The Reversed Field Pinch as a Magnetically Quiet and Non Chaotic Configuration

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Abstract. Recent progress in experiments open a path beyond the standard paradigm that a bath of magnetic turbulence is intrinsic to the reversed field pinch (RFP): quasi single helicity (QSH) states have been found in several RFP's. This motivates a thorough theoretical study of the single helicity (SH) states of the RFP which correspond to a laminar dynamo produced by a single mode, and an integrable magnetic field with good flux surfaces, a feature favourable to good confinement. Numerical simulations of visco-resistive MHD reveal a bifurcation from SH to multiple helicity related with temporal intermittency, which is ruled by the product of resistivity by viscosity. Furthermore a mechanism of magnetic chaos healing is shown to exist when the magnetic separatrix of the dominant mode of QSH states disappears due to a saddle-node bifurcation.

1. Introduction

The Reversed Field Pinch (RFP) is one of the configurations for magnetic confinement of thermonuclear fusion plasmas. Like in a tokamak, a toroidal current flows in the plasma, but the toroidal field has an amplitude similar to the poloidal one, and reverses in the outer region. Recent progress in experiments and theory open a path beyond the standard paradigm that a bath of magnetic turbulence is intrinsic to the RFP: in particular in RFX, the largest present RFP experiment, the existence of plasma states with a hot helical core has been proven by soft X-ray tomography [1,2,3]. These states correspond to a quasi single helicity (QSH) magnetic spectrum where one mode with m=1, and a given n dominates over all others. Such magnetic features have been seen transiently in several RFP's and during whole discharges in RFX. This motivates a thorough theoretical study of the single helicity (SH) states of the RFP. A further motivation is the link of these states with the q=1 mode of the tokamak and with the helical states of the stellarator. The theoretical approach reveals a bifurcation in visco-resistive MHD [4,5,6] related with temporal intermittency, which places this problem at the forefront of modern nonlinear dynamics. Furthermore a mechanism of magnetic chaos healing is shown to exist when the magnetic separatrix of the dominant mode of QSH states disappears due to a topological saddle-node bifurcation.

2. Pure Single Helicity States

First we recall that toroidal field reversal requires the loss of axisymmetry. This is easily shown for a cylindrical RFP. Indeed if cylindrical symmetry is assumed, the parallel Ohm's law implies that the reversal of the toroidal field means that of the parallel current. As, according to Ampère's law, the azimuthal component of the current is the opposite of the radial derivative of the axial field, the current reversal means that the axial field is minimum at its reversal point, a contradictory statement. It is interesting to notice that Cowling theorem [7] has been traditionally invoked to explain why an axisymmetric RFP is impossible. This theorem states that axisymmetric magnetic fields cannot be maintained by axisymmetric dynamo action, i.e. by a given axisymmetric velocity field. It does not apply to stabilised z-pinches, since a non reversed axisymmetric paramagnetic pinch may be sustained by an axisymmetric pinch velocity field. Indeed the poloidal part of the field of stabilised z-pinches is naturally provided through the forced toroidal (axial) current, their velocity field is not directly driven, and their kinetic energy is much smaller than their magnetic energy. However for the RFP a deformation of the plasma with at least one helicity must be present. The kink instability is the natural origin of this deformation since $q<1$.

The possibility of having a RFP plasma in a pure SH state was put forward since 1983 through two-dimensional numerical simulations [8,9,10,11,12] where a stationary RFP state was found by forcing SH. The SH states have a laminar dynamo produced by a single mode and its harmonics. They correspond to a magnetic field with good flux surfaces, a feature favourable to good confinement. They are not Taylor states, since the sign of their helical pitch is opposite to that in Taylor's theory [13]. In fact, an intuitive description of the magnetic field self-reversal process allows to view the SH state as the nonlinear state of a resistive kink mode self-stabilised by the outer toroidal field reversal, if toroidal flux conservation is imposed in the relaxation process [14]. In a tokamak the stabilisation of the m=1 kink mode is made possible by the plasma itself because the helical deformation it drives costs energy in the whole $q>1$ domain. No such mechanism is available in the RFP where $q<1$ everywhere, and the outer field reversal is necessary to stabilise the kink mode.

