

Polarization Current and the Neoclassical Tearing Mode Threshold in Tokamaks; Comparison of Experiment With Theory

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Abstract. Neoclassical tearing mode islands are one of the main causes of reduced performance at high β_θ in standard ELMy sawtoothing H-mode. The leading candidate for the threshold is the helical polarization/inertial current which arises from mode propagation at frequency ω in the $E_r=0$ guiding center frame of plasma flow. A threshold island width w_{p0l} is predicted, which is proportional to the ion banana width $\epsilon^{1/2}\rho_{\theta i}$ and also depends on ω . The polarization current is predicted to be stabilizing only for $0 \leq \omega \leq \omega_{i*}$, the ion diamagnetic drift frequency, and yields a *minimum* β_θ (below which the helically perturbed bootstrap current is too small to excite NTMs) that gives critical β_N scaling linearly with ρ_{i*} . A database compiled from the tokamaks ASDEX Upgrade (AUG), DIII-D and JET shows such a $\beta_{Ncrit} \propto \rho_{i*}$ is indeed observed for the $m/n=3/2$ NTM induced by a sawtooth crash. Typically, unstable seed island widths that grow are observed to be of the order w_{p0l} . Detailed measurements of mode propagation in the $E_r=0$ frame are also consistent with a polarization current threshold.

1. Introduction

Neoclassical tearing mode (NTM) islands are one of the main causes of reduced performance at high β_θ in both standard ELMy sawtoothing H-mode and in advanced tokamaks. Tokamak plasmas are metastable to neoclassical tearing modes in that the plasma must be perturbed beyond a threshold so that the helically perturbed bootstrap current can cause the mode to grow. A typical example from JET is shown in Fig. 1 in which a sub-threshold discharge remains metastable but a similar discharge with more power and higher beta is sufficiently disturbed by a $q=1$ “sawtooth” to induce an $m/n = 3/2$ NTM which reduces confinement by 20% [1–3]. Similar effects are observed on ASDEX Upgrade and DIII-D [4,5]. See Ref. [2] for definitions used in Eq. (1) and elsewhere.

The leading candidate for the threshold mechanism [6–8] is the helical polarization/inertial current which arises from mode propagation at frequency ω in the $E_r=0$ guiding center frame of plasma flow. A threshold island width w_{p0l} is predicted (Fig. 2), which is proportional to the ion banana width $\epsilon^{1/2}\rho_{\theta i}$ with a coefficient that increases several times if the ion collision frequency ν_i/ϵ exceeds

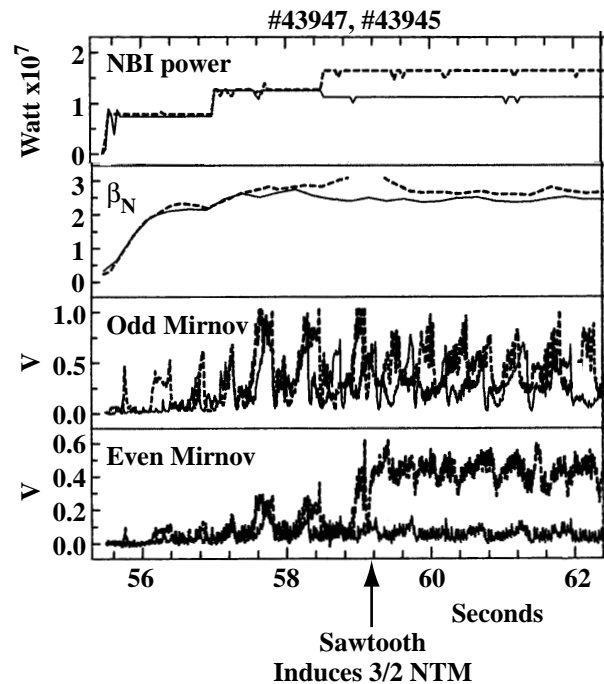


FIG. 1. JET: Neutral beam injected (NBI) power, β_N , odd and even toroidal mode number Mirnov (dB/dt) for two discharges, (1) solid line has no final step up in power and despite periodic sawteeth (jumps on odd Mirnov) remains stable to 3/2 NTM, (2) dashed line has extra step up in power, initially higher β_N but 3/2 NTM excited on sawtooth reduces β_N almost down to that of the discharge with lower power.

