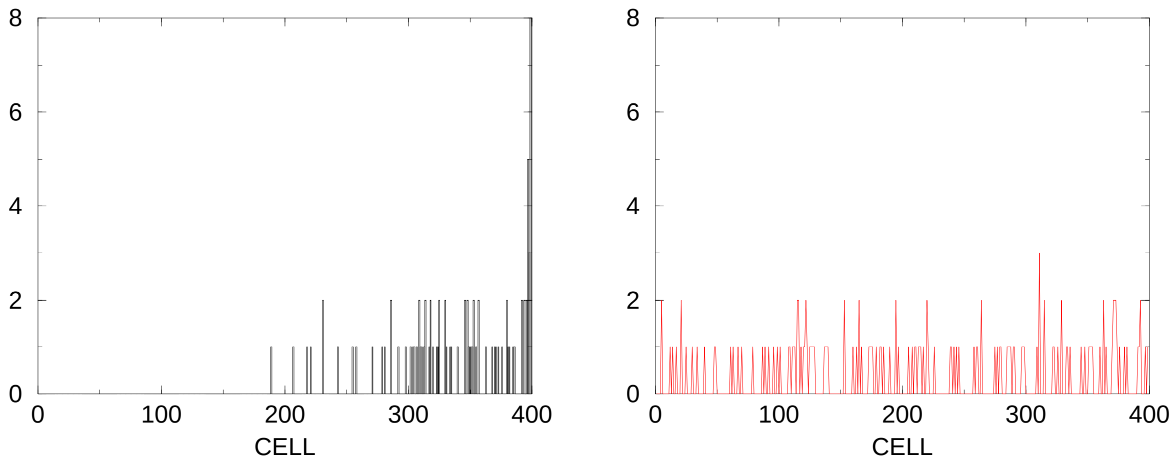

Figure 2: PDF of start-up locations for avalanches for $D_0/P_0 = 0.5$ (left) and $D_0/P_0 = 0$ (right).

3 CONCLUSIONS

As previously mentioned, it has been recently proposed that plasma transport (at least in L-mode discharges) might be dominated by a sub-critical turbulent mechanism of locally varying intensity (an even nature). The plasma would then stay in a SOC-like state, with avalanches of all sizes driving energy and particles out of any unstable location in order to bring the profiles below the instability threshold. However, no definitive experimental test to confirm this point is possible, since even the definition of what constitutes a SOC system is still a matter of discussion. Besides, some skepticism exists on the expectations of being able to test the system dynamics just by edge probing. As a consequence, a great controversy has developed within the fusion community, and clashing opinions exist on whether the available experimental evidence support or invalidate the SOC concept as a paradigm for plasma transport [3,8].

We think that some of the results of this paper might help answer some of this skepticism. It is clear from the simulations that, in a SOC-diffusive system the SOC-like region will extend inwards from the edge. Therefore, it seems to make a lot of sense to test for possible SOC behaviour by probing the edge region (which is certainly fortunate, considering the difficulty of taking measurements further inside the plasma). At the same time, we have been able to identify the trends that a SOC-diffusive system would follow when the relative magnitude of the diffusive to the avalanche transport is varied. Tests of this kind could be carried out at the edge of tokamaks or stellarators by artificially varying the relative intensities of both channels. This could be possibly done by taking fluctuation measurements at different radial positions, to take advantage of their different local dependence. Another (more complicated) way could be to take measurements at a fixed radial location for a set of discharges in which the configuration could be changed in a controllable way: for instance, reducing the magnetic well at the edge, that effectively would increase the turbulence levels due to resistive pressure-driven-turbulence. If this extended SOC paradigm applies, clear signatures of the interaction of the two channels should then be revealed from a statistical analysis of this edge data. These signatures should include the reduction of the $1/f$ region of the fluctuation power spectrum, the disappearance of algebraic tails in the fluctuation autocorrelation function. At the same time, the linear region in the R/S analysis should also vanish.

Finally, we will discuss what we think is the most important result of this paper: the central relevance of the edge for SOC-diffusive dynamics. In a pure SOC system, the edge is not particularly significant. But in SOC-diffusive systems, even for small diffusion, we have found that the edge comes to play a dominant role, with most of the big transport events triggered from it. These events penetrate very deeply in the system, making the core confining properties strongly dependent of the edge conditions. This might be the reason why core confinement in fusion devices is so critically affected by a good conditioning of the edge, as long known from many experiments. And it could probably shed some light on the role that neutrals might play as a trigger for catastrophic avalanches. At the same time, it gives us further confidence to pursue edge probing to characterize the SOC character of a confined plasma. As a last comment, the steepest edge region might be considered as a natural definition for a plasma edge.

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