Edge Transport Modulation by Coherent Shear Flows

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Abstract. Different from the core, in the transitional core/edge regime a rich variety of distinct radially and poloidally coherent flow structures have been found by numerical and analytic studies, ranging from solitary propagating zonal flows to sinusoidal structures with definite radial wavenumber. Toward the edge, the fluctuating flows get weaker until the coherent component of the flows vanishes completely, which is analytically analogous to the Bose-Einstein phase transition. The coherent flow structures strongly modulate the transport via wave-kinetic effects. The clear signature and the transport relevance of the coherent flows in the transitional regime make them an ideal target for experimental investigation.

1 Introduction

The energy confinement of tokamaks is mainly controlled by small scale ($\sim$cm) turbulence giving rise to the "anomalous transport". Analytical and computer-aided studies have shown that the anomalous transport in turn is often controlled by fluctuating "zonal flows" [1, 2, 3], vortices whose cross-sections are very elongated in poloidal (along the minor angle) direction and have radial scales similar to the ordinary turbulence. In the core, they are poloidally coherent (0,0)-modes but exhibit no pronounced structure in radial direction [4]. In contrast, in the transitional core/edge regime a rich variety of distinct radially and poloidally coherent flow structures have been found numerically, ranging from sinusoidal structures with definite radial wavenumber to solitary propagating zonal flows. Locally the flows strongly modulate the turbulence due to wave-kinetic effects. In conjunction with the toroidal coupling of pressure perturbations to the poloidal rotation (Stringer-Winsor term) this results in a new flow drive mechanism. Stepping farther out to the edge, the fluctuating flows tend to get weaker until the coherent flow component vanishes and only poloidally and radially incoherent flows remain.

For cost reasons, the domains of turbulence simulations are usually thin flux tubes along the magnetic field lines [5]. The flux tube dimensions perpendicular to the magnetic field are chosen to accomodate the prevalent turbulent structures. For the elongated zonal flows, however, these computational domains are not always adequate and the flows *appear* to extend across the complete flux tube. We analyze the question, whether this is true for arbitrarily large ratio of flux surface circumference to turbulence scale length (poloidally coherent flows) or whether the flows have a finite scale length in poloidal direction (poloidally incoherent flows). This question is particularly relevant for tokamak edge turbulence, which has very small scales in comparison with the flux surface circumference. For this purpose, domains up to the equivalent of a full flux surface of the JET tokamak have been used in turbulence simulations.

2 Basic considerations

Before the numerical results we discuss the basic nature of the excitation mechanism of the zonal flows. First of all we have to distinguish between a local or nonlocal mechanism in wavenumber space. In the first case the energy is transferred in small steps in k space towards smaller wave-numbers until the zonal flow poloidal wave-numbers are reached. In this case the zonal flows are really a part of the turbulence. This scenario is well known as the inverse energy cascade, and occurs, e.g., in 2D Navier Stokes turbulence. Nonlocal excitation is due to the nonlinear interaction of disparate scales. In this case, the flows are excited directly by the small scale eddies without intermediate scale structures as mediators. An example for this process is flow excitation by negative turbulence viscosity [1, 6, 7]. Since the tokamak zonal flows in any case have $k_\theta \ll k_r$ the nonlinear interaction terms are very small for local interactions, and we feel justified to assume a nonlocal generation process for the following, especially, since it is also more consistent with the flow wave-number spectra.

In the nonlocal scenario we can distinguish between incoherent and coherent generation of the flows. In the first case (analogous to spontaneous emission in atomic physics) the randomly interacting small scale eddies generate flows without any phase relation to the already existing flows. For the coherent generation, seed flows distort the small scale turbulence, which as a response amplifies the initial flows in a fixed phase relation to the original flows. The incoherent excitation is supposed to average out as the system size is increased compared to the eddy size. Therefore only coherently generated flows can survive in large enough systems such as realistic tokamaks. Usually we expect coherent and incoherent excitation to be present simultaneously. Their action on the set of eddies with large poloidal scale length can thus be described analogous to the quantum mechanical boson statics. Depending on the parameters, partial condensation into the (0,0) modes can occur (analogous to the Bose Einstein condensation). Only the condensed (0,0) flow component is independent of the system size, while the rest of the (0,0) flows disappears, as the incoherent forces average out.

As for the coherent turbulence response to the flows one can distinguish between a wave-kinetic response and a linear turbulence response. In the latter case, the turbulence slowly adjusts to the zonal flows, e.g., by a locally altered growth rate, while in the first case, the distortion of the turbulence eddies due to the flow directly changes their properties, e.g., by compressing or moving them in space.

