

Effects of Toroidal Geometry and FLR Nonlocality of Fast Ions on Tearing Modes in Reversed Field Pinch

V.V. Mirnov 1), F. Ebrahimi 2), C.C. Kim 3), J. R. King 1), M. C. Miller 1),
J. A. Reusch 1), J. S. Sarff 1), D. D. Schnack 1), C. R. Sovinec 1), and T. D. Tharp 1)

- 1) University of Wisconsin-Madison, and the Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas, USA.
2) Space Science Center, University of New Hampshire, Durham, USA
3) University of Washington, Seattle, USA

E-mail contact of main author: vvmirnov@wisc.edu

Abstract: Two types of tearing modes drive impulsive reconnection in the Madison Symmetric Torus (MST) RFP: $m=1$ modes in the plasma core and $m=0$ edge modes resonant at the reversal surface where the equilibrium toroidal field changes its sign. When $m=0$ modes are not present there is also no change in stored magnetic energy, ion temperature, or toroidal plasma rotation. Motivated by the special importance of this mode, we report here the first computational results for an edge-resonant, $n=1$ tearing mode in a toroidal geometry in combination with nonlinear simulations of its cylindrical analog and new analytic and numerical results for finite Larmor radius (FLR) stabilization of tearing modes by neutral beam injected (NBI) fast ions. The structure of the toroidal $n=1$ edge resonant mode is substantially different from the cylindrical $m=0$, $n=1$ mode and agrees well with experimentally measured profiles. Cylindrical single fluid and two fluid MHD computations show a strong axial squeezing of the $m=0$ magnetic island caused by the nonlinear coupling of harmonics with $m=0$ and $1 < n < 200$. Analytical solutions obtained in the asymptotic limit of large Larmor orbits predicts a significant stabilizing effect of fast ions on tearing modes in the MST NBI experiments.

1. Introduction

Strong ion heating and plasma momentum transport are observed during periodic magnetic relaxation events (sawtooth cycles) in self-organized high temperature plasmas such as the reversed field pinch (RFP). The sawtooth cycle is characterized by a slow growth phase followed by a short “crash” phase in which magnetic reconnection rapidly alters the field, transfers magnetic energy to ion thermal energy, and modifies mean plasma flows. Two types of tearing modes drive impulsive reconnection in the Madison Symmetric Torus (MST) RFP: $m=1$ modes in the plasma core and $m=0$ edge modes resonant at the reversal surface where the equilibrium toroidal field changes its sign. The core-resonant and edge-resonant modes have global structure, allowing them to interact nonlinearly. There is also geometrical coupling of the modes caused by the toroidal configuration of the system. It is seen experimentally that when $m=0$ modes are not present there is also no change in stored magnetic energy, ion temperature, or toroidal plasma rotation [1]. Motivated by the special importance of this mode for RFPs, we report here the first computational results for an edge-resonant, $n=1$ tearing mode in a toroidal geometry in combination with nonlinear simulations of its cylindrical analog and new analytic and numerical results for finite Larmor radius (FLR) stabilization of tearing modes by neutral beam injected fast ions. The key findings are: **(1)** the structure of the toroidal $n=1$ edge resonant mode is substantially different from the cylindrical $m=0$, $n=1$ mode and agrees well with experimentally measured profiles; **(2)** cylindrical single fluid and two fluid MHD computations show a strong axial squeezing of the $m=0$ magnetic island caused by the nonlinear coupling of a large number of edge resonant modes with $m=0$ and $1 < n < 20$; **(3)** analytical solutions obtained in the asymptotic limit of large Larmor orbits predicts a significant stabilizing effect of fast ions on tearing modes in the RFP. These predictions are in a qualitative agreement with the previous semi-analytical

approaches and recent measurements of neutral beam injection effect on $m=1$, $n=5$ tearing mode in MST. The analytical solution is also a key part of benchmarking program for numerical modeling using NIMROD with a pressure tensor calculated from the δf particle-in-cell method.

