Modeling of Noble Gas Injection into Tokamak Plasmas

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Abstract. Noble gas injection for mitigation of the disruption in DIII-D is simulated. The simulation of the first two stages is performed: of the neutral gas jet penetration through the background plasmas, and of the thermal quench. In order to simulate the first stage the 1.5-dimensional numerical code LLP with improved radiation model for noble gas is used. It is demonstrated that the jet remains mainly neutral and thus is able to penetrate to the central region of the tokamak in accordance with experimental observations. Plasma cooling at this stage is provided by the energy exchange with the jet. The radiation is relatively small, and the plasma thermal energy is spent mainly on the jet expansion. The magnetic surfaces in contact with the jet are cooled significantly. The cooling front propagates towards the plasma center. The simulations of the plasma column dynamics in the presence of moving jet is performed by means of the free boundary transport modeling DINA code. It has been shown that the cooling front is accompanied by strongly localized “shark fin-like” perturbation in toroidal current density profile. After few milliseconds the jet (together with the current perturbation) achieves the region where safety factor is slightly higher than unity and a new type of the non-local kink mode develops. The unstable kink perturbation is non-resonant for any magnetic surface, both inside the plasma column, and in the vacuum space. The mode disturbs mainly the core region. The growth time of the “shark fin-like” mode is higher than the Alfven time by a factor of 100 for DIII-D parameters. Hence, the simulation describes the DIII-D experimental results, at least, qualitatively.

1. Introduction

One of the most important problems for ITER is the problem of disruptions. Recent experiments in DIII-D tokamak have shown the mitigation of the deleterious effects from tokamak disruption by a high-pressure noble gas injection [1]. At the first stage of the experiment the penetration of the jet into plasma core without any significant MHD activity has been observed. Magnetic perturbations rise rapidly after jet arrival to some critical internal magnetic surface. The MHD instability causes the thermal quench at the second fast stage. This instability mixes the central plasma region and brings the noble gas ions into plasma center in a short time. The central temperature decreases by a factor of 2 or 3 after mixing. Strong radiation and following fast plasma cooling is typical for the third stage. The third stage has been investigated in Ref. [1] with zero-dimensional code KPRAD as well as the relativistic electron generation. The jet deep penetration into plasma core together with plasma-impurity mixing has not been discussed earlier.

Jet expansion along and across magnetic fields by means of 1.5-D LLP code was simulated in [2]. Results of this simulation demonstrate that the jet remains mainly neutral and is able to penetrate deep to the background plasma. The jet penetration to the plasma column cools plasma on the magnetic surfaces contacting with the jet. The corresponding perturbed temperature profile initiates current redistribution. Below the current profile redistribution is analyzed by two means. First it has been shown analytically that the temperature front is accompanied by the strongly localized current perturbation. Second, the numerically simulated “shark fin-like” form in the current profile has been obtained by means of DINA code [3]. With MHD-stability theory it has been shown that the "shark fin-like" profile becomes unstable when achieves the region with the safety parameter close to 1. The new type of kink mode arises. The mode is localized mainly inside the plasma column. The growth time of the "shark fin-like" mode is higher than the Alfven time by a factor of 100 for DIII-D parameters. The mode mixes the plasma column interior and carries noble gas ions into plasma center. The suggested pattern might explain experimental results [1]. The estimations show that the plasma at the third stage is optically thin, at least if the temperature is higher than few tens of eV. Hence, the KPRAD model [1] describes radiation cooling of plasmas adequately.

In this paper we pay attention mainly on the jet penetration and plasma mixing under DIII-D conditions. One can expect the similar results for ITER tokamak. However, the corresponding conclusions can be carried out after additional studying.

2. Model of jet penetration

The initially neutral jet in a tokamak plasma expands in B-parallel and B-perpendicular directions and becomes partially ionized. The ionized particles expand freely along the magnetic field lines while the cross-filed motion is affected by an interaction with the magnetic field. At some time instant, ionization sets in at the jet periphery and the expansion in the poloidal direction comes to a full stop. The respective stopping radius should be considered as the transverse jet size for further field-aligned simulations. Jet motion in the radial direction is provided by the polarization of the jet and the radial $\vec{E}\times\vec{B}$ drift (the initial jet velocity is $\vec{V}_0 = \frac{\vec{E}_0\times\vec{B}}{B^2}$). The polarization electric field of dipole type $E_0 = V_0 B$ should appear (see Fig.1).