In an electrostatic model there are essentially two drive terms for poloidal $\mathbf{E}\times\mathbf{B}$ flows. The first is the radial divergence of the Reynolds stress [8]

$$\delta_{Reynolds}\langle \dot{v_E}\rangle = \partial_x \langle v_E v_{r,\mathrm{ion,total}}\rangle \tag{1}$$

with v_E denoting the poloidal $\mathbf{E}\times\mathbf{B}$ velocity, $\langle . \rangle$ denoting the flux surface average, and $v_{r,\mathrm{ion,total}}$ denoting the total radial ion velocity including diamagnetic and $\mathbf{E}\times\mathbf{B}$ velocity. The second is the torque on the plasma column due to the force of the magnetic field inhomogeneity on the $(1,0)$ pressure perturbations, the so called Stringer Winsor (SW) term

$$\delta_{StringerWinsor}\langle \dot{v_E}\rangle = \langle p \sin\theta\rangle, \tag{2}$$

with p being the pressure, and θ the poloidal angle. The SW term can give rise to the Stringer spin up instability [9]. In the turbulence studies it is however found that the Stringer spin up growth rate is always negligibly small due to the slow parallel sound wave which is needed as a mediator to generate the pressure fluctuations driving the flows via (2).

3 Numerical Model

Numerical turbulence simulations have been carried out using the three dimensional electrostatic drift Braginskii equations with isothermal electrons (a subset of the equations of Ref. [10]) for parameters resembling the resistive ballooning regime (a) and the transitional core/edge

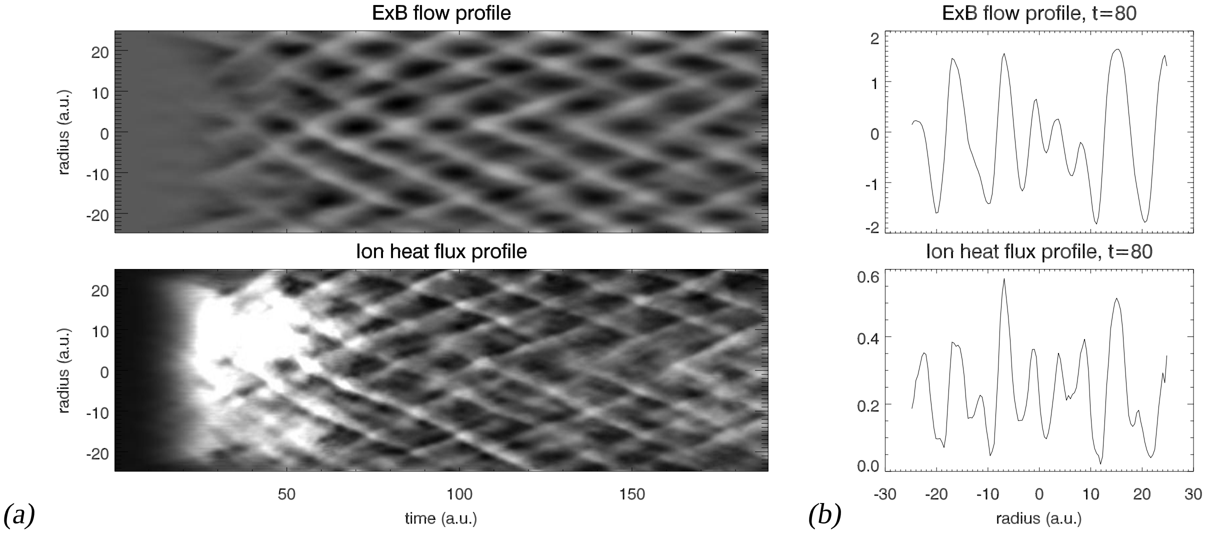

Figure 1: $\mathbf{E}\times\mathbf{B}$ *flow profile and heat flux profile as function of time (a) and at* $t = 80$ *(b).*

