Nonlinear Dynamics of Magnetic Islands Imbedded in Small Scale Turbulence of Edge Tokamak Plasmas

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Abstract. A magnetic island can develop in the vicinity of a flow instability and/or a turbulent region. Understanding of the interaction between islands and flows is still an open question, the underlying mechanisms depending on many parameters among which the nature of the instability, the size of the island and the close presence or not of other islands. In this work, we present three separate studies, showing the influence of this parameters on the triggered magnetohydrodynamic (MHD) activity.

1. Introduction

Pressure gradient instabilities can lead to the generation of turbulent flows and are located in the vicinity of resonant surfaces. The role of such instabilities and, more generally, of the microturbulence island dynamics still needs investigations. Several experiments report the coexistence of both microturbulence and MHD activities showing some correlated effects[1, 2, 3]. For instance, such activities are observed in reversed shear plasmas in presence of a transport barrier related to zonal flows and micro-turbulencet[4]. The rotation of magnetic islands can lead to the sub-critical excitation of magnetic islands. The rotation can be modified by the presence or not of instabilities and/or turbulence. In this paper, we investigate the rotation of the drift-tearing mode, focusing first on the role of the viscosity and the self-generated zonal flow on the rotation frequency. The direction and the amplitude of the island rotation appears to be strongly influenced by the viscosity (section 2). Linear and nonlinearly generated diamagnetic velocities are also at the origin of the rotation of the plasma. In a second study (section 3), we are interested in small islands in interaction with an interchange turbulence. We obtain and characterize regimes where nonlinear diamagnetic effects are more important than the zonal flow. In the last section (section 4), we show that the nonlinear interaction of two neoclassical tearing modes can generate strong energetic fluctuations and enhance the MHD activity, even if the resonnant surfaces are well separated .

2. Influence of the viscosity on the rotation of an island

The rotation of magnetic islands is an important factor for the sub-critical excitation of magnetic islands via the polarization current[5, 6]. Moreover, once the rotation of magnetic islands is locked by the resistive wall, the rotation of plasmas is also slowed down, which causes the degradation of the confinement: for example, the breaking of the transport barrier and continuously the destabilization of the turbulence and the MHD activity.

In this study, we investigate the rotation of the drift-tearing mode. To this aim, we use a four-field reduced set of two-fluid equations and examine the dependence of the rotation frequency of magnetic islands on viscosity. Detailed study of nonlinear interactions in determining the rotation frequency of magnetic island is presented.

Large aspect ratio tokamak plasmas ($\varepsilon = a/R_0 \ll 1$, where ε, a and R_0 are the inverse aspect ratio, the minor and the major radii, respectively) in the cylindrical coordinate (r,θ,z) are considered. We introduce a reduced set of two-fluid equations derived from Braginskii's two-fluid equations[9], which describes the drift-tearing mode (the linearly unstable classical tearing mode combined with pressure gradient). The model equations are

$$\frac{D}{Dt}\nabla_\perp^2\phi = \nabla_\parallel j_\parallel + \mu\nabla_\perp^4\phi, \tag{1}$$

$$\frac{\partial}{\partial t}A = -\nabla_\parallel(\phi - \delta p) - \eta_\parallel j_\parallel + \alpha_T\delta\nabla_\parallel T, \tag{2}$$

$$\frac{D}{Dt}n + \beta\frac{D}{Dt}p = \delta\beta\nabla_\parallel j_\parallel + \eta_\perp\beta\nabla_\perp^2 p, \tag{3}$$

$$\frac{3}{2}\frac{D}{Dt}T - \frac{D}{Dt}n = \alpha_T\delta\beta\nabla_\parallel j_\parallel + \varepsilon^2\chi_\parallel\nabla_\parallel^2 T + \chi_\perp\nabla_\perp^2 T, \tag{4}$$