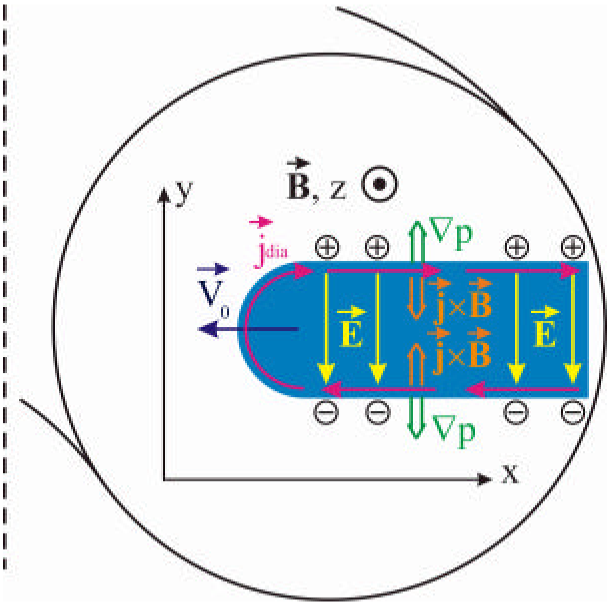

Fig.1. *Force balance and electric field in the jet penetrating into the tokamak.*

Calculations are performed for the parameters typical for DIII-D and ITER tokamaks. Jet parameters for DIII-D are taken from the experiment [1]. Jet expansion starts at $t=0$ with given cloud density $n_0 = 4\cdot 10^{24} m^{-3}$ (DIII-D), $n_0 = 4\cdot 10^{25} m^{-3}$ (ITER), temperature $T_0 = 300K$, cloud size $z_0 = r_0 = 7.5cm$ (cylindrical symmetry is assumed) and ionization degree $a = 0$ (this value corresponds to a jet temperature of about 300*K*). Jet velocity is $V_0 = 250 m/s$. The modeling is performed until the jet reaches the tokamak center, provided there is no radial deceleration, $t_{center} = a/V_0$, where a is the tokamak minor radius. (For DIII-D it means 2.5 ms, for ITER – 10 ms). Background density and temperature profiles are supposed to be known.

Numerical simulations are performed in the directions along and across magnetic field lines. Results obtained show that the jet do not freely expands in poloidal direction because the ionization degree of the order of 5% reached at the end of the calculation at the jet periphery is enough to confine the jet due to interaction with magnetic field. This conclusion is verified by the time evolution profile of the transverse jet size presented in Fig. 2. Here and below all results are shown for DIII-D background plasma parameters. From Fig. 2 one can see that the transverse size does not change significantly (by a factor of two) during jet penetration.