regime (b), where most of the heat flux is caused by ITG modes. The nondimensional parameters in case (a) were $\alpha_d = 0.2$, $\varepsilon_n = 0.08$, $q = 5$, $\tau = 1$, $\eta_i = 1$, $\hat{s} = 1$, and in case (b) $\alpha_d = 0.4$, $\eta_i = 3$ and the other parameters the same. The radial domain width was $24L_{\mathrm{RB}}$ (a) and $48L_{\mathrm{RB}}$ (b), for the poloidal coherence studies, the width L_θ perpendicular to $\mathbf{r}$ and $\mathbf{B}$ was changed between $24L_{\mathrm{RB}}$ (only for (a)), $192L_{\mathrm{RB}}$, $384L_{\mathrm{RB}}$, and $768L_{\mathrm{RB}}$ corresponding to a tokamak with minor radius $r = L_\theta q/2\pi$. For a definition of these parameters see Ref. [10]. The parameters of the largest domains are consistent with the physical parameters $R = 3$ m, $r = 1.5$ m, $L_n = 12$ cm, $n = 3.5\times10^{19}\ \mathrm{m}^{-3}$, $Z_{\mathrm{eff}} = 4$, $B_0 = 3.5$ T, and for (a) $T = 100$ eV, $L_{\mathrm{RB}} = 5.1$ mm, $\rho_s = 0.58$ mm and for (b) $T = 200$ eV, $L_{\mathrm{RB}} = 3.6$ mm, $\rho_s = 0.82$ mm. The perpendicular grid resolution was $5.3/L_{\mathrm{RB}}$ (a) and $2.6/L_{\mathrm{RB}}$ (b). Parallel to the magnetic field 12 points per poloidal connection length were used. The largest runs had a grid of $128\times4096\times12$. To check convergence, runs with 32 points along the magnetic field per poloidal connection length have been performed, too.

4 Radial flow structures in the transitional regime

Fig. 1 (a) shows the flux surface averaged ion heat flux and the flow amplitude as a function of radius and time for a simulation run with the reference parameter set (b). After a transient initial phase the flows together with the average turbulence level approach a relatively well defined stationary equilibrium value. Apparently, the major part of the transport fluctuations is controlled by the flows as each moving flow is accompanied by a transport front. From an instantaneous plot of the radial flow pattern and the corresponding heat flux profile [Fig. 1 (b)] one concludes, that the transport is nearly completely trapped in the regions with flow into the electron diamagnetic direction (positive sign).

It has been found, that each flow (and transport) maximum is accompanied by a radial double layer of the up-down asymmetric part of the ion heat flux $\langle v_r T_i \sin\theta\rangle$, with the radial $\mathbf{E}\times\mathbf{B}$ velocity v_r and the ion temperature T_i. This asymmetric transport component is therefore exactly in the right radial position to drive the flows via equation (2). According to the energy balance the drive of the flows due to this asymmetric transport component in conjunction with the SW term has been found stronger than the Reynolds stress drive.

These observations have been found qualitatively unchanged in the range $0.4 < \alpha_d < 3.6$, $0.04 < \varepsilon_n < 0.2$, $2 < q < 5$, $1 \leq \eta_i \leq 5$. There are however strong variations of the amplitude of the flows and the strength of the turbulence modulation. For the resistive ballooning regime case (a), we do not get the concentration of turbulence within the flow maxima in a certain direction. Instead, the turbulence is simply suppressed in the regions of maximum flow shear. The absence of the coherent flow drive in the ballooning case leads to radially incoherent flows.

To clarify the nature of the heat flux modulation in the transitional regime, several numerical experiments have been performed. Most important it is necessary to check on one hand that the zonal flows are the cause of the flux modulation and that both are not the consequence of a third phenomenon, and on the other hand that the modulation is not a side effect of the flows, such as a compression effect caused by the toroidal geometry which actually modulates the transport. For this purpose arbitrary radial flow profiles were enforced onto the system in a fully turbulent state, while simultaneously the density and temperature averages over the toroidal angle, $\langle n\rangle_\phi(r,\theta), \langle T_i\rangle_\phi(r,\theta)$, were kept fixed at the respective flux surface averages.

To approximate the stationary turbulent state of the system, first a propagating sinusoidal flow profile with similar amplitude, frequency and propagation velocity was used. This robustly results in the same transport modulation as in the full system for a broad range of flow amplitudes, frequencies and propagation velocities. I.e., the flows are indeed the cause of the transport modulation.

To find out, whether the modulation can be explained by a local change of the linear growth rate similar to the one analyzed in Ref. [11], the turbulence fluctuations were set to zero everywhere in a fully turbulent state except for a single flow peak. The (artificial) propagating flow is then still causing a transport front, which would be impossible, if only the linear growth rates are modified. In that case a local modulation of the turbulence intensity would have been expected. Instead, the outcome of the experiment suggests that a wave-kinetic effect modulates the heat flux.