where $j_\parallel = -\nabla_\perp^2 A$, $\alpha_T = 0.71$, $\frac{D}{Dt} = \frac{\partial}{\partial t} + [\phi,\ ]$, $\nabla_\parallel = \frac{\partial}{\partial z} - [A,\ ]$, $\nabla_\perp = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial\theta}$, $[f,g] = \hat{\mathbf{z}}\cdot\nabla f\times\nabla g$. $(\hat{\mathbf{r}},\hat{\boldsymbol{\theta}},\hat{\mathbf{z}})$ indicate unit vectors. $\{\phi,A,n,T,p\}$ indicate electrostatic potential, vector potential parallel to the ambient magnetic field, electron density, electron temperature and electron pressure defined by $p = n+T$, respectively. $\{\mu,\eta_\parallel,\eta_\perp,\chi_\parallel,\chi_\perp\}$ are ion viscosity, parallel resistivity, perpendicular resistivity, parallel and perpendicular thermal conductivity, respectively. In the dissipationless limit, the energy is conserved. $\{\delta,\beta\}$ indicate ion skin depth normalized by minor radius and plasma beta value at the plasma center, respectively. The normalization is $\{\varepsilon v_A t/a \to t,\ r/a \to r,\ z/R_0 \to z\}$, where v_A is Alfvén velocity. The perturbed quantity $f(r,\theta,z,t)$ is assumed to vary as $f_0(r) + \sum_{m,n}\tilde{f}_{m,n}(r,t)\exp\{i(m\theta - nz)\}$, where m and n are the poloidal and toroidal mode number. The perturbation $\tilde{f}_{m,n}(r)$ satisfies boundary conditions: $\tilde{f}_{m,n}(0) = \tilde{f}_{m,n}(1) = 0$ for $m,n \neq 0$ and $\partial\tilde{f}_{0,0}/\partial r|_{r=0} = \tilde{f}_{0,0}(1) = 0$ (the center and edge of plasma correspond to $r = 0$ and $r = 1$). The equilibrium quantities are chosen such that $q(r) = 1.5 + 0.5\left(\frac{r}{r_s}\right)^3$ ($q(r)$ stands for the safety factor defined by $\frac{1}{q(r)} = -\frac{1}{r}\frac{d}{dr}A_0(r)$), $n_0'(r) = -\frac{2\beta}{\varepsilon}r$, $T_0'(r) = -\frac{2\beta}{\varepsilon}r$, where the prime indicates the radial derivative. The equilibrium current $j_0(r)$ is evaluated by the cylindrical MHD equilibrium. The stability parameter for the tearing mode is given by $\Delta' = 17.78 > 0$. In the numerical simulation, we set $r_s = 0.6, \varepsilon = 0.2, \beta = 0.01$ and $\delta = 0.01$ and default values of the transport coefficients are chosen as $\mu = 10^{-5}, \eta_\parallel = 10^{-5}, \eta_\perp = 2\times10^{-5}, \chi_\parallel = 1, \chi_\perp = 10^{-5}$. In this study, we adopt a predictor-corrector scheme to calculate time evolutions, and a spectral method to evaluate nonlinear terms, where only modes which satisfy $m/n = 2$ (resonant on the $q = 2$ surface) is considered, for simplicity. The radial grid has 512 meshes, and the time step is 0.01. For the spectral resolution, $-4 \le n \le 4$ Fourier modes are considered.

In our parameter regime, only $(m,n) = (2,1)$ mode is unstable and other higher modes (the drift wave) are stable. In the nonlinear simulation, magnetic island is excited at the rational