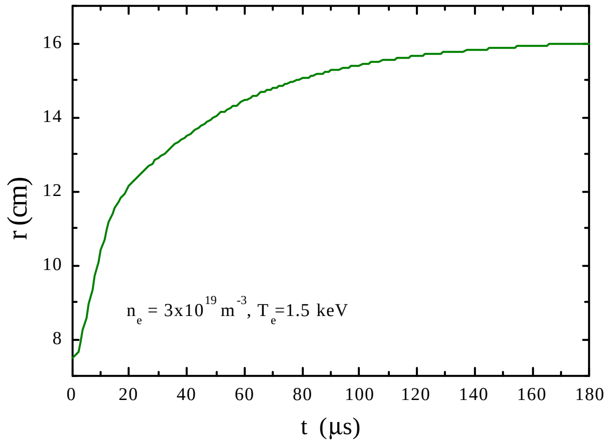

Fig. 2 *Temporal evolution of the transverse jet size.*

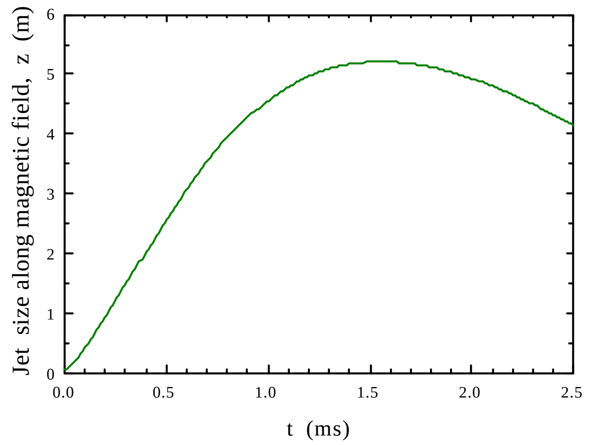

Fig. 3 *Temporal evolution of the longitudinal jet size.*

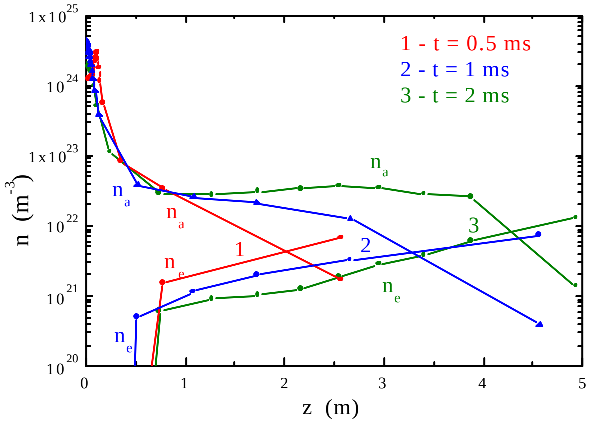

Fig. 4 *Density profiles along* $\vec{B}$ *for different moments.*

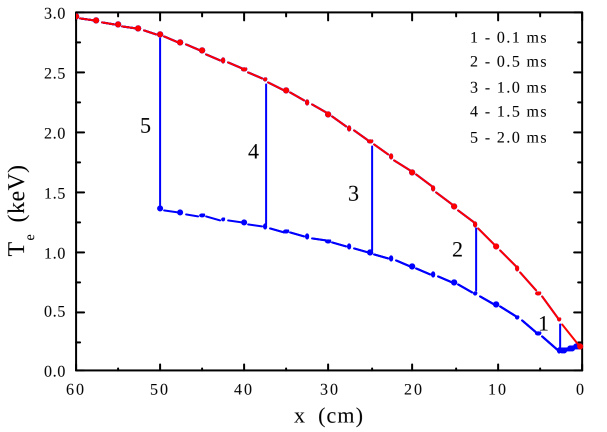

Fig. 5 *Temperature profiles for different moments.*

Simulation results along $\vec{B}$ are presented in Figs. 3, where shown is the evolution of the longitudinal jet size. The electron n_e and neutral density n_a profiles are shown in Fig. 4. It is seen that for the parameters of experiment [1] the most part of the jet remains neutral. The neutral jet, initially injected in the radial direction, is able to move further in the radial direction with the constant velocity. The mechanism is connected with polarization poloidal electric field and is discussed in [2].

While penetrating into the central tokamak regions the jet cools the hot background plasma. Resulting background temperature profiles are presented in Fig. 5, where $x = 0$ corresponds to the plasma boundary.

Similar profiles are used then as the input ones for the simulation of the plasma current profile evolution with DINA code.

3. "Shark fin-like" current profile simulation

With DINA code a simulation of plasma current profile evolution in #90204 DIII-D shot and the Start of Burn operation point in ITER 15 MA plasma inductive scenario have been performed. Step-like propagation of cooled electron temperature profile corresponding to Fig. 5. is used. The value $Z_{eff} = 2$ was taken. Plasma density $6\cdot 10^{19}$ m^{-3} for DIII-D and $8\cdot 10^{19}$ m^{-3} for ITER are supposed to be uniform in computation area. Plasma conductivity includes neoclassical effect of trapping particles. In Fig. 6 the time evolution of plasma current profile for this case is presented. During evolution a total plasma current is assumed to be fixed.