The loss of axisymmetry of the magnetic surfaces induces a modulation of the current density along field lines. This modulation is driven by an electrostatic electric field produced by charge separation. This electric field and the induction electric field produce an $\boldsymbol{E}\times\boldsymbol{B}$ velocity field which is the dynamo velocity field of the RFP [14]. Therefore the origin of the dynamo in the SH state of the RFP is a mere consequence of the pinch effect and of the breaking of axisymmetry due to the resistive kink mode: the helical magnetic equilibrium has an electrostatic helical counterpart which provides the helical part of the dynamo velocity field, a slaved laminar field. This picture is the physical interpretation of the scheme proposed in reference [6] for the calculation of SH states.

The SH ohmic states of the RFP were looked for in the framework of the resistive MHD model in cylindrical geometry with and without pressure [6,15,16]. In the force free case we solved the Grad-Shafranov equation in helical coordinates by assuming a polynomial dependence of $\lambda=\boldsymbol{J}\bullet\boldsymbol{B}/\boldsymbol{B}^2$ on the helical flux function χ. This equation was found to have two basins of solution: in the first one the axisymmetric part of the helical flux function, χ_o, has a local maximum in the plasma region (at the resonance radius), while χ_o is a monotonic function of r in the second one. The two basins correspond to a resonant or non-resonant helical term respectively.

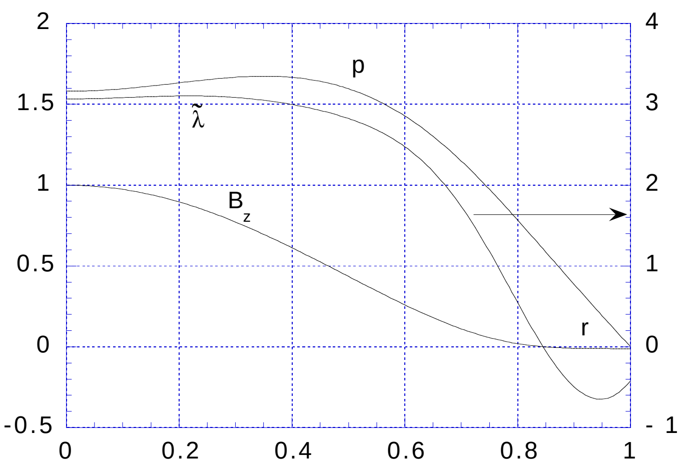

FIG 1: $B_z^{(0,0)}$, $\tilde{\lambda}$, and p (p amplified by a factor 60) as a function of r for Θ=1.78, F=-0.06 and a volume averaged β=0.1, where Θ and F are the usual pinch and reversal parameters for RFP configurations.

When the pressure is taken into account the Grad-Shafranov equation was solved in helical coordinates by assuming a polynomial dependence of $\tilde{\lambda} = \mathbf{J}\cdot\mathbf{B}/B^2 - p'\,g/B^2$ on the helical flux function χ, where p is the pressure, $p' = dp/d\chi$, and $g = mB_z - krB_J$ is the helical magnetic field; the pressure was assumed to depend linearly on the helical flux function, $p(\chi) = p_o + p_1\,\chi$, where $p_o = -p_1\,\chi_o(a)$ since p must vanish at the plasma boundary. It was then possible to find ohmic solutions where $\langle B_z \rangle$ (axial magnetic field averaged over the helical flux surface) does not reverse in the outer plasma region, while the axisymmetric field $B_z^{(0,0)}$ does (figure 1); λ was found to be almost constant far from the edges. Figure 2 displays the corresponding poloidal contour plot of the helical flux function $\chi(r,u) = \chi_o(r) + \chi_1(r)\ \cos(u)$ in z=0 poloidal section. It corresponds to a resonant case, but the level of perturbation is high enough for the separatrix to vanish (see section 4).

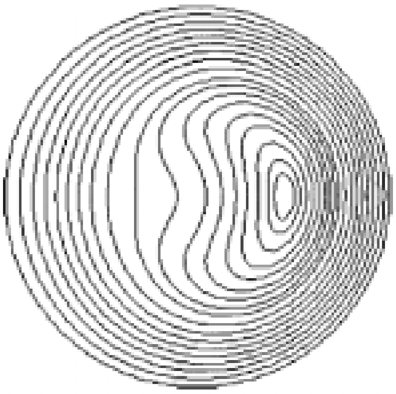

FIG. 2. Poloidal section of single helicity magnetic surfaces computed through the helical Grad-Shafranov equation and Ohm's law.