$$(1a) \frac{\tau_R}{r^2} \frac{dw}{dt} = \Delta' + \varepsilon^{1/2} (L_q/L_p) \beta_\theta \left(\frac{1}{w} - \frac{w_{pol}^2}{w^3} \right)$$

$$(1b) \beta_{\theta, \min} \approx \frac{3\sqrt{3}}{2} [-\Delta' r / (\varepsilon^{1/2} L_q/L_p)] \left(\frac{w_{pol}}{r} \right)$$

$$(1c) w_{pol}^2 \approx 1.64 [(\omega_{i*} - \omega) / \omega_{e*}^2] (L_q/L_p) (\varepsilon^{1/2} \rho_{\theta i})^2$$

... for low collisionality

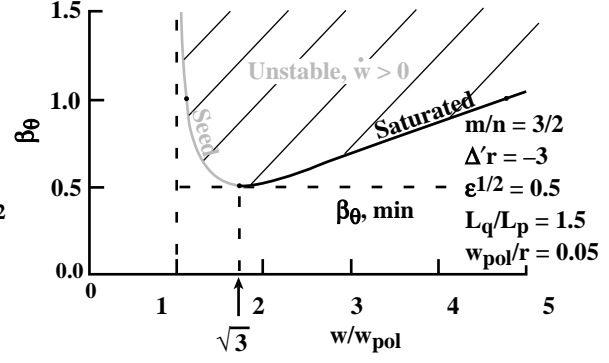


FIG. 2. Helically perturbed bootstrap current can excite neoclassical tearing mode. Unstable region is bounded by $dw/dt = 0$ from the modified Rutherford equation shown on the left.

ω , and which also depends on ω . For example, the threshold is zero for $\omega=0$ (thus no polarization current) or for $\omega=\omega_{i*}$, the ion diamagnetic drift frequency. The original theory predicted propagation in the electron drift direction which would be stabilizing, i.e., a threshold island width would exist for $\omega < 0$. However, reappraisal of the theory in a sheared slab geometry identified an additional contribution to the perturbed polarization/inertial current which reverses its overall effect on island stability leading to a threshold for $0 < \omega < \omega_{i*}$. Other theoretical work has also questioned the polarization current as a threshold for NTMs [9,10]. However, numerous experiments have found the polarization threshold model as superior in scaling and magnitude of critical beta as compared to the incomplete pressure flattening threshold model [11]; COMPASS-D [12], DIII-D [5], JET [1], ASDEX Upgrade [3,13], JT-60U [14], and TEXTOR [15]. Confirmation of the polarization threshold is a key issue for extrapolation to the beta limit of reactor-grade tokamaks.

2. Comparison of Experiment With Theory

A. Scaling of critical beta

Threshold scaling data is consistent with predictions of the polarization current theory which (Fig. 2) yields a *minimum* critical β_θ *below* which the helically perturbed bootstrap current is too small to excite NTMs (assuming that ω/ω_{i*} yields a stabilizing threshold). This would give a linear scaling of critical β_N ($\propto \beta_\theta$) with ρ_{i*} ($\propto \rho_{i0}/a$) in the low collisionality regime. A database was compiled from the tokamaks ASDEX Upgrade (AUG), DIII-D and JET in lower single-null divertor configuration (Fig. 3) with $q_{95} \geq 3$. Such a $\beta_{N, \text{crit}} \propto \rho_{i*}$ is indeed observed experimentally in tokamaks for the $m/n=3/2$ NTM induced by a sawtooth crash as shown in Fig. 4. The different scaling in collisionality between tokamaks is discussed in Ref. [2] and may result from the different seed island scaling in larger devices with higher magnetic Reynolds number [16] which could obviate the $\beta_N \propto \rho_{i*}$ scaling.

B. Unstable seed islands

One expects, that depending on β_θ , the NTM grows when $w_{\text{seed}}/w_{\text{pol}} > 1$ with the “seed” branch (Fig. 2) having $\sqrt{3} > w_{\text{seed}}/w_{\text{pol}} > 1$. Here the island width is determined from the Mirnov data, with correction using electron cyclotron emission (ECE) measurements [2,17]. A toroidal array of Mirnov probes on the outboard midplane is used to measure dB_θ/dt . The

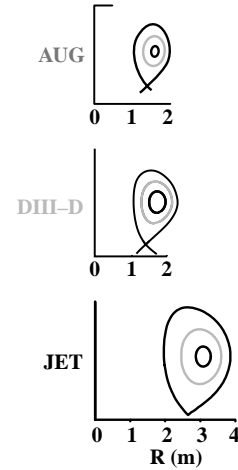


FIG. 3. Separatrix as well as $q=1$ and $q=3/2$ surfaces for ASDEX Upgrade, DIII-D, and JET, showing relative sizes.

