Optimization of Tokamak Poloidal Field Configuration by Genetic Algorithms

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Abstract. The design of the poloidal field system in tokamaks is a lengthy procedure due to functional dependence of design parameters on pre-assumed plasma parameters. Therefore changing an individual design parameter could in principle influence the overall design. This complicated task could easily lead to non-optimal final results. In the paper it is shown that genetic algorithms, as powerful optimization methods, are useful in approaching an optimal design. The work may be extended to the whole plasma scenario and magnetic configuration in other small and large scale systems.

1. Introduction

One of the most important physics issues of tokamak design is the design of the poloidal field system. This includes the number and location of magnetic coils, their individual current, and the discharge scenario. The overall plasma shape and performance are determined by the configuration and scenario of the poloidal field coils. Therefore, one should be careful in selecting the final design parameters. Obviously, the performance of the resulting plasma and design goals are not related to the poloidal field configuration with simple relations. The relevant design parameters include the energy confinement time, plasma current, beta, temperature and so on. Indeed the named parameters are all elaborate functionals of the whole poloidal field system, and even of each other. This explains the high complexity of the tokamak design process.

Usually, different magnetic configurations are examined until design goals are satisfied. This requires the pre-assumption of plasma shape, order of plasma current and temperature, etc., then advancing into the poloidal field design level which satisfies this configuration, then simulating plasma scenarios and choosing the best one. This loop is closed and the above procedure is repeated until the optimal design is reached. Of course, there is no procedural or systematic way to make sure of the result achieved is optimal.

Fortunately, there are a number of well-established optimization algorithms based on deterministic and non-deterministic approaches. The deterministic algorithms are usually suitable for mathematically well-defined problems with one local minimum, while non-deterministic algorithms are efficient for complicated problems. Non-deterministic algorithms usually go through a heuristic, but procedural, search for the best matching conditions.

The genetic algorithms [1] are in principle capable of solving any optimization problems by iterative procedures. What a genetic algorithm does is to find the global maximum (or minimum) of a given goal function. This goal function should be carefully defined and may involve the consequences of any design parameter (or inputs), and even budget limitations. The main power and flexibility of genetic algorithms are derived from the fact that the number of inputs is not limited in any way. Hence, the genetic algorithm converges, no matter what has been the goal function.

In this work, we report the application of genetic algorithms in optimizing the tokamak poloidal field system. We demonstrate that the algorithm predicts a better configuration for one of the existing machines. As a simple example, the optimization problem is assigned to the number, position and current of poloidal field coils, with the goal function being simply the plasma energy confinement time. To simplify the numerical approach, the Neo-Alcator scaling law [2] is used. The main reason for this choice is that this scaling law depends only on geometric and electromagnetic properties of the plasma, as discussed below. The results are forced to stay within general tokamak stability limits, i.e. maximum plasma current, minimum and maximum plasma magnetic shears, maximum plasma density, and maximum plasma beta.

Moreover, this choice removes the necessity of including the plasma neo-classical transport effects which complicate the calculations (because most of the more complicated energy confinement scalings require the total heating power, whose exact calculation requires at least a knowledge of the time-domain evolution of the plasma). This special choice obviously does not affect the totality of this approach, since for instance the ITER scaling laws could be used if it were decided to utilize a neo-classical transport and equilibrium code. Similarly, the whole plasma scenario may be optimized when a good time-domain equilibrium code is accessible.

2. Theory

The free-boundary equilibrium of a tokamak plasma is described by the Grad-Shafranov equation in terms of the magnetic poloidal flux $\boldsymbol{\Psi}$ as [3]

$$\frac{1}{r}\Delta^{*}\Psi = -\mu_0 J_{\phi}\,. \qquad (1)$$

Here, J_ϕ is the toroidal current density and Δ^* is the elliptic Grad-Shafranov operator

$$\Delta^{*} = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\,. \qquad (2)$$

Exact analytical solutions to the Grad-Shafranov equation of the form

$$\Psi(r,z) = \int_{-\infty}^{\infty}\int_{0}^{\infty} G(\mathbf{r},\mathbf{r}')J_{\phi}(\mathbf{r}')dr'dz' \qquad (3)$$

exist, in which $G(\mathbf{r},\mathbf{r}')$ is referred to as the Green function. This Green function reads [4,5]

$$G(\mathbf{r},\mathbf{r}') = \mu_0\frac{\sqrt{rr'}}{2\pi}\mathrm{Q}_{1/2}\left(1+\frac{|\mathbf{r}-\mathbf{r}'|^2}{2rr'}\right), \qquad (4)$$

where $\mathrm{Q}_{1/2}(\cdot)$ is the Legendre function of the second kind, which may be efficiently evaluated [6]. This offers a simple way to evaluate of the magnetic poloidal flux $\boldsymbol{\Psi}$ due to the magnetic poloidal field system.