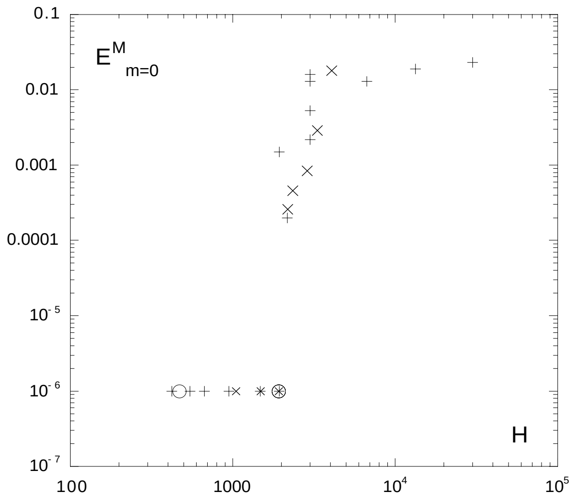

FIG.3. Transition to single helicity as a phase transition with the energy of the m=0 modes as order parameter, and the Hartmann number as control parameter; the crosses (resp. the x's) correspond to S=3x10^4 (resp. 3.3x10^3), and the circles to simulations where a small MH perturbation was added to an initial SH state; by convention 10^{-6} corresponds to a vanishing energy.

The boundary conditions of the theoretical SH states imply the existence of a continuous distribution of helical boundary currents. The existence of cuts in the shells of present RFP's prevents these currents from properly flowing, and induces return currents generating a MH error fields. Unless a good correction of this error field is performed, the shell acts like an ergodic divertor exciting a broad spectrum of resonant modes in the plasma core. This suggests an evolution of the RFP into a forced SH RFP where most of the helical boundary currents are provided by external windings. Such a RFP would still produce most of the confining magnetic field by plasma currents, but it would be intermediate between the tokamak (because of the toroidal current) and the stellarator (because of helical external windings).

3. Transition from Single to Multiple Helicity in Visco-resistive MHD

The simplest visco-resistive MHD model to study RFP dynamics is [5,17]

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \mathbf{J}) \qquad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{J} \times \mathbf{B} + \nabla^2 (\nu \mathbf{v}) \quad (2)$$

with $\mathbf{J} = \nabla \times \mathbf{B}$ and $\nabla \cdot \mathbf{B} = 0$. Here time and velocity are normalized to the Alfvén time and velocity respectively, and the other variables to macroscopic values: in these units η is the inverse Lundquist number, $\eta=\tau_A/\tau_R\equiv S^{-1}$ and ν corresponds to the inverse magnetic Reynolds number, $\nu=\tau_A/\tau_V\equiv R^{-1}$, for a scalar kinematic viscosity.

Since 1990 a transition from MH to SH has been known to occur in this model when viscosity increases at fixed S [4,5,6]. A recent scaling approach to this model reveals that the Prandtl number acts only through the inertia term [18]. When this term is negligible the dynamics is ruled by the Hartmann number $H=(\eta\nu)^{-1/2}$ only. This occurs for the dynamics of the RFP, as shown by 3D numerical simulations of the model. Therefore it was interesting to revisit the SH/MH transition with H as unique control parameter. An order parameter for the transition can be found by noticing that $E_{m=0}{}^{M}$, the energy of the m=0 mode, vanishes in SH states due to the lack of coupling of different m=1 modes. Figure 3 displays this energy as a function of H. The result is seen to be independent of S. When H is small, two basins of SH are shown to coexist (m=1, n=11 and 12). In the vicinity of H=2500 the system displays a temporal intermittency whose laminar phases are of QSH type. For higher H's the system reaches a multiple helicity (MH) state whose features, in particular magnetic chaos, are analogous to the traditional turbulent state of RFP plasmas. We notice that the SH/MH transition is analogous to a second order phase transition where $E_{m=0}{}^{M}$ is the order parameter, H the control parameter, and where the intermediate QSH regime corresponds to the critical divergence of the correlation scales.