2. Toroidal Edge Resonant $n=1$ Mode

In typical MST discharges, it is believed that the edge resonant tearing mode is linearly stable and driven by coupling to the core modes. The coupling can be caused by toroidal effects or nonlinear interaction of the modes. In order to distinguish between these two mechanisms, we need to know the detailed structure of the toroidal edge resonant mode. This information is also valuable for correct interpretation of the magnetic probe measurements performed near plasma edge in MST [2]. At MST's magnetic Prandtl number, $P_m \sim 0.2$, and Lundquist number, $S \sim 10^6$, full nonlinear modeling of the sawtooth cycle in a toroidal geometry is computationally demanding and is only now becoming possible with state-of-the-art codes. To avoid these difficulties a new alternative approach is suggested. We report here magnetic $n=1$ mode structures calculated in cylindrical geometry using a linear resistive MHD eigenvalue code [3] and in a toroidal geometry from linear initial-value single-fluid simulations by NIMROD [4]. Though the edge resonant mode is nonlinearly driven in the experiment, the comparison with linear theory is justified by noting that slowly varying magnetic perturbations have robust global spatial structures. Since the timescales of the sawtooth cycle are much longer than the Alfvénic timescales, the magnetic profiles are described by the Newcomb-like marginal ideal MHD equations [5,6] and, therefore, weakly sensitive to the mechanism – linear or nonlinear – of the mode excitation. These arguments are confirmed by a strong quantitative agreement between the experimentally measured profile and the normalized toroidal computation as illustrated in Figure 1.

Both of these calculations (cylindrical and toroidal) require evaluation of an equilibrium that is unstable to the $n=1$ mode. It has been shown [7] that there is a wide class of equilibrium configurations for which the edge resonant mode is spontaneously unstable. A profile that

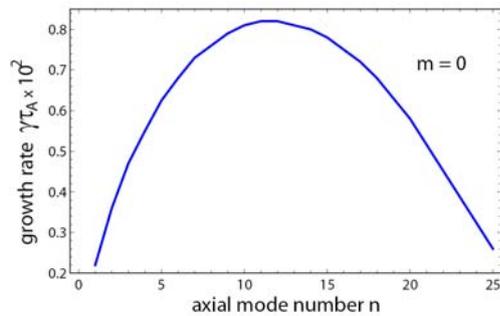


FIG. 2. Growth rate of $m=0$ mode as a function of axial mode number n

$S=10^5$, $\beta \sim 0.1\%$ and aspect ratio $R/a = 3$, the growth rate of this purely current driven tearing mode increases with n up to some peak value at $n \sim 10-15$ and then drops to zero at $n \sim 30$

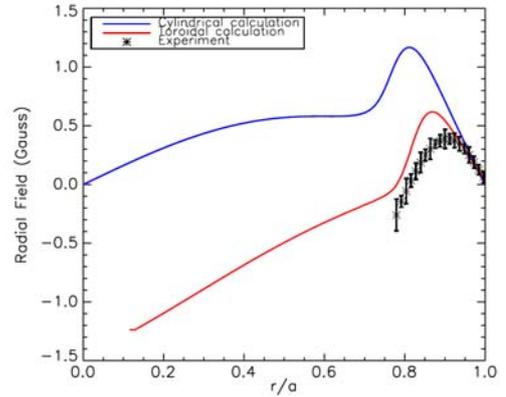


FIG. 1. Profiles of magnetic component B_r for $n=1$ mode from experimental measurements and numerical modeling

both supports an unstable $m=0$ mode and has an equilibrium close to that of the experiment (with pinch parameters $\Theta=1.8$, reversal parameter $F = -0.22$ and reversal surface at $r_s/a=0.76$), was chosen to evaluate the linear tearing instability for both cylindrical and toroidal geometries. This allows us to pick up an exponentially growing edge resonant tearing mode and identify its spatial structure. For $m=0$ modes, both cylindrical and toroidal computations show unusual dependence of the growth rate on the toroidal mode number n . For the given force-free equilibrium, Lundquist number

(see Figure 2). This dependence is opposite to a well known property of core-resonant tearing modes $m=1$, where the smallest toroidal wave numbers are most unstable.