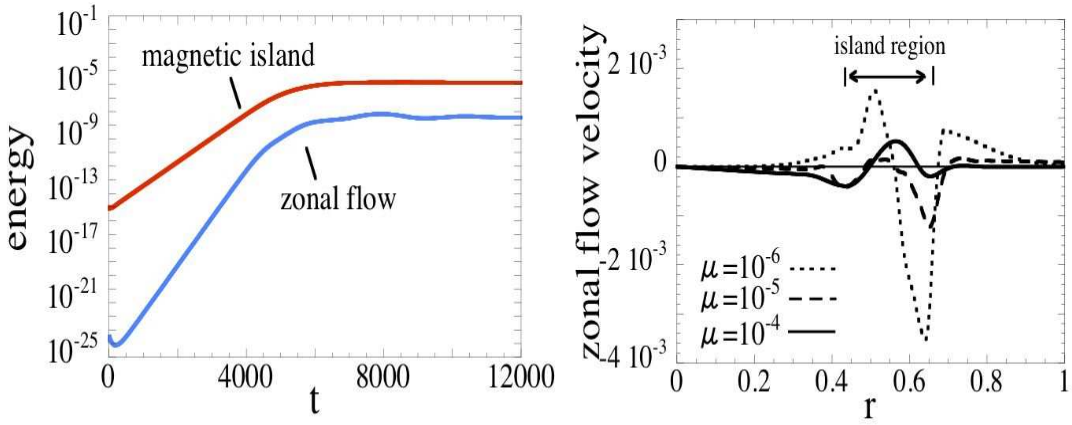

Figure 1: *Left: The time evolution of energy. Right: The radial pattern of zonal flow.*

surface $r = 0.6$, and the so-called Rutherford growth regime and the nonlinear saturation are observed. Figure 1 shows the time evolution of magnetic island and zonal flow. The growth rate of zonal flow is double of that of magnetic island, i.e. zonal flow is quali-linearly excited by magnetic island.

Figure 1 shows also the ion viscosity dependence of radial pattern of zonal flow velocity in the nonlinear saturation level. Three cases are plotted: $\mu = 10^{-6}$, $\mu = 10^{-5}$ and $\mu = 10^{-4}$. It is noted that the modification of saturation width of magnetic islands by ion viscosity is neglegible. It is found that, the amplitude of zonal flow is monotonically decreasing function of ion viscosity. In addition, the radial pattern of zonal flow inside magnetic islands is changed, according to the increase in ion viscosity. Zonal flow is generated by Reynolds stress and Maxwell stress, associated with electrostatic and electromagnetic fluctuations of magnetic islands, respectively. In our analysis, it is found that the strength ratio of Reynolds stress to Maxwell stress is a monotonic increasing function of ion viscosity; Reynolds stress is stronger in small ion viscosity regime, on the other hand, Maxwell stress is dominant in large ion viscosity regime. Therefore, the radial pattern of zonal flow is controlled by ion viscosity through changing the balance between Reynolds stress and Maxwell stress[8].

Figure 2 shows the rotation frequency of magnetic islands with different ion viscosity at the nonlinear saturation level. The positive and negative signs correspond to the direction of the electron and the ion diamangetic drift, respectively. It is found that the rotation frequency exhibits a monotonic increasing function of ion viscosity. Especially, depending on ion viscosity, the direction of the magnetic islands rotation is changed from negative to positive. We also examine the resistivity dependence, and it is found that the dependence of the rotation frequency on the magnetic Prandtl number $\mu/\eta_{\parallel}$ shows a monotonic increase.

From the $(2,1)$ component of Ohm's law, the rotation frequency of magnetic islands ω_r is approximately given by the summation of diamangetic drift and zonal flow as $\omega_r \approx \langle\tilde{\omega}_*\rangle + \langle\tilde{\omega}_{\mathrm{E\times B}}\rangle$, where $\tilde{\omega}_* = -\delta k_\theta \left\{ n_0' + \tilde{n}_0' + (1+\alpha_T)(T_0' + \tilde{T}_0') \right\}$, $\tilde{\omega}_{\mathrm{E\times B}} = k_\theta \tilde{\phi}_{0,0}'$, $k_\theta = 2/r$ and $\langle\ \rangle$ indicates the radial average inside magnetic islands[7]. The diamagnetic drift frequency has the positive contribution, and zonal flow give the main mechanism to change the direction of rotation.

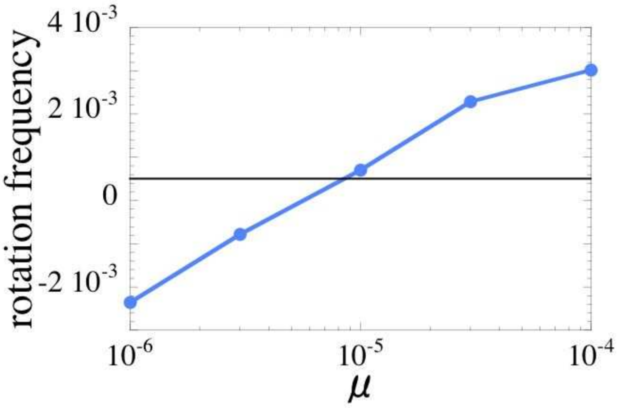

Figure 2: *The rotation frequency of magnetic islands* ($\langle\tilde{\omega}_*\rangle_{t=0} = 5.4\times10^{-3}$).