Inertia is also negligible in other devices than the RFP. This questions the tradition of scaling magnetic fusion quantities against S, and suggests that H should be used instead. In turn this raises the difficult issue of the definition of viscosity in magnetic fusion plasmas. The importance of H in fusion physics was raised previously in references [19,20,21,22].

4. Resilience of Single Helicity States to Chaotic Perturbations

The resilience to chaotic perturbations of a one-parameter one-degree-of-freedom Hamiltonian dynamics increases when its corresponding separatrix vanishes due to a saddle-node bifurcation. This is important for the magnetic chaos of QSH states of the RFP [23]. Indeed for a high enough amplitude of a resonant SH mode, the magnetic separatrix of this mode bifurcates out, which makes this SH mode more resilient to the chaos induced by the smaller modes with other helicities of the QSH state [23]. This brings a rationale for the confinement improvement of helical domains experimentally found for QSH plasmas [2,3]. Such a feature would not be expected from the classical resonance overlap picture as the separatrix disappearance occurs when the amplitude of the dominant mode increases.

5. Issues about Taylor's Theory

Taylor's theory (TT) [13] applied to the RFP is consistent with the facts that both experimentally and numerically λ is approximately constant in the centre of the machine, and that the energy decreases faster than the magnetic helicity, at least during the beginning of the relaxation (small scale dissipation). TT as variational search for minimum energy states contains also the stability results of previous ideal and resistive MHD stability theories of λ=const profiles [24]. However several points limit the applicability of this theory. Some are classical [24] like (*i*) the fully relaxed state with λ=const. is inconsistent with the boundary condition for a resistive plasma in contact with a rigid, perfectly conducting boundary; this effect is amplified in real experimental plasmas with cold edges, as the large edge resistivity then causes the current density to be small or zero near the wall; (*ii*) the helical state of TT has a sign of the pitch which would correspond to external resonant modes never seen in simulations or real experiments; this is in particular the case for the theoretical SH states and the experimental QSH states which correspond instead to internal modes; (*iii*) the predicted saturation of Θ is not observed; more strikingly, in the MST experiment the shell is close to the plasma, in agreement with the geometrical assumption of TT, nevertheless the minimum value of Θ that leads to reversed configurations corresponds to the maximum value allowed in TT (this fact is related to the non constancy of λ); (*iv*) TT does not account for pressure gradients. Other inconsistent points with TT may be added: (*i*) experimentally a linear relation is observed between F and Θ [25], one more fact related to the non constancy of λ; (*ii*) due to Ohm's law, if $\nabla\times\mathbf{B}=\lambda\mathbf{B}$, then λ must come close to zero at the reversal [6]; (*iii*) reaching a minimum energy state is natural for a closed system, but not for an open system; in particular there is a classical result in electrotechnics that an open ohmic system maximises its magnetic energy.

The loss of axisymmetry is an essential feature of the RFP which is intrinsically due to the saturation of a (spectrum of) kink mode(s) obtained through field reversal. To the contrary TT proposes a relaxation leading in particular to axisymmetric reversed states. Experimentally going from QSH to MH means that the helical deformation of the plasma becomes more toroidally localised [3]. This shows a continuity between QSH and MH. Furthermore the nature of the intermediate states between SH and MH of numerical simulations show the existence of a continuity between SH and MH states as well: the system alternates between MH phases and QSH phases. Would MH be related to a principle of minimum energy, how could the system leave MH to become QSH ? Therefore the non applicability of TT to SH questions its applicability to MH. It is interesting to notice that recent visco-resistive MHD numerical simulations of magnetic configurations encountered in solar physics show that the plasma relaxes to a state far from a Taylor state [26].

6. Conclusions

In the future the issue of stability, accessibility and robustness of SH states should be addressed by incorporating new elements: shell radius larger than plasma radius, heat transport (filamentation effects might be present), role of the pinch parameter and of the aspect ratio. In particular linear stability theory should be developed for helically symmetric profile (it is striking to notice that the Cowling type of argument has been known for long to rule out axisymmetric profiles for the RFP, but that linear stability theory for the RFP has been developed for such profiles only). Future work should be dedicated to assess the value of viscosity to incorporate in the Hartmann number so as to predict the scaling of this number reactorwise. It would be interesting to assess whether the domain of SH is extended in a forced SH RFP due to the presence of the right magnetic boundary conditions.

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