The behaviour of the ITG modes can be qualitatively understood already in the simple drift wave model of [1]. The dispersion relation of adiabatic driftwaves is

$$\omega = \frac{\omega_d k_\theta}{1+(k\rho_s)^2} \tag{3}$$

resulting in the radial group velocity

$$v_g = \frac{\partial\omega}{\partial k_r} = -\omega\rho_s^2 \frac{k_\theta k_r}{1+(k\rho_s)^2} \tag{4}$$

A zonal flow v_E distorts a linear drift wave according to

$$\partial_t k_r = -k_\theta v_E', \qquad k_y = const \tag{5}$$

The acceleration of the group velocity is therefore always in the direction of increasing flow velocity in electron diamagnetic direction,

$$\partial_t v_g = \omega\rho_s^2 \frac{k_\theta^2 v_E'}{1+(k\rho_s)^2}. \tag{6}$$

Therefore the turbulence modes are attracted towards shear flow maxima in electron diamagnetic direction, which explains the numerical observations.

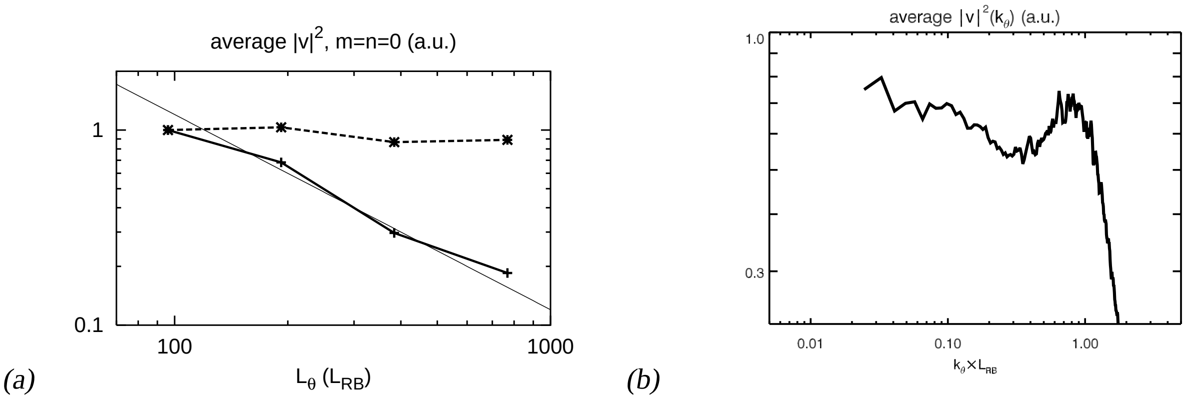

Figure 2: *Part (a): Mean square $m = n = 0$ shear flow amplitude per poloidal area as a function of poloidal domain size L_θ for case (a) (solid) without condensation and case (b) (dashed) exhibiting condensation; the thin line is proportional to $1/L_\theta$. Part (b): Mean square shear flow amplitude as a function of k_θ for case (a) for $L_\theta = 768L_{\mathrm{RB}}$. Note the greatly different scale lengths of high-k_θ turbulence and low-k_θ shear flows.*

The up-down asymmetric modulation of the transport is analogous to adiabatic wave compression. For the drift waves, the action invariant [12] is

$$\phi^2(1+(k\rho_s)^2)^2, \tag{7}$$

where ϕ is the fluctuating potential of the drift wave. Hence decreasing k_r increases the drift wave amplitude. According to (5) this is the case for $k_r k_\theta v_E' > 0$. Due to magnetic shear, the mode structure is approximately

$$k_r \sim k_\theta \hat{s}\theta. \tag{8}$$

Amplification occurs thus for $\hat{s}\theta v_E' > 0$, which is exactly what is observed, even for simulation runs with negative magnetic shear.

5 Poloidal coherence of zonal flows

After the study of transport modulation and the radial flow structures, we turn now to the question of flow coherence in the poloidal angle direction.

The dependence of the average $(0,0)$ shear flow energy density on the domain size, L_θ, is compared for the resistive ballooning (a) and the transitional regime (b) in Fig. 2 (a). In contrast to case (b), the $(0,0)$ mode energy density decreases proportional to $1/L_\theta \propto 1/a$, apparently because the flows are incoherently generated and the random forcing averages out for increasing domain size. In another perspective, the flows in (a) exhibit a finite poloidal scale length, and there is no condensed (0,0) component of the flow spectrum. The intensity decay with increasing domain width results because the given shear flow energy density is distributed equally among an increasingly dense set of modes.

