Interactions between Neoclassical Tearing Modes and their Stabilization by an Externally Applied Helical Field

Q. Yu[1], S. Günter, K. Lackner

MPI für Plasmaphysik, EURATOM Association, D-85748, Garching, Germany

Q. Yu's e-mail address: qiy@ipp.mpg.de

The interaction between the neoclassical tearing modes (NTMs) of different helicities is investigated theoretically. It is found that once two magnetic islands are close, the more unstable island survives and suppresses the less unstable one, in agreement with the experimental results. The mechanism responsible for this suppression is the decreased fundamental harmonic pressure perturbation of the NTM in the presence of magnetic perturbations of different helicity. Based on the mechanism found, the effect of static helical magnetic fields on the nonlinear growth of neoclassical tearing modes (NTM) is investigated, and the NTM is found to be stabilized by an externally applied helical field of a different helicity if the field magnitude is sufficiently large, suggesting a very simple method for stabilizing the NTM.

1. Introduction

For a high β tokamak plasma the perturbed bootstrap current can drive magnetic islands to grow, leading to the neoclassical tearing mode (NTM)[1,2]. NTMs have been found experimentally to limit the maximum plasma pressure well below the predictions of ideal magnetohydrodynamic (MHD) calculations in large tokamaks[3-7].

In recent years extensive research efforts have been devoted to this mode. However, one issue regarding the NTMs is of great concern: whether NTMs of different helicities driven by the pressure gradient can develop simultaneously at their corresponding rational surfaces, and lead to an enhanced transport or even to disruptions. This would be a very serious problem, especially for a tokamak reactor. As the β values for the mode onset have been found to be proportional to ρ^* (the ion Larmor radius normalized to minor radius) [8], in a reactor NTMs could occur for much lower β values than in present day tokamaks.

In the present paper the interaction between the NTMs is investigated. It is found that whenever the amplitude of the m/n=2/1 mode becomes sufficiently large, the 3/2 mode decays, where m and n are the poloidal and toroidal mode numbers, respectively. Similar to the suppression of the 3/2 mode by the 2/1 mode, the 4/3 mode is found to be suppressed by the more unstable 3/2 mode. The numerical results agree with both the analytical results and the experimental observations on ASDEX Upgrade[9]. The mechanism responsible for the suppression of one NTM by another has been identified to be the decreased pressure perturbation in the presence of magnetic perturbations of different helicities.

Based on the mechanism found, the effect of an externally applied static helical magnetic field on the NTM is studied. It is found that the NTM is suppressed by a helical field of a different helicity if its magnitude is sufficiently large[10], suggesting another possible method for stabilizing NTMs in addition to the localized RF current drive[11].

---

[1] Institute of Plasma Physics, Chinese Academy of Science, 230031, Hefei, P. R. China

2. Model

The basic equations describing the NTMs are Ohm’s law, the equation of motion, and the pressure evolution equation,

$$\frac{\partial\psi}{\partial t}+\mathbf{B}\cdot\nabla\phi = E-\eta(j-j_b), \tag{1}$$

$$\rho(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla)\nabla^2\phi = \mathbf{e}_t\cdot(\mathbf{B}\cdot\nabla j)+\rho\mu\nabla^4\phi, \tag{2}$$

$$\frac{3}{2}(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla)p = \nabla\cdot(\chi_b\nabla_b p)+\nabla\cdot(\chi_\perp\nabla_\perp p)+Q, \tag{3}$$

where $\mathbf{B}=B_{0t}+\nabla\psi\times\mathbf{e}_t$ and $\mathbf{v}=-\nabla\phi\times\mathbf{e}_t$ are the magnetic field and plasma velocity, respectively, ψ and ϕ the magnetic flux function and the stream function. $j=-\nabla^2\psi$ is the plasma current density along the $\mathbf{e}_t$ (toroidal) direction, $j_b=-g(\sqrt{\varepsilon}/B_p)dp/dr$ is the bootstrap current density, g is a function of the minor radius r which becomes zero as r approaches the edge, $\varepsilon=r/R$ is the inverse aspect ratio, and B_p is the poloidal magnetic field. ρ is the plasma mass density, μ the plasma viscosity, p the plasma pressure, and χ the transport coefficient. The subscripts b and $\perp$ denote the parallel and perpendicular components, and Q and E are the heating power and the equilibrium electric field, respectively. Since the equilibrium toroidal magnetic field is approximated to be a constant, the toroidal mode coupling is neglected. The bootstrap current is the only toroidal effect included in the model. Eqs. (1)-(3) have been used earlier to investigate the nonlinear evolution of the single and double NTMs[12-14].