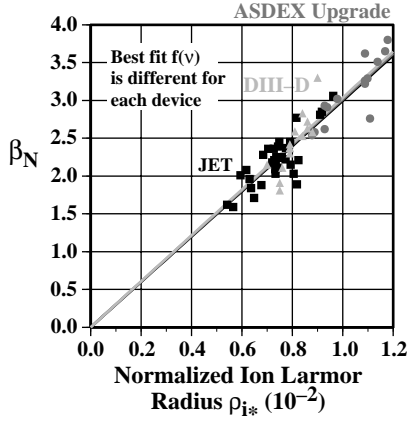


FIG. 4. Critical β_N corrected by each best fit function of collisionality ν vs. ρ_{i*} . Note that $f(\nu)$ is different for each device. ■ JET, ● ASDEX Upgrade, ▲ DIII-D.

integrated \tilde{B}_θ amplitude for the $n=2$ signal is shown versus time in Fig. 5. The value of \tilde{B}_θ that grows is the “seed level” and converted into seed island width by

$$w_{\text{seed}} \approx \left(\frac{16rR|\tilde{B}_r|}{3sB_T} \right)^{1/2}$$

$$\text{with } |\tilde{B}_r| \approx \frac{1}{2} \left(\frac{b}{r} \right)^4 |\tilde{B}_\theta|_{\text{wall}} \quad (2)$$

This width is compared to the low collisionality $[(\nu_i/\epsilon)/\omega_{e*} < 0.3]$ predicted polarization threshold $w_{\text{pol}} \approx 1.64^{1/2} (L_q/L_p)^{1/2} \epsilon^{1/2} \rho_{\theta i}$ assuming mode propagation is stabilizing. The estimated unstable seed islands that must exceed the threshold are found to be of order of the predicted low collisionality regime polarization threshold island (Fig. 6).

C. Island propagation

A hidden variable is the relative propagation frequency ω of an island at small amplitude, i.e. upon initiation, with respect to the guiding center frame in which the local radial electric field $E_r=0$. When the island streams through the plasma in this frame ($\omega \neq 0$) (Fig. 7), polarization drift occurs due to ion inertia and quasi-neutrality gives rise to a return polarization current. The helical polarization current contribution to dw/dt in Eq. (1a) characterized by w_{pol} thus depends on propagation of the island at frequency ω with respect to the ion drift ω_{i*} [8] with w_{pol} as in Eq. (1c) $\propto f^{1/2}(\omega) = [(\omega\omega_{i*} - \omega^2)/\omega_{e*}^2]^{1/2}$. The predicted stabilizing region is for $0 < \omega < \omega_{i*}$ as shown in Fig. 8.

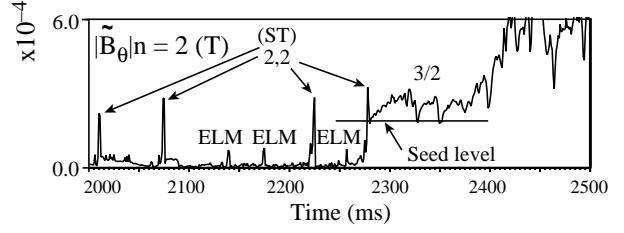


FIG. 5. $n=2$ Mirnov amplitude \tilde{B}_θ versus time in DIII-D. Peaks for sawteeth (ST) and ELMs are noted as well as “seed level” after the ST that induces a growing $m/n = 3/2$ NTM.

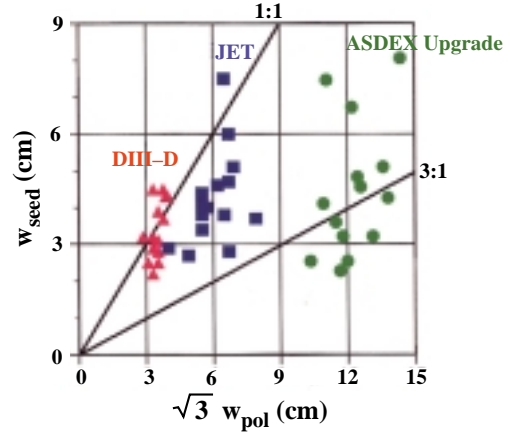


FIG. 6. Comparison of measured unstable $3/2$ seed island width (estimated from Mirnov seed level and calibrated to ECE radiometer measurements of width of large, saturated island) to $\sqrt{3}$ the predicted polarization threshold island from the low collisionality regime (assuming stabilizing mode propagation).