In conclusion, the rotation frequency of the drift-tearing mode strongly depends on the self-generated zonal flow, where the competition between Reynolds stress and Maxwell stress forms the energy sink of zonal flow, and the amplitude of zonal flow is controlled by the magnetic Prandtl number. In future works, such effects of self-generated zonal flow on the rotation of magnetic islands should be examined in more generalized cases, especially for the neoclassical tearing mode.

2. Impact of the nonlinear diamagnetic effects on a small island

In this section, we study the interaction of a magnetic island with small scale turbulence. More precisely, we focus on the effect of a pressure gradient, introducing an interchange like mechanism, on a magnetic island driven by a tearing instability. We also provide some insights on the origin of the nonlinearly generated island poloidal rotation.

A 2D slab reduced MHD based model, where interchange and tearing instability mechanisms are present, is adopted. This is in fact a model similar to the one used by Ottaviani [7], except we include curvature terms [10]. Typically, it involves a set of coupled equations for the electrostatic potential ϕ, the pressure of the electron p and the magnetic flux ψ:

$$\partial_t\nabla_\perp^2\phi+\left[\phi,\nabla_\perp^2\phi\right] = \left[\psi,\nabla_\perp^2\psi\right]-\kappa_1\partial_y p+\nu\nabla_\perp^4\phi, \quad (5)$$

$$\frac{\partial}{\partial t}p+[\phi,p] = -v_\star\left((1-\kappa_2)\frac{\partial\phi}{\partial y}+\kappa_2\frac{\partial p}{\partial y}\right)+C^2\left[\psi,\nabla_\perp^2\psi\right]+\chi_\perp\nabla_\perp^2 p, \quad (6)$$

$$\partial_t\psi+[\phi-p,\psi] = -v_\star\partial_y\psi+\eta\nabla_\perp^2\psi, \quad (7)$$

For simplicity, a focus on the interaction of basic mechanisms is done. In particular, the linear diamagnetic effect is suppressed, some interchange modes are unstable and a strong coupling between the pressure and the magnetic flux is chosen (high beta regime). Compare with the results of the previous paragraph, we should underline that we do not have parallel thermal conductivity terms, $\nu=\chi_\perp=\eta=10^{-4}$ and $\Delta'=6$. We also have 96 modes in the poloidal direction and the radial grid has 128 points. We use an order 4 Runge-Kutta temporal scheme and brackets are computed by means of a Harakawa scheme.

The important point is that the nature of the dynamics depends on the choice of the parameters.