The curves shown in Figures 3,4, illustrate a substantial difference between the cylindrical and toroidal computations of the spatial mode structures. Although the toroidal $n=1$ mode is constructed as a direct analog of the cylindrical $m=0$, $n=1$ mode, it has a substantial $|m|=1$ harmonic content arising from the toroidal effects at the MST aspect ratio $R/a=3$. The asymmetry is unexpectedly strong for the radial component B_R plotted in Figure 3 as a function of major radius R at $Z=0$ ($B_r^{(inboard)}/B_r^{(outboard)} \sim 6$). The effect of geometry and, correspondingly, the difference between cylindrical and toroidal profiles is much smaller for the vertical component B_Z shown in Figure 4 as a function of Z at $R=R_0$. The asymmetry is less significant for the poloidal and toroidal components ($\sim 1.5-2$).

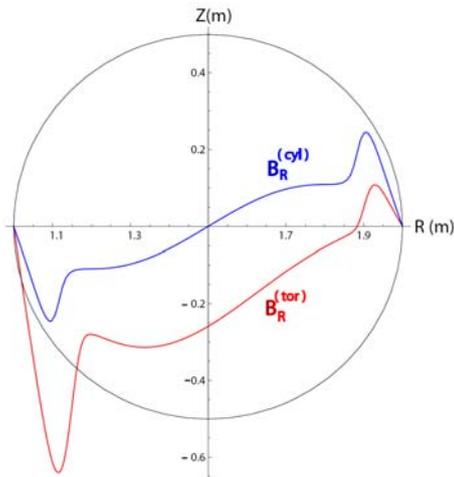


FIG. 3. Radial (horizontal) magnetic component $B_R(R,0)$ from cylindrical (blue) and toroidal (red) computations

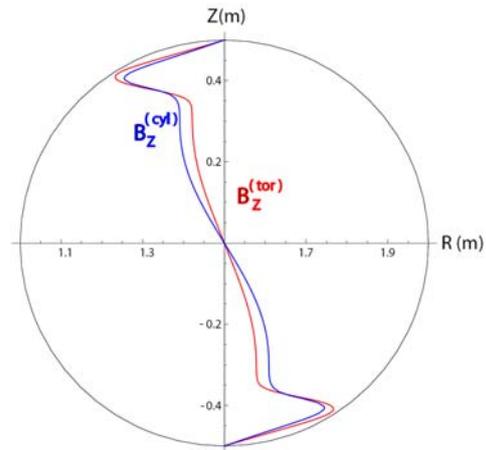


FIG. 4. Vertical component $B_Z(R_0,Z)$ for cylindrical (blue) and toroidal (red) cases

3. Nonlinear Cylindrical Simulations

Two different 3D resistive MHD computational models were used to study the role of nonlinear coupling in the excitation of the $m=0$ mode and magnetic reconnection near the reversal surface in the RFP. The standard way of visualization of magnetic reconnection in toroidal systems is a plotting of island configuration and positions of X and O-points in the poloidal (r, θ) -plane. This projection is not appropriate to the geometry of the edge-resonant $m=0$ tearing mode in RFP. Near the plasma edge, the guide magnetic field is in the θ -direction and magnetic islands are formed in the (r, z) -plane, which is the plane of reconnection and is poloidally symmetric. A typical picture of perturbed magnetic surfaces is shown in Figure 5 for $m=0$, $n=1$ mode in a periodic cylinder.

A single fluid DEBS simulation is performed in a periodic cylinder with $P_m \sim 250$ and $S=10^6$. At these parameters, well-defined, quasi-periodic sawteeth, similar to those seen in MST, are observed. In addition, a simulation with a compressible