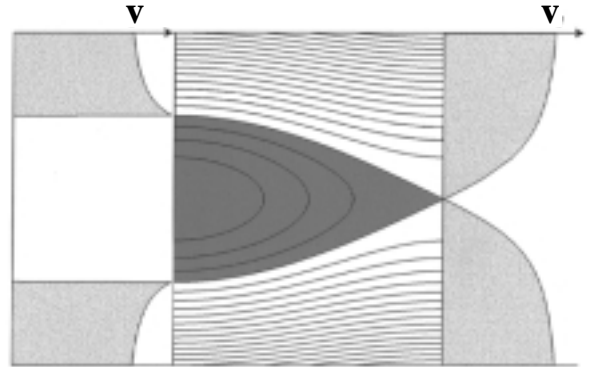


FIG. 7. Schematic of an island propagating at V through a plasma along with the assumed very different radial profiles of perturbed flow at the island O and X-points.

Detailed measurements in DIII–D of mode propagation in the $E_r=0$ frame are consistent with a polarization current threshold. Great care is made to precisely define and measure: (1) the Mirnov frequency of the island in the laboratory frame, ω_{Mirnov} , (2) E_r at the island rational surface and thus the frequency of the frame in which $E_r=0$

$$\frac{\omega_{E_r=0}}{2\pi} = \frac{nE_r}{2\pi R B_\theta} = \frac{-nv_\phi}{2\pi R} + \frac{nv_\theta (B_\phi/B_\theta)}{2\pi R} + \frac{n\nabla p_i}{2\pi Z_i n_i R B_\theta} \quad (3a)$$

and the local ion and electron diamagnetic drift frequencies

$$\frac{\omega_{i*}}{2\pi} = \frac{m}{2\pi r} \frac{(T_i/L_{pi})}{B_\phi} + \frac{n}{2\pi R} \frac{(T_i/L_{pi})}{B_\theta}, \quad \frac{\omega_{e*}}{2\pi} = \frac{-m}{2\pi r} \frac{(T_e/L_{pe})}{B_\phi} - \frac{n}{2\pi R} \frac{(T_e/L_{pe})}{B_\theta}, \quad (3b)$$

with $\omega = \omega_{\text{Mirnov}} - \omega_{E_r=0}$. The geometry is shown in Fig. 9.

The key theoretical parameter at issue of ω/ω_{i*} is measured for $m/n = 5/4, 4/3,$ and $3/2$ tearing modes after a sawtooth crash which acts as the seed for the onset of $4/3$ and $3/2$ tearing modes, the $3/2$ mode eventually growing to a much larger amplitude. The $5/4$ mode had originally been excited two sawteeth earlier at lower beta. In a second, otherwise identical DIII–D discharge, later additional heating power and higher beta also produced an $m/n = 2/1$ mode just after a later sawtooth crash during the large saturated $3/2$ mode. Analysis is done at 6 ± 1 ms post crash for the $m/n = 5/4, 4/3,$ and $3/2$ modes, i.e. when the $4/3$ and $3/2$ islands are still small and at 9 ± 5 ms post crash for the $m/n = 2/1$ mode before it has grown to large amplitude. The relative propagation is shown in Fig. 10.

All of these modes have ω/ω_{i*} consistent with a stabilizing polarization threshold according to the most recent theory [8].