3. Results

3.1 Interactions between Neoclassical Tearing Modes

On ASDEX Upgrade NTMs of different helicities m/n=5/4, 4/3, 3/2, 2/1 have been observed, mostly coupled to an (m-1)/n kink mode. These NTMs, however, never occur simultaneously. They often appear as a sequence, starting with the mode of highest mode numbers. Such a sequence is shown in Fig. 1, in which the time evolution of the relative perturbed poloidal field amplitudes of the 3/2 and the 2/1 NTMs are shown in arbitrary units. It is seen that, as soon as the 2/1 mode starts growing, the 3/2 mode amplitude decreases.

To understand the experimental results, Eqs. (1)-(3) are solved simultaneously using an initial value code TM. This code had been used to simulate the nonlinear evolution of the single and double NTMs[12-14]. In the calculations, $S=\tau_R/\tau_A=5\times10^6$ and $\tau_\mu=(0.1\text{-}1)\tau_R$ are taken, where $\tau_A=a/v_A$, $\tau_R=a^2\mu_0/\eta$, $\tau_\mu=a^2/\mu$, and a is the minor radius. A sufficiently large value for $\chi_b/\chi_\perp$ is taken. For the q-profile used, the distance between the rational surfaces of interest is approximately the same as that of the corresponding experimental q-profile.

For a single m/n=3/2 mode, the time evolution of the normalized island width, $w_{3/2}/a$, is shown by the solid curve in Fig. 2. The q=3/2 surface is at $r_{3/2}=0.575a$, and the local bootstrap current density fraction $f=j_{BS}/j_0=0.11$ at $r_{3/2}$. The 3/2 island grows and saturates after some time. If an 2/1 initial perturbation is introduced at $t=0.01\tau_R$, keeping all the other input parameters the same, the time evolution of $w_{2/1}/a$ and $w_{3/2}/a$ are shown by the dashed curves. The q=2/1 surface is at $r_{2/1}=0.70a$, and f=0.097 at $r_{2/1}$. It is seen that as $w_{2/1}$ becomes large $w_{3/2}$

decays. When both the 3/2 and the 2/1 initial perturbations are introduced at t=0, similar results to Fig. 2 are found: $w_{3/2}$ first grows and then decays as $w_{2/1}$ becomes large.

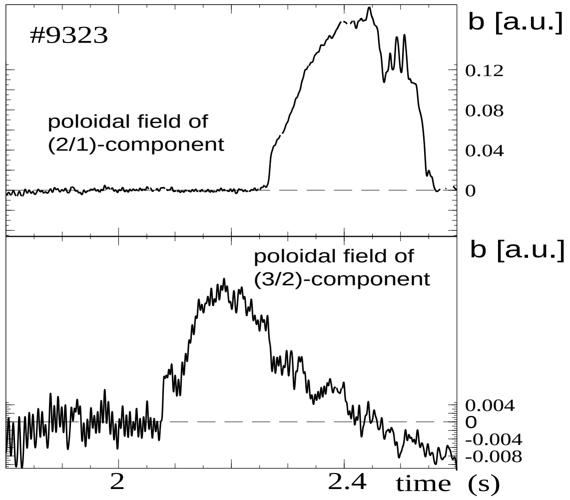

Fig.1 The time evolution of the 2/1 and 3/2 poloidal field amplitude, measured by Mirnov coils.

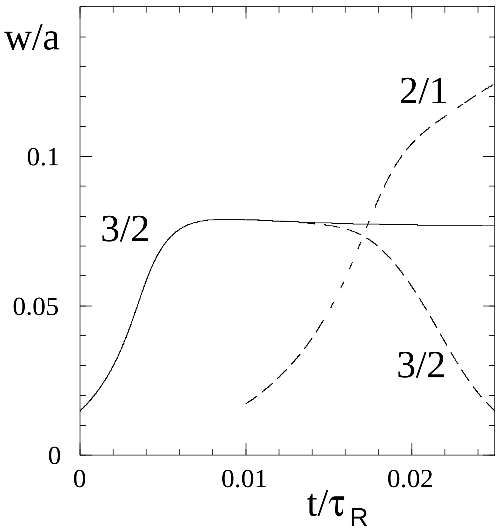

Fig.2 The time evolution of the island width of a single 3/2 NTM (solid curve), and after the onset of an additional 2/1 mode (dashed curves).

Analytical work has been carried out for understanding the numerical results[9,10]. It is found that, the fundamental harmonic pressure perturbation of the NTM is decreased when there is a helical magnetic field of a different helicity, no matter the helical field comes from another NTM or from external coils. The decrease of the pressure perturbation leads to the decrease of the corresponding bootstrap current perturbation and, therefore, the decrease of the drive of the NTM.