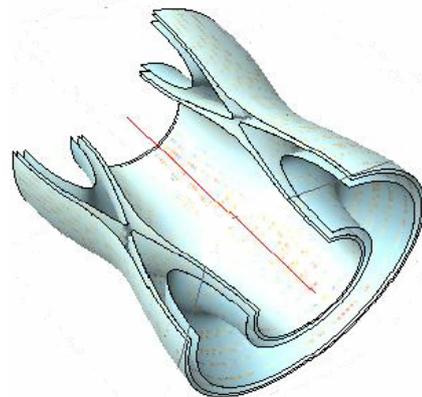


FIG. 5. 3D picture of magnetic perturbations driven by $m=0$, $n=1$ tearing mode

simulation with a compressible

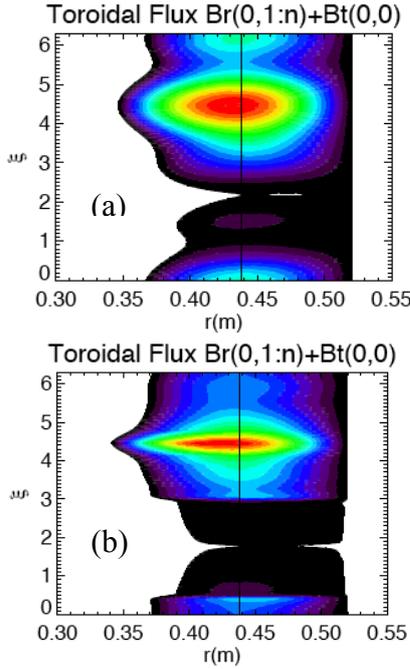


FIG. 6. Magnetic field lines (contours of toroidal flux) in (r, ζ) -plane (ζ is the toroidal angle). (a) $n=1-4$. (b) $n < 200$

caused by the nonlinear coupling of edge resonant modes with $m=0$ and large n numbers up to $n \sim 200$. The resulting nonlinear island structure is illustrated in Figure 6(b).

In the MST experiments, integration of the measured toroidal field from the wall using only the equilibrium field and the $n=1$ mode contribution results in the toroidal flux associated with the $n=1$ reconnection. Contours of this flux yield a visualization of 2D experimental field line picture, shown in Figure 7(a). The addition of the mode contributions from $n=2,3,4$ provide a more complete configuration of the magnetic field, seen in Figure 7(b). In particular the addition of these modes does not destroy the overall shape of the $n=1$ island, indicating that a coherent $m=0$ dominated structure exists in MST plasmas. Note, that inclusion of $n=2-4$ components leads to experimentally observed compression of the O-point and deformation of the structure of the X-point that is consistent with the aforementioned computational results from DEBS and NIMROD.

4. FLR Tearing Mode Stabilization by Fast Ions

A new capability for MST RFP experiments is the use of a long pulse (20 msec) of energetic

two-fluid model, including the generalized Ohm's law and ion gyroviscosity, is done with the NIMROD code at $S=5 \times 10^3$, $P_m=1$, $\beta=0.1$ and ion skin-depth $d_i/a=0.17$. At these parameters, the two-fluid simulations capture significant features of electron-ion decoupling across magnetic islands and permit exploration of the role of the Hall term and plasma compressibility in the dynamics of reconnection.

The reconnection field structure is visualized in the (r,z) -plane by plotting contours of toroidal flux obtained from DEBS and NIMROD outputs. Both DEBS and NIMROD computations contain a large number of interacting core and edge modes and exhibit similar nonlinear behavior. The mode interaction is shown to have a strong impact on the $m=0$ magnetic and flow structures in the vicinity of the reversal surface. Taking into account only the mean (equilibrium) toroidal field and a few field components ($n=1-4$) yields an island that is elongated in the axial direction, see Figure 6(a). During a relaxation event, significant compression in the axial direction of the $m=0$ magnetic island and flow structure through the presence of multiple n modes was found. This effect is

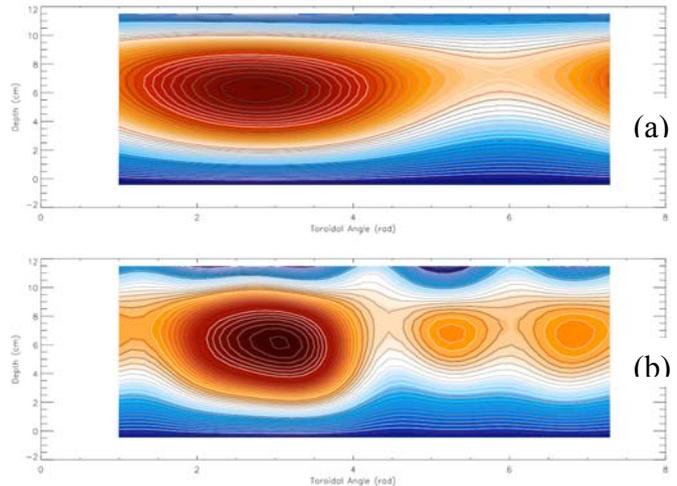


FIG. 7. Experimental measurements of edge-localized magnetic structures. (a) contribution from $n=1$ mode. (b) result of superposition of four modes $n=1-4$