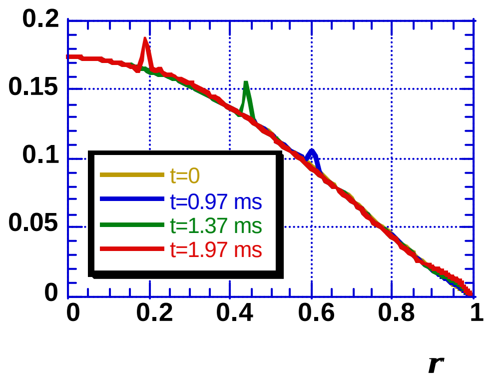

Fig. 6 *Current profile evolution*

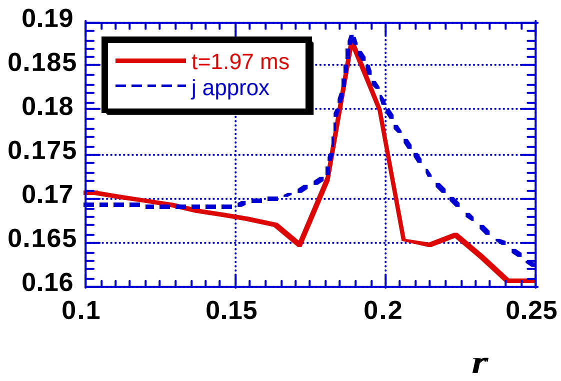

Fig. 7 *The "shark fin-like" perturbation of the initial current profile*

One can see the current spike propagating together with the temperature front.

The spike shape is transformed from triangular for earlier moments to the "shark fin" for latest moments (Fig. 7). The "shark fin" may be approximated by the expression

$$dj \approx \begin{cases} 6.6\cdot 10^{-3}\cdot\left(1-1.25\left(\sqrt{1-\mathbf{r}^4/b^4}\right)\right) & if\ \ 0\le \mathbf{r}\le b \\ 1.25\cdot 10^{-2}\cdot\exp\left(-C\left(\mathbf{r}-b\right)\right); & if\ \ b\le \mathbf{r}\le 1 \end{cases} \quad (\mathrm{A/m^2}) \tag{3.1}$$

(Blue line in Fig. 7) Here $\mathbf{r} = r/a$, b determines the position of the spike, $C \approx 35$. In ITER profile for selected scenario the spike shape keeps the triangular form. The relative magnitude is small.

The appearance of the spike may be understood by a simple linear theory for the ideally conducting plasma. Using Maxwell equations

$$m_0\cdot \mathrm{curl}\ \vec{B} = \vec{j}\ ,\ \mathrm{curl}\ \vec{E} = -\frac{\partial \vec{B}}{\partial t},$$

plasma momentum equation $Mn\dfrac{\partial \vec{V}}{\partial t} = -\nabla p + \left[\vec{j}\times\vec{B}\right]$, and assuming the infinite conductivity, $\vec{E} + \left[\vec{V}\times\vec{B}\right] = 0$ one can find:

$$Mn\frac{\partial V_r}{\partial t} = -\frac{\partial p}{\partial r} + j_q B_j - j_j B_q ,$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(rV_r B_j\right)\right) = \frac{1}{m_0}\frac{\partial j_j}{\partial t} .$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(rV_r B_q\right)\right) = \frac{1}{m_0}\frac{\partial j_q}{\partial t} .$$

Here the subscripts r, q, and j mark radial, poloidal and toroidal components, respectively. The cylindrical symmetry is assumed. One can assume the initial magnetic field and current to be independent on r at the spike size. Also, the cylindrical surface curvature may be ignored to the zero approximation. Hence, all spike functions can be assumed to be the functions of the new variable $x = r - V_0 t$. It yields for perturbed value of the current density:

$$dj_j = -\frac{1}{1 - V_0^2 / c_A^2} \cdot \frac{B_q^0}{\left(B_q^0\right)^2 + \left(B_j^0\right)^2}\frac{\partial dp}{\partial x} .\ \text{Here}\ c_A^2 = \frac{B_j^2}{m_0 M\, n} >> V_{jet} .$$

One can see that the spike magnitude is determined by the pressure perturbation gradient and almost independent on V_0. Hence, the magnitude of the spike coincides with the toroidal component of the diamagnetic current. Estimation of this current spike using the simulation parameters results in the value of about 10% of the Ohmic current.. The accurate shape of the “shark fin” is obtained from the non-linear numerical simulation (red line in Fig 7.) This current perturbation is the toroidal component of the transversal current. The transversal current cannot be compensated by the inductive parallel one. On the other hand, the local bootstrap current inducing by the step-pressure gradient is compensated by the parallel inductive current. Time scale of the step pressure gradient location is much smaller than the skin time. Hence, the bootstrap current effect may be ignored.