Inside the plasma, however, the toroidal current density may be described in terms of the magnetic poloidal flux as [2]

$$J_{\phi} = J_0\left(\frac{\Psi-\Psi_B}{\Psi_m-\Psi_B}\right)^{\nu}\left(\frac{r}{R_0}\beta_j + \frac{R_0}{r}(1-\beta_j)\right). \qquad (5)$$

Here, J_0 is the current density on the magnetic axis of the plasma, and Ψ_B and Ψ_m are the values of magnetic poloidal flux on the boundary and magnetic axis of plasma, respectively. R_0 is the major radius of the plasma. β_j is a parameter close to the plasma poloidal beta (typically around 10%), and ν is a positive real number expressing the flatness of the current density profile. The overall solution of (1) subject to (5) gives the plasma equilibrium inside the plasma, while the boundary value Ψ_B should match the resulting flux from the poloidal system as computable through (3).

The computation results are subject to the simplified stability conditions for a toroidal tokamak plasma [2]. They are

$$\frac{q_a}{q_0} > 2,\ q_a > 2, \text{ and } q_0 > 1, \tag{6}$$

for stability against the kink, internal kink, and spiral instabilities,

$$\beta < 3\times10^{-6}\frac{I}{aB_t} = 15\frac{a}{R_0 q_a}, \tag{7}$$

as the maximum permissible plasma beta β, and

$$n < 1.5\times10^{20}\frac{q_a B_t}{R_0}, \tag{8}$$

for the maximum plasma density n. In the above relations, q_a, q_0, a, and B_t are the values of the safety factor on the edge and center of the plasma, plasma minor radius, and toroidal magnetic field at the center of the plasma, respectively (all parameters are in metric SI units). Typically, any equilibrium solution not matching the above criteria would be eliminated from the numerical optimization procedure.

In our optimization scheme, the Neo-Alcator scaling law for the energy confinement time τ_E is used to judge the confining properties of the resulting plasma, that is [2]

$$\tau_E = 5\times10^{-22} naR_0^{\ 2} q_* A_i^{1/2}. \tag{9}$$

Here, A_i is the atomic mass of the plasma, being a constant of analysis. Also, q_* is given by

$$q_* = 5\times10^6 \frac{a^2 B_t}{R_0 I_p}\frac{1+\kappa^2\left(1+2\delta^2-1.2\delta^3\right)}{2}. \tag{10}$$

Thus this scaling law gives a rough estimation of the energy confinement time τ_E by looking into the geometric and electromagnetic properties of the plasma, i.e. the minor and major radii a and R_0, plasma elongation and triangularity κ and δ, total plasma toroidal current I_p, and maximum plasma density n. These values are either fixed, or derivable from a free-boundary equilibrium analysis as discussed above.

Our genetic code is based on the algorithm described in [1]. The genetic algorithm accepts a set of input variables with given dynamic ranges. Then it tries to maximize (or minimize) a known function, referred to as the goal function. The goal function could be in principle any non-linear functional combination of functions, which are computable from the input variables and some constants. The genetic algorithm is especially useful when no closed form exists which relates the final variables or functions to the input. In such cases, there usually exist a number of local maxima. The genetic code is thus able to locate the global maximum through iterative special reproduction, mutation, and crossover processes on the so-called genes. The genes are here actually the positions and currents of the poloidal coils. As well, the effect of a larger influent variable, even the total weight and construction cost could be determined, by means of a proper choice of goal function. Needless to say, the goal function must provide a logical connection between the output variables from the plasma analysis code, addressing the plus and minus features of the plasma. More discussion on the genetic algorithms can be found in Ref. [1].

The definition of the problem is completed by defining the inputs to the genetic algorithm, that is the number, individual position and current of the poloidal field coils. For further simplicity we assume that the poloidal coils are placed on a vacuum vessel or toroidal field coils with a prescribed geometry. It might be considered that the geometrical form of the vacuum vessel or toroidal coils is already determined through mechanical considerations for a minimal stress body. This form is usually found by solving a differential equation [7]. Moreover, it is supposed that the machine is symmetric with respect to its equatorial plane such that the solution region across the plasma cross section is halved.