The k_θ spectrum of the poloidal flow velocity v_E, for the $L_\theta = 768L_{\mathrm{RB}}$ runs [for case (a) see Fig. 2 (b)] exhibits a rise at low k_θ associated with the shear flows, different from the microturbulence fluctuations at $k_\theta \sim 1$. The square amplitude of the $m = 0$ mode in case (a) and (b) is 0.3 and 7 times, respectively, the total shear flow amplitude, suggesting strong condensation for (b). In both cases, the typical poloidal scale length of the $m \neq 0$ shear flows is roughly a factor

10 greater than the scales of the turbulence. Failure of the computational domain to accomodate the finite scales of the uncondensed shear flows in case (a) results in an overestimate of the shear flow amplitude, and hence in an underestimate of the anomalous transport. The particle flux for $L_\theta = L_r = 24 L_{\mathrm{RB}}$ was found to be 25% lower than for $L_\theta = 768 L_{\mathrm{RB}}$.

The poloidal zonal flow spectra can be understood in a model, in which their poloidal and radial wavenumber spectra are controlled by the interplay of damping by the collisional electron response and ion dissipation, the linear response of the turbulence to the flows, and the excitation of zonal flows by random fluctuations. Under certain conditions, a finite fraction of the flow amplitude *does* condense into zonal flows with zero poloidal and toroidal mode numbers (m,n), *regardless of the system size*. The mechanism is analogous to the Bose-Einstein condensation (BEC). The three effects acting on the flows take the role of absorption, stimulated, and spontaneous emission. For the BEC, a finite fraction of the quanta is eventually scattered into the ground state because the state density *near* the ground state is too low to hold an arbitrary amount of quanta under the prevalent conditions. In the zonal flow case, the total shear flow energy is regulated by the turbulence such that the average growth rate of the turbulent modes is zero. It can be shown [13] that a finite fraction of the zonal flows "condenses" into (0,0) modes as soon as a threshold in required flow amplitude is exceeded, and the $m \neq 0$ modes are unable to receive it.

6 Conclusions and consequences

Radially coherent zonal flows have been observed in numerical turbulence simulations of the transitional regime between core and edge. Due to their clear signature – coherent oscillations with well defined frequency and wavenumber – they should be an interesting target for experimental scrutiny. The flows are essentially geodesic acoustic modes, which is possible because the parallel sound wave is too slow to short-circuit the pressure perturbation associated with the GAMs. The flows are primarily driven by the up-down asymmetric component of the anomalous transport in conjunction with the Stringer Winsor effect, and not by Reynolds stress or indirectly by the mediation of the parallel sound wave as in the conventional Stringer spin up instability.

The flows strongly modulate the transport locally in space and time by wave-kinetic effects. The maxima and minima of the flux surface averaged transport are associated with the maxima of the flows in electron and ion diamagnetic direction, respectively. This is caused by an attractive force exerted by the flow in electron diamagnetic direction upon the turbulence. Furthermore, wave compression by the flow shear leads to the up-down asymmetry in transport which drives the flows. Both effects are absent for vanishing diamagnetic drift velocity and gyro radius, such as in the resistive ballooning regime, resulting in only weakly driven incoherent flows.

Regarding the poloidal coherence, it has been shown numerically that in the case of incoherent flow drives, the shear flows controlling the turbulence are not only $(0,0)$ modes but rather consist of a spectrum of poloidal mode numbers. The $(m,n) \neq (0,0)$ flows differ from drift waves or convective cells by their large poloidal [10 times larger than the turbulence (Fig. 2 [b])] and parallel scale length, while their perpendicular scale length is similar to that of the turbulence. In the limit of large system size, a non-zero (0,0)-mode amplitude develops only if the shear flows undergo a condensation into these modes, analogous to the Bose–Einstein condensation. Up to now, in flux tube based turbulence computations the zonal flows were implicitly assumed to be global modes. With domain widths inadequate to the rather large poloidal scales of the zonal flows, the uncondensed phase of the zonal flows *appears* to have zero poloidal and toroidal mode number. Such modes do not experience the resistive damping, which would reduce the flow amplitude in a full system. Hence, the simulations tend to overestimate the total flow amplitude. Because often the anomalous transport critically depends on the zonal flows, for quantitative results, flux tube simulations have to be checked for influences of a finite poloidal scale length of the zonal flows.

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