(25 keV) high current (40A) neutral atoms for plasma heating. This motivates our interest in the stabilization of the tearing modes by the fast minority species. The effect of fast ions can

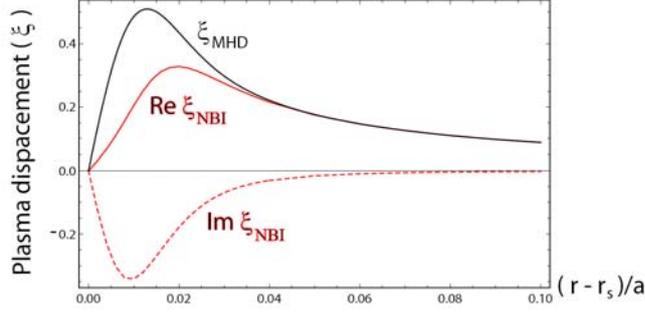


FIG. 8. Profiles of eigenfunctions for plasma displacement with FLR stabilization (red) and without stabilization (black, MHD case)

be both destabilizing (fishbone instability) and stabilizing (FLR mechanisms). In tokomaks, the high energy component decouples due to the large banana orbits of the energetic ions. In the RFP, the magnetic field is weaker and the corresponding gyroradius is much larger than in a tokomak, so that the FLR effects become dominant. In addition to this, tearing modes are strongly localized to the mode rational surfaces. Their narrow spatial scale emphasizes the importance of the FLR physics for RFPs. Our program of FLR studies includes the use of NIMROD with a hybrid drift-kinetic-MHD model to treat the full Lorentz equation of fast ion motion. This version of NIMROD has been benchmarked against M3D for the stabilization of $m=1, n=1$ internal kink modes in tokomaks [9]. We present here a simplified analytical model which allows us to analytically understand the evolution of the tearing modes in neutral-beam-injected RFPs. The model is also useful for benchmarking an existing approximate semi-analytic approach [10], as well as nonlinear simulations of fast particle with NIMROD, modified for full orbit kinetics to account for pressure tensor perturbations from high-energy ions [11].

In order to construct an analytically solvable model we consider the asymptotic limit of strong FLR effects, when the high energy component is completely decoupled from the bulk plasma. In this limit, the non-local effect of large Larmor orbits appears in the equations macroscopically within the scope of fluid formalism without solving the drift kinetic equation. Indeed, electron density, n_e , is a sum of bulk and hot ion densities, n_i and n_h respectively, where $n_h \ll n_i$. In this situation, the electron current, $-|e| n_e \mathbf{E} \times \mathbf{B} / B^2$, is not compensated by the bulk ion current, $|e| n_i \mathbf{E} \times \mathbf{B} / B^2$, yielding uncompensated cross-field current, $-|e| n_h \mathbf{E} \times \mathbf{B} / B^2$:

$$\mathbf{j}_{\perp}^{(add)} = -\frac{en_h}{cB(0)^2} \mathbf{E} \times \mathbf{B}^{(0)} = -en_h \mathbf{v}_{\perp}$$

If high energy ions moved according to the $\mathbf{E} \times \mathbf{B}$ drift, the additional electron current would be compensated by the fast ion response. We consider here the opposite situation where the Larmor orbits are large, and the response of high energy ions is strongly reduced. Considering the limiting case, the contribution from the fast ions to the cross-field current is set to zero. Then, the total current in plasma consists of ion polarization current, parallel current, and aforementioned additional current existing due to decoupling of the fast ions:

$$\mathbf{j} = \mathbf{j}_{\perp}^{(pol)} - en_h \mathbf{v}_{\perp} + j_{\parallel} \mathbf{b}$$

be both destabilizing (fishbone instability) and stabilizing (FLR mechanisms). In tokomaks, the high energy component decouples due to the large banana orbits of the energetic ions. In the RFP, the magnetic field is weaker and the corresponding gyroradius is much larger than in a tokomak, so that the FLR effects become dominant. In addition to this, tearing modes are strongly localized to the mode rational surfaces. Their narrow spatial scale emphasizes the