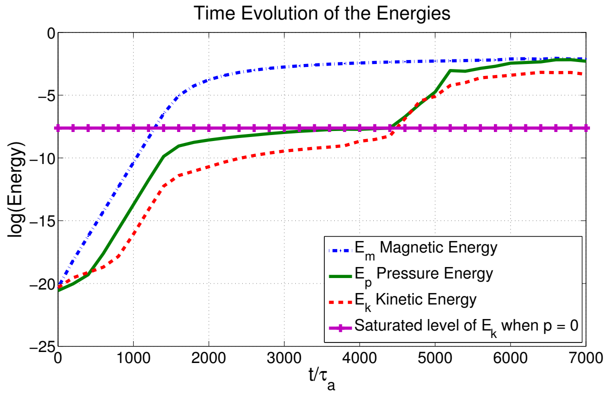

Figure 3: *Time Evolution of the Energies*

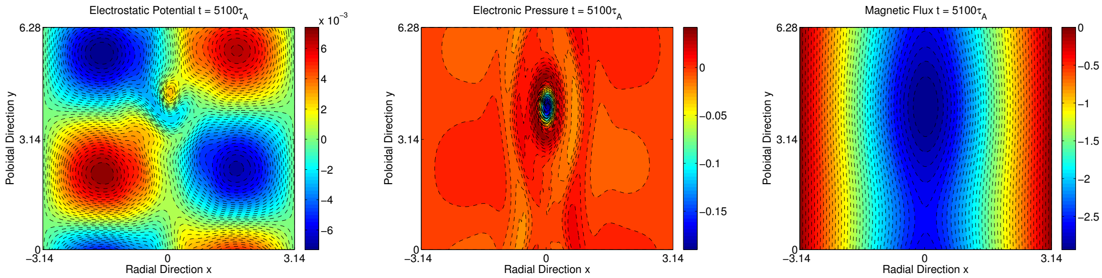

Figure 4: *Snapshot of the fields after the bifuration*

Analytically, we can show that it exists a critical resistivity value η_c such that if $\eta > \eta_c$, the nature of the linear tearing instability is roughly the one classically obtained by setting $p = 0$; but if $\eta < \eta_c$, the coupling between the pressure and magnetic flux is such that the island formation is driven by the pressure perturbation and not the flow. In the ideal limit, it is obtained $\eta_c = 0.58\,\Delta'^{-1/2}C^{5/2}$, where the coupling parameter is $C = \frac{\beta_e}{2(\Omega_i \tau_A)^2}$. β_e, Ω_i and τ_A are respectively the ion cyclotronic frequency and the Alfvén time. Setting parameters such that $\eta < \eta_c$, in figure (3) we show the time evolution of the magnetic, pressure and kinetic energies. Four regimes are observed. First, a linear regime where the magnetic island is formed. Second, the system reaches a quasi-plateau phase characterized by a slow growth of the energies. Then, the interchange unstable modes growth and a bifurcation occurs. Finally, the system reaches a new kind of dynamics. During the two first regimes, we can note that, from an energetic point of view, magnetic flux and pressure both dominate, the kinetic energy being too weak to let the flow stabilizes the island. In fact, owing to the high value of the coupling parameter, the linear and nonlinear growths of the magnetic island are controlled by an interplay between the pressure and the magnetic flux. More precisely, the magnetic island is maintained by pressure cells located in the vicinity of the separatrices. Moreover, theses pressure cells first generate a poloidal motion of the island and, second compress the current sheath. In these two regimes, the electrostatic potential does not play any fundamental role in the dynamics. However, during the second phase, a part of the kinetic energy is not dissipated into the magnetic island. As a result, the kinetic energy increases gradually. In fact, far from the island, the current is not significant and the flow is fed mainly by two mechanisms: incomming flow generated into the island and electrostatic interchange process. Initially, the latter is energetically the weakest. At the end of the second regime, the flow has enough energy to let a coherent large scale interchange process to occur outside the island and becomes in competition with the tearing

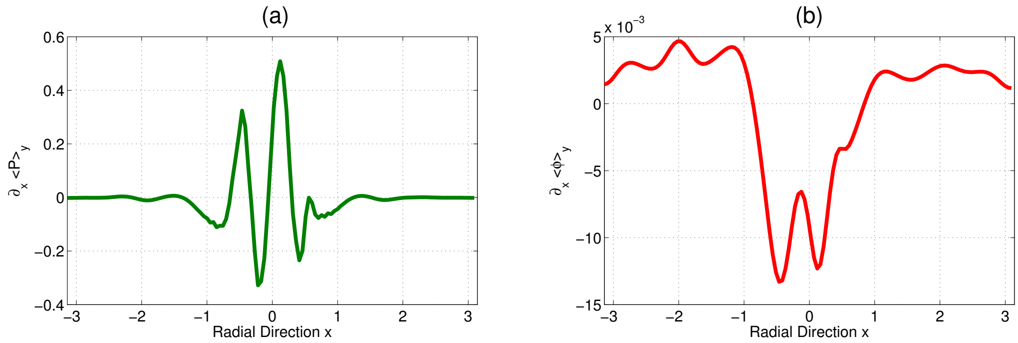

Figure 5: *Diamagnetic velocity (a) and zonal flow (b) versus radial direction at* $t = 5500\tau_A$

structure, both having pressure cells. This competition leads to a strong generation of small scales which, back, participate at the dynamics.