4. Ideal MHD stability.

The current column stability in a strong longitudinal magnetic field $B_j >> B_q$ has been investigated by many authors. The generation of the localized flute-like modes, internal kink modes, and tearing modes is possible if the resonant helical disturbance inside the plasma column ($0 < r_0 < a$) appears,

$$k_{||}(r_0) = \frac{1}{R}\left[mm(r_0) - n\right] = 0 . \qquad (4.1)$$

Here R is the major radius, and $m = \frac{RB_q}{rB_j}$. If the resonance exists in the vacuum space ($a < r_0 < r_w$, where r_w is the conducting wall radius) the non-local kink mode with the perturbed boundary may be driven.

The large scale MHD instability in DIII-D noble gas injection experiments is a real challenge for the stability theory. The initial current profile is stable. “Shark fin-like” perturbation is not able to disturb the current profile sufficiently to produce the resonant surface in the vacuum space. Hence, the external kink mode cannot be excited. The resonant surface may occur near the magnetic axis if the spike approaches very closely to it (see Fig. 8). Hence, tearing and other modes may be excited. However, tearing modes are too slow. Other instabilities are the small scale ones.

As is shown below, the new type of the ideal kink mode with perturbed boundary may be excited. The unstable perturbation is non-resonant for any magnetic surface, both inside the plasma column and in the vacuum space. For simplicity, modes $m \geq 2$ are considered in order to neglect toroidal effects.

In this case the helical perturbation of magnetic field may be described by the following dimensionless equation [4]:

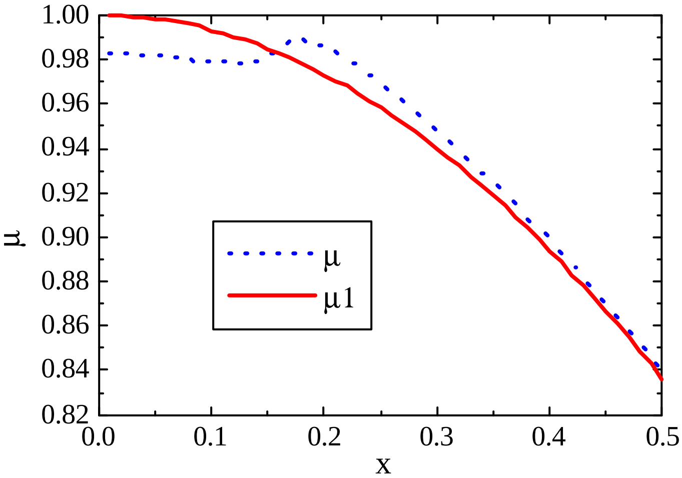

Fig. 8. *Radial dependence of the initial (solid line) and perturbed (dotted line) m values*

$$\frac{\hat{g}^2}{\hat{k}_\parallel}\Delta_\perp \frac{y}{\hat{k}_\parallel} + r^2 \Delta_\perp y = r\left(\frac{m}{\hat{k}_\parallel}\frac{d\hat{j}}{dr} + \left(\frac{2B_q^2}{B_j^2} + \frac{d\ln B_j}{dr}\right)\frac{mm_0}{B_q^2 \hat{k}_\parallel^2} m^2 \frac{dp}{dr}\right)y \,. \qquad (4.2)$$

Here y is the normalized electromagnetic potential, $\hat{g} = \frac{gR}{V_A}, g$ is the growth rate, $\hat{j} = \frac{m_0 R}{cB_j} j$, $\hat{k}_\parallel = k_\parallel R$, $m = \frac{1}{r}\int_0^r \hat{j}\tilde{r}\, d\tilde{r}$, $r = r/a$, and $\Delta_\perp = \frac{1}{r}\frac{d}{dr} r \frac{d}{dr} - \frac{m^2}{r^2}$.

For the marginal stability $g = 0$ the equation (4.2) is reduced to the Schrödinger equation. It is more convenient to transform it into the first order nonlinear equation:

$$\frac{d\Phi}{dy} - m^2 + \Phi^2 = \left(\frac{m}{\hat{k}_\parallel}\frac{d\hat{j}}{dy} + \left(\frac{2B_q^2}{B_j^2} + \frac{d\ln B_j}{dy}\right)\frac{mm_0}{B_q^2 \hat{k}_\parallel^2} m^2 \frac{dp}{dy}\right) \qquad (4.3)$$

together with the boundary condition $\Phi(-\infty) = m$.