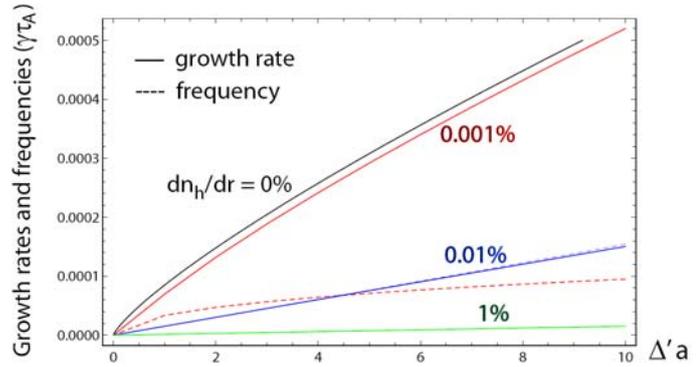


FIG. 9. Growth rate (solid line) and frequency (dashed line) vs tearing mode stability factor Δ' for different fast ion density gradients: 0% (no stabilization), $10^{-3}\%$ (weak stabilization), $10^{-2}\%$ (moderate stab.), 1% (strong stab.)

The additional current is proportional to the small factor n_h , but it also contains a large factor of the cross-field plasma flow velocity. The combination of these factors effectively reduces electric field across the reconnection layer, decreases the vortex flows and slows down the development of the instability. Taking into account the quasineutrality condition (divergence-free equation for the current) yields modified MHD vorticity equations, similar to Ref.[8], where effect of the high energy component is expressed locally in terms of the fast ion density:

$$-\nabla \cdot \mathbf{j}_{\perp}^{(pol)} + e \nabla \cdot n_h \mathbf{v}_{\perp} = \frac{(\mathbf{B}^{(0)} \cdot \nabla)}{B^{(0)}} j_{\parallel} + B_r \frac{d}{dr} (j_{\parallel}^{(0)} / B^{(0)})$$

We consider cylindrical force-free screw pinch equilibrium with uniform pressure, periodic boundary conditions and perturbations of the form:

$$f(r, \theta, z) = f(r) \exp(-i\omega t + im\theta - inz/R)$$

Keeping only leading terms with the highest radial derivatives gives a reduced form of the vorticity equation. This equation is used for the analysis of the boundary layer problem:

$$-i\rho\omega^2 \frac{\partial^2 \xi_r}{\partial r^2} - i \frac{e\omega \xi_r k_{\perp} B^{(0)}}{c} \frac{dn_h}{dr} = -\frac{F}{4\pi} \frac{\partial^2 B_r}{\partial r^2} + \frac{B_r k_{\perp} B^{(0)}}{c} \frac{d}{dr} (j_{\parallel}^{(0)} / B^{(0)})$$

Plasma displacement ξ_r and magnetic field perturbation B_r are also connected by the induction equation:

$$B_r - iF\xi_r = \delta^2 \nabla^2 B_r, \quad \delta^2 = \frac{i\eta c^2}{4\pi\omega}, \quad F = \mathbf{k} \cdot \mathbf{B}^{(0)}$$

Performing Taylor's expansion in the vicinity of the mode rational surface and assuming constant - ψ approximation yields the canonical form of the layer equations in normalized variables:

$$\xi'' - \left(x^2 + i \frac{k d_i S^{1/2} d\Lambda}{k'_{\parallel} \gamma^{1/2} dr} \right) \xi = -x, \quad B_r'' = \frac{\gamma^{3/2} S^{1/2} B_r}{k'_{\parallel}} (1 - x\xi)$$

where the effect of fast ions enter the equations via the density gradient $d\Lambda/dr$ ($\Lambda = n_h/n$). The first equation has a form of a nonuniform parabolic cylinder equation. We solve it for $\xi(r)$ (see, Figure 8) by applying a Fourier transform in the x-direction. The inner and outer solutions are matched by making use of the second equation and the tearing mode stability factor Δ' :

$$\Delta' = \frac{1}{B_r(0)} \left(\frac{dB_r}{dr} \Big|_{r_{s+0}} - \frac{dB_r}{dr} \Big|_{r_{s-0}} \right)$$