In addition to the stabilizing mechanism described above, there is another possible mechanism for the suppression of the 3/2 mode by the 2/1 mode: the reduction of the β (or f) value at $r_{3/2}$ due to the 2/1 island. For distinguishing between these two effects, test calculations have been carried out, taking the m/n=3/2(2/1) magnetic perturbation component to be zero in calculating $p_{2/1}(p_{3/2})$. In this case $w_{3/2}$ only slightly decreases. On the other hand, if the perturbation of $p_{0/0}$ due to the 2/1 island is neglected, $w_{3/2}$ still decreases significantly. Therefore, the suppression of the 3/2 mode by the 2/1 mode is mainly due to the suppression of $p_{3/2}$ by the perturbed magnetic field of a different helicity[9]. A simple analytical approach leads to the same results[9, 10].

The 3/2 mode is not necessarily always suppressed by the 2/1 mode. For the above cases, f at $r_{2/1}$ was close to that at $r_{3/2}$, and the 2/1 mode is more unstable than the 3/2 mode. With a different equilibrium pressure profile such as f=0.12 at $r_{3/2}$ and f=0.032 at $r_{2/1}$, the 2/1 mode is suppressed by the 3/2 mode. Therefore, for the two neighboring NTMs, we can conclude that the more unstable mode will suppress the less unstable one. A similar interaction between the 4/3 and 3/2 mode is also found. The 4/3 mode is found to be suppressed by the more unstable 3/2 mode. Inclusion of differential rotation between the two islands leads to essentially the same results. When additional modes like m/n=6/4, 4/2, 5/3, 7/4, 8/5, 9/5, resulting from the non-linearity and the coupling between the 3/2 and the 2/1 mode, are included, the modelling results are approximately the same as those with only three Fourier components, m/n=0/0, 3/2 and 2/1.

3.2 Stabilization of NTMs by an Externally Applied Static Helical Field

Since the NTMs provide the β-limit for tokamak plasmas, their stabilization is of great concern. It had been shown in the recent experiments that NTMs can be stabilized by localized Electron Cyclotron Current Drive (ECCD)[11,14]. Based on the results discussed above, here the effect of a static helical magnetic field on the nonlinear evolution of the NTMs is investigated.

For the externally applied helical field not to result in magnetic islands inside the plasma and considering $\psi_{m/n}\sim r^m$ inside the plasma, it is desirable to select its mode numbers to be $m/n<1$ with m=1 or 2. Here the effect of an m/n=2/4 static helical field on an m/n=3/2 NTM is studied. The m/n=2/4 static helical field is introduced in the numerical modelling by taking the boundary condition $\psi_{2/4}(a)\neq 0$.

In Fig. 3 the time evolution of the normalized m/n=3/2 island width, w/a, is shown for $\Psi\equiv\psi_{2/4}(a)/a|\mathbf{B}_0|=0$, 0.5×10^{-3} and 10^{-3}, respectively, with f=0.21. It is seen that w grows and saturates for $\psi_{2/4}(a)=0$. For small $\psi_{2/4}(a)$, w still grows, but more slowly. A sufficiently large $\psi_{2/4}(a)$ leads to the decay of the 3/2 island.

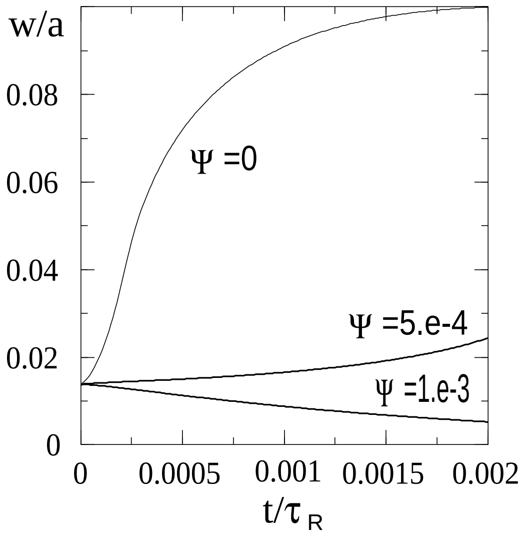

Fig.3 The 3/2 island width time evolution for different amplitudes of an m/n=4/2 helical field, applied at t=0.