Finally the two structures are not compatible and a transition occurs. This is around $t \sim 5100\tau_A$ in the figure (3) where an abrupt growth of the kinetic and pressure energies is observed. The dynamical system finally adopts a new behaviour. On figure (4) snapshots of ϕ, p and ψ at $t = 5100\tau_A$ are presented. It shows that the competition between interchange and tearing structures leads to a drastic change of the presure topology. In fact, after the bifurcation, a pressure island appears inside de the magnetic island. Let us precise that whereas the magnetic island is a quasi linear structure mainly linked to the mode $k_y = 1$, the pressure island is a fully nonlinear structure characterized by an increase of many modes energie ($k_y < 8$) after the bifurcation.

The rotation of the magnetic island is of course affected by these processes and the formation of the structures. Actually, from the beginning of the nonlinear regime ($t = 2000\tau_A$), a poloidal acceleration of the magnetic island is observed. Moreover, at the bifurcation ($t = 5100\tau_A$), a strong increase of the velocity and a change of direction of the nonlinear poloidal rotation occurs. Some insights into the origin of this island poloidal rotation can be obtained. Indeed, as a first approximation, we can summarize the sources of the poloidal motion of the island to the action of both the self-generated zonal flow, $v_{zon} = \frac{\partial}{\partial x} < \phi >_y$, and self-generated diamagnetic flow, $v_{dia} = \frac{\partial}{\partial x} < p >_y$ (brackets mean an average over the poloidal direction). In order to understand more precisely the role of these flows in the island rotation, diamagnetic velocity (a) and zonal flow (b) versus radial direction at $t = 5500\tau_A$ are presented on the figure (5). We can note that in terms of amplitude the diamagnetic velocity is largely higher than the zonal flow and so mainly governed the nonlinear island poloidal rotation.

3. Nonlinear interaction of multiple NTMs in tokamaks

Neoclassical tearing modes (NTMs) are driven by the perturbed bootstrap currents and can limit the normalized plasma β_N of a tokamak. Recent experiments on ASDEX and JET show evidence of mode coupling effects influencing the evolution of NTMs [11, 12]. Past studies suggest that the interaction can arise due to harmonic coupling of the waves [12, 13] or from stochastic coupling due to overlapping of the modes [14].

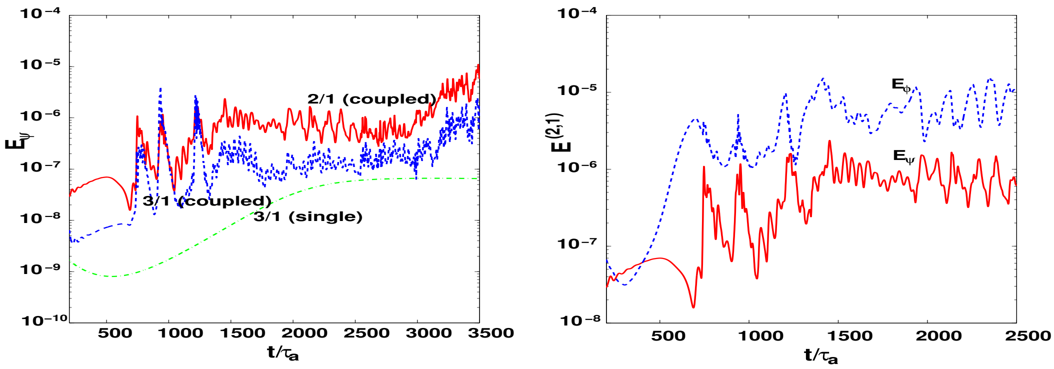

Figure 6: *Evolutions of magnetic energies with time for coupled and single NTMs (on the left) and comparison of magnetic and flow energies (on the right) for* $\Delta' < 0$*, finite* μ_e