This procedure yields an analytical dispersion relation for the growth rate γ :

$$\Delta' = \frac{\sqrt{2}\pi\gamma^{5/4} S^{3/4}}{k'_{\parallel}{}^{1/2}} \left(\frac{a}{2} \right)^{1/2} \frac{1}{G(a)}, \quad a = i \frac{k d_i S^{1/2} d\Lambda}{k'_{\parallel} \gamma^{1/2} dr}$$

where

$$G(a) = \frac{a^{1/2} \Gamma(1/4 + a/4)}{2 \Gamma(3/4 + a/4)}; \quad G(a) \rightarrow \frac{a^{1/2} \Gamma(1/4)}{2 \Gamma(3/4)} \quad a \ll 1, \quad G(a) \rightarrow 1 \quad a \gg 1.$$

Two different regimes of instability are possible. They are determined by the value of the factor $a(\gamma)$ which also depends on the gradient of fast ion density, dn_h/dr , at the mode rational surface. The limiting case of small $|a| \ll 1$ corresponds to the standard resistive MHD regime of instability with the growth rate γ_{MHD} . The stabilizing effect of high energy particles plays no role in this case. In the opposite limit, $|a| \gg 1$, the growth rate becomes complex (oscillatory mode) with an absolute value which is significantly reduced in comparison with γ_{MHD} :

$$\gamma_{NB} = \frac{(1+i)}{\sqrt{2}\pi} \frac{\Delta' k'_{\parallel}}{k d_i S (d\Lambda/dr)^{1/2}}$$

Since the stabilizing effect depends on the magnitude of fast ion density gradient, the critical value of this factor can be found by equating $\gamma_{NB} = \gamma_{MHD}$:

$$\left(\frac{d\Lambda}{dr}\right)^{(cr)} \simeq \frac{\Delta'^{2/5} k_{\parallel}'^{6/5}}{k d_i S^{4/5}}$$

The dependence of the real and imaginary parts of γ on Δ' is plotted in Figure 9 for a wide range of density gradient values (smaller and larger than its critical value). Estimating dimensionless parameters as $\Delta' \sim 10$, $k_{\parallel}' \sim 0.3$, $k d_i \sim 0.2$, $S=10^6$, yields a critical gradient, $d\Lambda/dr = 5 \times 10^{-3} \%$, which is significantly smaller than the fast ion density gradient in the MST NBI experiments ($d\Lambda/dr \sim (1-10)\%$). This means that the regimes with oscillatory character of instability and strong reduction of the growth rate are relevant to the MST parameter range. These predictions are qualitatively consistent with recent high power neutral-beam-injection experiments on MST where a reduction of the amplitude and velocity of the core-resonant $m=1$, $n=5$ tearing mode was observed.

5. Summary

Three computational and analytical problems important for the physics of RFP operation are analyzed. First, numerical calculation of the spatial structure of toroidal tearing modes in the RFP reveals unexpectedly strong toroidal asymmetry of the radial magnetic component of the edge-resonant $n=1$ mode. The analog of this mode in the cylindrical model of the RFP is the azimuthally symmetric $m=0$ mode which does not exist in tokamaks and is a specific feature of the RFP magnetic configuration. This mode plays an important role in the nonlinear coupling between core and edge resonant modes. The nonlinear interaction between these modes is believed to be responsible for the global multi-helicity magnetic reconnection, ion heating, and momentum transport in the RFP. The information about the spatial structure of the toroidal $n=1$ mode allows us to evaluate the strength of the geometrical (toroidal) coupling between core and edge modes and to incorporate this factor empirically into the existing cylindrical RFP models. Strong inboard-outboard asymmetry of the radial magnetic component may also have a significant impact on the poloidal asymmetry of the plasma transport in stochastic magnetic field caused by the overlapping of the magnetic islands.

Robust linear stability of $m=0$ mode was demonstrated in the past for the class of equilibrium current profiles described by so-called two-parameter α -model [12]. A more flexible four-parameter expression for the current profile [7] yields a wide range of RFP magnetic configurations (close to that of the experiment) for which $m=0$ modes are linearly unstable. Using three different codes (an eigenvalue code, and cylindrical and toroidal initial-value NIMROD codes) we have obtained (see Sec.2) an unusual for force-free configurations result that the growth rate of spontaneously unstable $m=0$ tearing modes increases with the axial mode number n and the most unstable modes correspond to large values of $n \sim 10-15$. If the $m=0$ mode were linearly unstable in the experiment, one could expect the spectrum to be localized at $n \sim 10-15$. However, the spectrum of the edge-resonant modes observed in MST experiments is a decaying function of n with a well determined maximum at $n=1$ that is reliably measured for, at least, first four harmonics with $n=1-4$. We consider this newly derived dependence of the growth rate on the toroidal mode number n , as an additional argument in favor of nonlinear drive of edge-resonant modes in MST.