3. Conclusions

Experimental data from NTMs in the tokamaks ASDEX Upgrade, DIII–D, and JET were compared to predictions of the latest polarization threshold theory. There is consistency in that: (1) a nearly linear critical beta with normalized ion Larmor radius is found (particularly worrisome for future devices but also dependent on seed island scaling), (2) the best measured estimates of seed islands which grow are of the order predicted by the polarization threshold and (3) island propagation (upon initiation) in the guiding center frame where $E_r = 0$ is at a fraction of the ion diamagnetic drift consistent with a predicted stabilizing sign of polarization current. A key unresolved theoretical problem is to

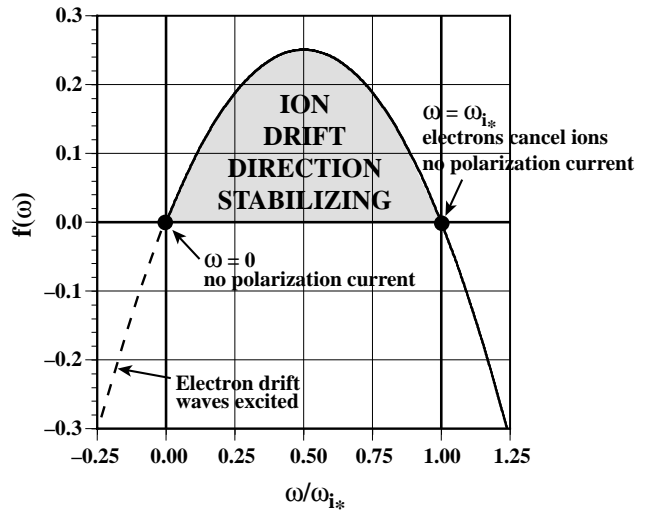


FIG. 8. $f(\omega)$ characterizes the sign and magnitude of the polarization current contribution to Δ .

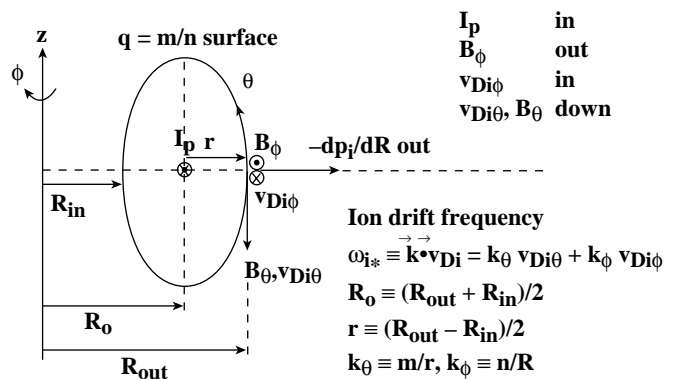


FIG. 9. Geometry defining the propagation directions.

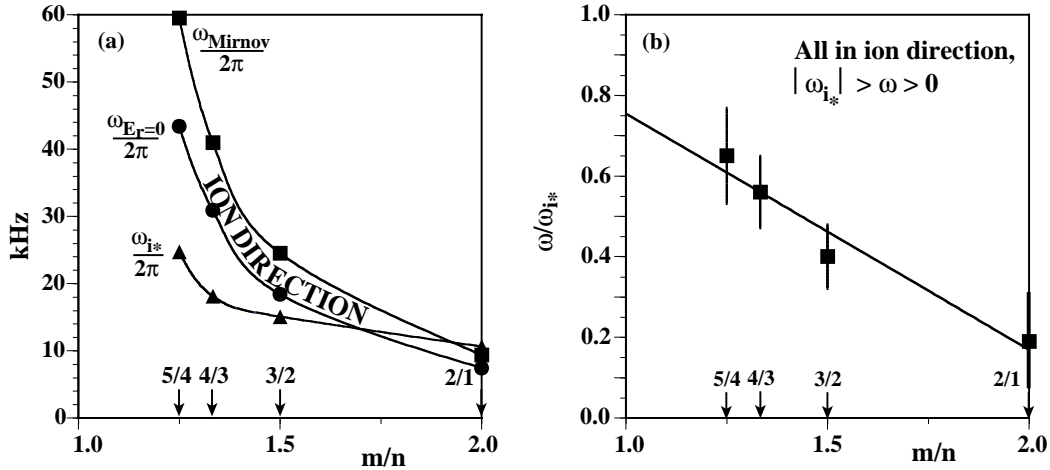


FIG. 10. (a) Propagation of islands, just after initiation. The Mirnov frequency in the lab frame, in all cases is just greater than the frequency of a frame in which $E_r = 0$, and in all cases is in the ion drift direction. (b) The relative island propagation is at a fraction of the ion drift and decreases at larger minor radius.

predict this propagation and how it scales, particularly in rf-heated discharges rather than the co-injected beam-heated discharges reported on here.

Acknowledgments

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