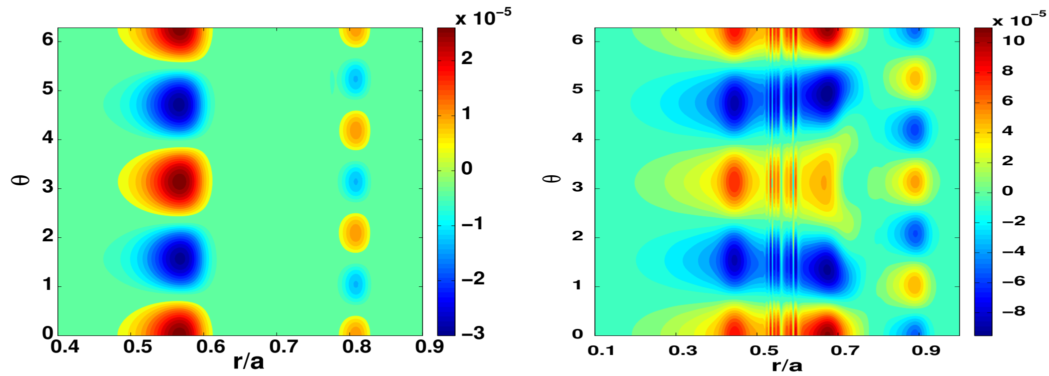

Figure 7: Φ *contours of coupled tearing modes with* $\Delta' > 0$*,* $\mu_e = 0$ *(on the left) and with* $\Delta' > 0$*, finite* μ_e *(on the right) at saturation*

Here we have carryied out numerical simulation studies of multiple NTMs using a fully toroidal code NEAR which solves a set of generalized reduced MHD equations [15, 16]. The model equations are,

$$
\begin{aligned}
&\frac{\partial\psi}{\partial t} - \nabla_{\parallel}\phi = \eta J_{\parallel} - \frac{1}{ne}\mathbf{b_0}\cdot\nabla\cdot\pi_{\mathbf{e}} \\
&\frac{dU}{dt} = \boldsymbol{B}_0\nabla_{\parallel}\left(\frac{J_{\parallel}}{B_0}\right) + \nabla\cdot\frac{\mathbf{B_0}\times\nabla p}{B_0^2} \\
&\frac{dp}{dt} + \Gamma p\nabla\cdot\mathbf{V} = -(\Gamma-1)\nabla\cdot\mathbf{q} \\
&\frac{dV_{\parallel}}{dt} = -\nabla_{\parallel}p
\end{aligned}
\tag{8}
$$

where, $U = \nabla\cdot\left(\frac{\nabla\phi}{B_0^2}\right)$, $J_{\parallel} = \nabla_{\perp}^2\psi$, $\mathbf{q} = -\chi_{\perp}\nabla_{\perp}p - \chi_{\parallel}\nabla_{\parallel}p$, $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla$, $\mathbf{V} = \frac{\mathbf{B_0}\times\nabla\phi}{B_0^2} + \mathbf{V}_{\parallel}$ and other notations are standard. We have taken (m,n)=(2,1) and (3,1) as perturbed modes (m is poloidal and n is toroidal mode number) to ensure well separated resonant surfaces. We have a circular equilibrium obtained from TOQ code [17] with R/a $\sim$ 10, S ($=\tau_R/\tau_A$) is 10^5 and toroidal $\beta_0 = 0.009$. This equilibrium is stable for the (2,1) and (3,1) classical tearing modes ($\Delta' < 0$). These modes can be destabilized by the neoclassical driving term which is proportional to viscosity μ_e. The left panel of Fig.6 shows that coupled NTMs lead to oscillations in the energy while there are no such oscillations for a single (3,1) NTM evolution. We have noted that the nature of the oscillations depends on the plasma β_p values. The right panel of Fig.6 shows that coupled NTMs also generate large perpendicular flows. Unlike the pure classical tearing modes case in left panel of Fig.7, the flows in presence neoclassical viscosity term are not restricted to their respective resonant surfaces as seen in right panel of Fig.7. Small scale flows appear inside the resonance layer and an expanded flow pattern exists outside the resonant surfaces providing coupling between the two modes. It is to be noted that none of these phenomena appear for linear runs and these are basically nonlinear effects. We have also observed that similar oscillations with neoclassical term for an equilibrium which is slightly unstable ($\Delta^{'} > 0$). In absence of neoclassical term we have not seen any such oscillations. The nature of these oscillations have some similarities with GAMs i.e. geodesic accoustic modes [18, 19]. Just like GAMs, the oscillations are accompanied or triggered by higher $\phi^{(1,0)}$, $p^{(0,0)}$ and $p^{(1,0)}$ modes. The characteristic frequency of the oscillations, calculated from the power spectrum of energy evolution of Fig.6 is found to be in the range of values typical for GAMs.

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