Using different computational tools in cylindrical geometry, we find that the cascade of harmonics with large axial mode numbers leads to a deformation of the edge-localized magnetic island. The island is squeezed in the toroidal direction. Existence of the island is associated with magnetic reconnection driven by nonlinear beating of the core-resonant modes near plasma edge. The compression (squeezing) of the island takes place in the axial

direction so that the resulting structure (O-point and two X-points) is localized within short interval of the toroidal angles $\sim 60^\circ$. These computational results are confirmed by experimental measurements of magnetic fluctuations and plasma flows in MST. The narrow angular structure assumes an existence of a broad n spectrum. The physics of nonlinear interactions leading to the broadening of the spectrum is not clear and requires more computational work.

New experiments with high power neutral beam injection in MST motivate our interest in stabilization of the tearing modes by finite Larmor radius effects. An analytical model is developed based on the assumption of strong FLR effect. In the presence of a large fast ion density gradient, an oscillatory instability with a strongly reduced growth rate is predicted. Since the model is based on the assumption of the strong FLR effects, it gives an upper limit for the expected stabilization effect. More detailed analysis will be done with the use of: (1) an approximate semi-analytic approach [10]; (2) nonlinear simulations of fast particles with NIMROD, modified for full orbit kinetics to account for pressure tensor perturbations from high-energy ions. The analytical model is a key element in benchmarking both computational approaches. The reduction of the growth rate is qualitatively consistent with the semi-analytical model but the oscillatory character of the instability was not found in Ref. [10]. Note that the modified NIMROD equations differ from the semi-analytical simulation, which incorporates the high energy component into Ampere's law while ignoring the high energy pressure tensor in the equation for the bulk plasma. We will benchmark these two approaches to verify their consistency.

References

- [1] GANGADHARA, S., et al., "Ion heating during reconnection in the Madison Symmetric Torus reversed field pinch", *Phys. of Plasmas* **15**, 056121 (2008).
- [2] THARP, T. D., et al., "Measurement of impulsive reconnection driven by nonlinear Hall dynamics", submitted to *Phys. Plasmas* (2010).
- [3] SVIDZINSKI, V.A., et al., "Modeling of ion heating from viscous damping of reconnection flows in the reversed field pinch", *Phys Plasmas* **15** 065511 (2008)
- [4] SOVINEC, C. R., et al., "Nonlinear magnetohydrodynamics simulation using high-order finite elements", *J. Comput. Phys.* **195** 355 (2004)
- [5] CONNOR, J. W., et al., "Tearing modes in toroidal geometry", *Phys. Fluids* **31**(3) 577 (1988).
- [6] ZANCA, P., et al., "Reconstruction of the magnetic perturbation in a toroidal reversed field pinch", *Plasma Phys. Control. Fusion* **46** 1115 (2004).
- [7] MIRNOV, V.V., et al., "Two fluid dynamo and edge-resonant $m=0$ tearing instability in reversed field pinch", *Fusion Energy 2006* (Proc. 21st IAEA Int. Fusion Energy Conf., Chengdu, TH/P3-18, 2007),
- [8] BERK, H. L., et al., "Theoretical interpretation of Alfvén cascades in tokamaks with nonmonotonic q profiles", *Phys. Rev. Lett.* **87**, 185002 (2001).
- [9] KIM., CHARLSON C., "Impact of velocity space distribution on hybrid kinetic-magnetohydrodynamic simulation of the (1,1) mode", *Phys. Plasmas* **15** 072507 (2008)
- [10] SVIDZINSKI, V.A., PRAGER, S.C., "Effects of particles with large gyroradii on resistive magnetohydrodynamic stability", *Phys. Plasmas* **11** 980 (2004)
- [11] KIM, C.C., et al., "Preliminary simulations of FLR effects on RFP tearing modes", *J. Fusion Energy* **27**, 61 (2008).
- [12] ORTOLANI, S., SCHNACK, D., "Magnetohydrodynamics of Plasma Relaxation", Singapore (1993).