Here $\Phi = d\ln y / dy$, and $y = \ln r$. The stability criterion based on the general quantum mechanics principle may be formulated as following: plasma current column is unstable if and only if the solution of equation (4.3) has a negative singularity in any point between $-\infty$ and zero. For the first time the criterion has been introduced in Ref. [5].

The marginal stability has been found numerically with the equation (4.3) under the assumption $dB_j / dy = 0$ for the mode $m = 2,\ n = 2$. The initial current profile has been approximated by the expression:

$$j(r) = j_0\left(1 - r^2\right)^{1.35} + dj\ ,$$

where dj is determined by (3.1).

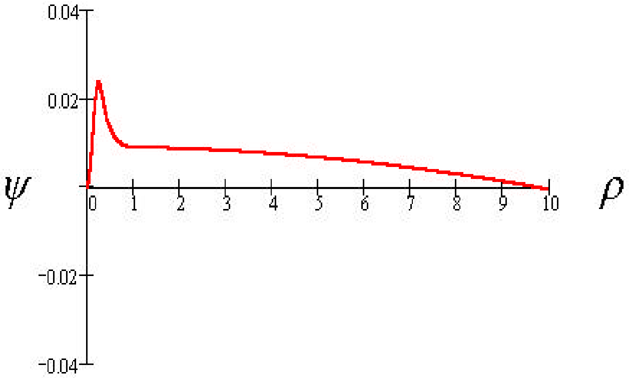

Fig. 9. *Eigenfunction corresponding to the marginal stability.*

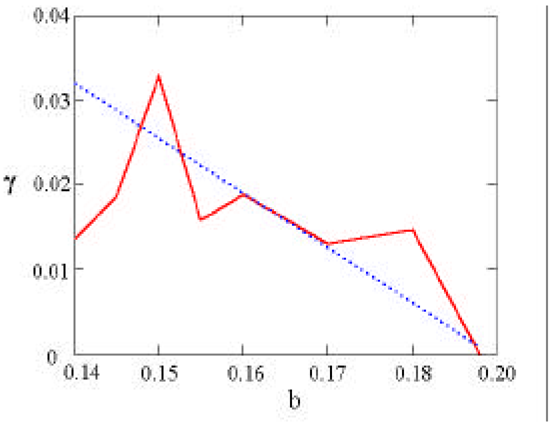

Fig. 10. *Dimensionless growth rate v/s the spike position*

The eigenfunction corresponding to the marginal stability has been obtained from (4.2). It is shown in Fig.9.

The growth rate of the mode may be estimated solving (4.3) with the right hand multiplied by factor $\hat{g} / k_{\min} + \hat{k}_{||}$. Here $k_{\min}$ corresponds to the minimal magnitude of $\hat{k}_{||}$ inside the spike. The result is shown in Fig. 10 (solid line). The dispersion of points is connected with numerical errors. Dotted line shows the approximation $\hat{g} = 0.123 - 0.65b$.

One can see that the instability appears when the temperature front penetrates inside plasma column to the distance from the magnetic axis $r \approx 0.2$. The result corresponds qualitatively to the experimental one $r \approx 0.3$ [1].

Dimensional growth rate may be estimated as $g \approx \sqrt{mk_{\min}} / t_A$. Here t_A is the Alfven time. For our example, $k_{\min} \approx 10^{-2}$, $A \approx 10^{-2}$, and $\tau_A \approx 10^{-7}\ s$. Hence $\gamma \approx 10^5\ s^{-1}$. This result is in agreement with experimental data, at least, qualitatively.

5. Summary

1.The noble gas jet remains mainly neutral and thus is able to penetrate up to the central region of the tokamak in accordance with experimental observations on DIII-D.
2. The "shark fin-like" spike on plasma current profile should be formed due to the step-like plasma temperature profile formed by the jet.
3. "Shark fin-like" spike excites the new type of instability after achieving the internal plasma region. For DIII-D conditions the growth time is estimated as 10^{-5} s.

4. The mode mixes the plasma column interior and carries noble gas ions into the plasma center. The suggested pattern might explain experimental results on DIII-D.

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7**. References**

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