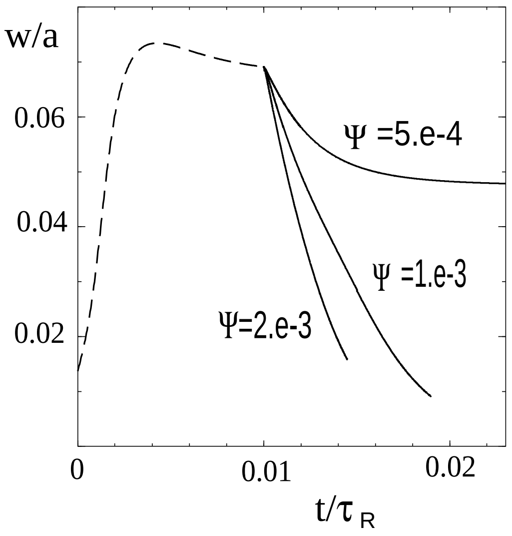

Fig.4 The 3/2 island width time evolution without the m/n=2/4 helical field (dotted curve) and with different amplitudes of an m/n=4/2 helical field applied at $t/\tau_R = 0.01$.

The dotted curve in Fig. 4 shows the growth of the 3/2 island alone for $t=0\text{-}0.01\tau_R$ with f=0.1. The island grows and saturates. At $t=0.1\tau_R$ an 2/4 helical field is turned on with $\Psi=0.5\times10^{-3}$, 10^{-3}, and 2×10^{-3}, respectively. It is seen that a larger $\psi_{2/4}(a)$ lead to a stronger and faster decay of the 3/2 island.

Introducing a poloidal plasma rotation or more Fourier components into the calculation, the results essentially do not changed. With a larger $\chi_\perp$, the 3/2 mode decays faster. The effect of an m/n=2/6 or 2/8 helical field on the m/n=3/2 mode is found to be the same as that of the m/n=2/4 field. The m/n=2/1 NTM also becomes stabilized by a sufficiently large m/n=2/4 or 2/3 helical field.

4. Summary

Using a nonlinear MHD code in toroidal geometry XTOR[15], the decrease of the 3/2 pressure perturbation by an 2/1 magnetic field is also found, in agreement with the numerical results presented here.

In summary, we find that

1) Once two magnetic islands are close, there is a survival competition between the two neighboring islands. The more unstable island survives and suppresses the less unstable one, indicating that NTMs will not lead to an enhanced transport by producing many small islands at their corresponding rational surfaces. The strength of the interaction between two neoclassical magnetic islands is found to be determined by their relative amplitude and distance. The mechanism responsible for this suppression is the decreased fundamental harmonic pressure perturbation of the NTMs in the presence of magnetic perturbations of different helicity. The theoretical results agree with the corresponding experimental observations on ASDEX Upgrade.

2) NTMs can be stabilized by an externally applied static helical magnetic field of a different helicity if its magnitude is sufficiently large, suggesting a very simple method for stabilizing NTMs. The required helical field magnitude is found to be proportional to the local bootstrap current density fraction and the island width. For typical ASDEX Upgrade parameters and for stabilizing the m/n=3/2 mode, the required helical field magnitude is found to be 0.1 percent of the toroidal magnetic field.

Acknowledgment: We are grateful to M. Maraschek and A. Gude for their help in checking ASDEX Upgrade experimental results and to R. Arslanbekov and H. Lütjens for their help in using XTOR code. Q. Yu is partially supported by the "One Hundred Excellent Project" of Chinese Academy of Science.

[1] National Technical Information Service Document, No. DE86008946 (University of Wisconsin Plasma Report UWPR 85-5 by W. X. Xu and J. D. Callen) .
[2] R. Carrera, R. D. Hazeltine, and M.Kotschenreuther, Phys. Fluids **29**, 899 (1986).
[3] Z. Chang *et al.*, Phys. Rev. Letts, **74**, 4663(1995).
[4] H. Zohm *et al.*, Plasma Phys. Control. Fusion, **37**, A313(1995).
[5] R. J. La Haye *et al.*, Nucl. Fusion **37**, 397(1997).
[6] O. Sauter *et al.*, Phys. Plasmas **4**, 1654(1997).
[7] S. Günter *et al.*, Plasma Phys. Control. Fusion, **41**, B231(1999).
[8] S. Günter *et al.*, Nucl. Fusion **38**, 1431(1998).
[9] Q. Yu, *et al.*, Interaction between neoclassical tearing modes, to be published in Nucl. Fusion, 2000.
[10] Q. Yu, S. Günter, K. Lackner, Phys. Rev. Letters, **85**, 2949(2000).
[11] H. Zohm *et al.*, Nucl. Fusion **39**, 577(1999).
[12] Q. Yu and S. Günter, Phys. Plasmas **5**, 3924(1998).
[13] Q. Yu and S. Günter, Nucl. Fusion **39**, 487(1999).
[14] Q. Yu, S. Günter *et al.*, Phys. Plasmas **7**, 312(2000).
[15] H. Lütjens and JF. Luciani, Phys. Plasmas **4**, 4192(1997).