3. Results

In this section the results of the program for the Damavand tokamak, which is now in operation at the Nuclear Fusion Research Center of the Atomic Energy Organization of Iran, is described. The Damavand tokamak, as the modified Russian TVD-11 tokamak [8], has an elongated plasma (elongation up to 4) at an aspect ratio of about 5, as illustrated in Fig. 1 [9] (the limiters are not shown).

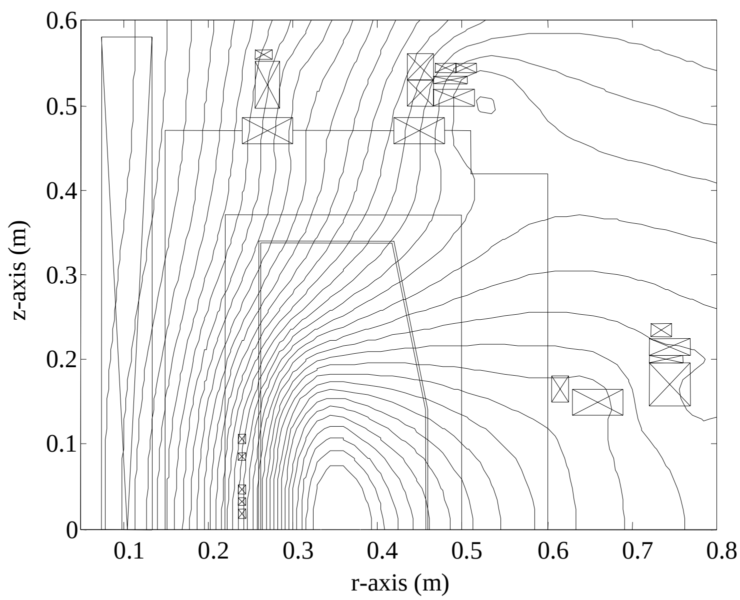

FIG. 1. Damavand tokamak cross section.

The Neo-Alcator scaling predicts an energy confinement time of about 0.18ms for Damavand with the above configuration, which is a fairly low figure for a tokamak with a comparable size. Our code, however, predicts that the following modified poloidal configuration for Damavand could result in a 25% increase in the energy confinement time up to 0.24ms (Fig. 2). The coils are supposed to be placed just over the toroidal coil which have a fixed shape and have equal dimensions. It is seen that the resulting plasma is a little more elongated but less triangular. However, the resulting simple poloidal field system is not necessarily easily achievable in practice, because most of the feedback control system must be modified accordingly, and some additional coils might be still added. In future publications, this work will be extended to more complicated goal functions.

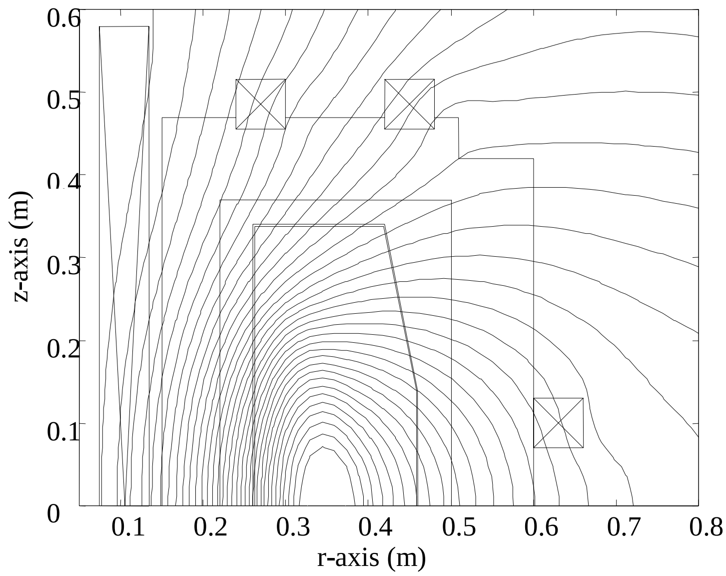

FIG 2. Optimized poloidal field for Damavand.

4. Conclusions

A novel approach to algorithmic design of the tokamak poloidal field system was introduced based on genetic algorithms. The developed code accepts the number, individual position and current of the poloidal field coils, with the assumption that they are placed on the vacuum vessel or toroidal coils with a prescribed form. A numerical code calculates the corresponding equilibrium, thus enabling the most important geometric and electromagnetic properties of the plasma to be estimated. If the configuration is unstable, it is ignored. Otherwise, the energy confinement time is estimated according to the Neo-Alcator scaling law, as a simplified choice. The code then tries to find the design with optimal confinement properties by finding the global maximum of the goal function within the given limits for the machine.

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