Effects of strong $\mathbf{E} \times \mathbf{B}$ flow on gyrokinetics

N. Miyato[1)], B. D. Scott[2)], S. Tokuda[3)] and M. Yagi[4,1)]
1) Japan Atomic Energy Agency, Naka, Ibaraki, Japan
2) Max-Planck Institut für Plasmaphysik, Garching, Germany
3) Research Organization for Information and Science, Nakameguro, Tokyo, Japan
4) Kyushu University, Kasuga, Fukuoka, Japan
e-mail: miyato.naoaki@jaea.go.jp

Based on the phase space Lagrangian Lie-transform perturbation method and the field theory, a reduced kinetic model with large $\mathbf{E} \times \mathbf{B}$ flow beyond the standard ordering ($V_{\mathrm{E\times B}}/v_{th} \sim O(\rho/L)$) is constructed by modifying the guiding-centre phase space transformation. The model can be regarded as a natural extension of the standard model without flow since the symplectic part of the Lagrangian is the same as the standard one formally. Some aspects of the model are revealed and effects of the flow are discussed in course of comparison with the standard model. The push-forward representation of general particle fluid moment is presented in the subsonic flow case. In sonic flow case, corrections to the reduced quasi-neutrality condition due to the $\mathbf{E} \times \mathbf{B}$ flow are found by variational derivation of the push-forward representation of particle density.

1 Reduced kinetic model with large $\mathbf{E} \times \mathbf{B}$ flow

A guiding-centre phase space Lagrangian with large E×B flow whose speed is comparable to the ion thermal velocity is derived by modifying the standard guiding-centre transformation. The guiding-centre Lagrangian for a particle with mass m and charge e is given by [1]

$$L_p = e\mathbf{A}^* \cdot \dot{\mathbf{X}} + (m/e)\mu\dot{\xi} - H, \tag{1}$$

with the Hamiltonian,

$$H = e\varphi + \epsilon\left(\frac{m}{2}U^2 + \mu B - \frac{m}{2}V_E^2\right) + \epsilon^2\frac{m}{2e}\left(\mu + \frac{mV_E^2}{2B}\right)\mathbf{b}\cdot\nabla\times\mathbf{V}_E, \tag{2}$$

where $\mathbf{X}$ is the guiding-centre position, U is the guiding-centre parallel velocity, μ is the guiding-centre magnetic moment, ξ is the guiding-centre gyroangle, $\mathbf{A}^* = \mathbf{A} + \epsilon(m/e)U\mathbf{b}$, $\mathbf{A}$ is an equilibrium vector potential, $\mathbf{V}_E = \mathbf{b}\times\nabla\varphi/B$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity and $\epsilon \sim \rho/L \ll 1$ is a small parameter. It is noted that $\mathbf{A}^*$ in the present model is formally the same as the standard gyrokinetic one without large flow [2], while an additional flow term appears in $\mathbf{A}^*$ in conventional formulations with large flow [3–8]. Therefore the present model can be regarded as a natural extension of the standard model. As a result, Hamilton equations derived from the above Lagrangian keep their standard general form even when $\mathbf{V}_E$ is time-varying. Besides, Jacobian of the phase space, $\mathcal{J} = B_\parallel^*/m$ ($B_\parallel^* \equiv \mathbf{b}\cdot\mathbf{B}^*$ with $\mathbf{B}^* \equiv \nabla\times\mathbf{A}^*$), is the same as the starndard one as well. The conventional formulations with large flow cause no problems for time independent flow. For time-varying flow, however, changes from the standard ones are inevitable: additional time derivative terms appear in the Hamilton equations and the Jacobian acquires time dependence. Although the similar Lagrangian is also found in [9], we proceed to higher order and complete the perturbation analysis. Moreover, based on the above reduced Lagrangian, we construct an energy-conserving reduced Vlasov-Poisson system through the field theory [10]. Since the model can treat large flow observed in transport barrier regions such as an ITB in reversed shear tokamaks and a tokamak edge in an H-mode regime where the standard model is not valid, the model makes it possible to investigate the dynamics of transport barrier including ITB formation and the L-H transition.

2 Single particle motion in a strong electric field

Based on the guiding-centre Lagrangian with large $\mathbf{E}\times\mathbf{B}$ flow, we investigate the guiding-centre motion in an axisymmetric system.

2.1 Constants of the guiding-centre motion

First of all, μ is a constant of motion by construction of the guiding-centre model. Next, if the Lagrangian does not depend on the toroidal angle ζ, the Euler-Lagrange equation or Noether's theorem says that its canonically conjugate momentum

$$p_\zeta \equiv \frac{\partial L_p}{\partial \dot{\zeta}} = eA^*_\zeta = eA_\zeta + mUb_\zeta \tag{3}$$

is a constant of motion, where A_ζ and b_ζ are covariant ζ components of $\mathbf{A}$ and $\mathbf{b}$, respectively. The toroidal angular momentum p_ζ is the same as the standard one in the no flow case since the symplectic part of the Lagrangian is common. Although the Hamiltonian is different from the standard one, it is possible to recover the standard form by neglecting the second order term in Eq. (2) and defining an effective potential as

$$\varphi^* = \varphi - \frac{m}{2e}V_E^2. \tag{4}$$

Then we can follow the standard analysis for guiding-centre orbits [11, 12] .

2.2 Guiding-centre orbits

Here we consider a radial electric field only. The radial displacement of the guiding-centre orbit is obtained from conservation of the guiding-centre energy and the toroidal angular momentum. In the large aspect ratio limit it is given by [11]

$$\Delta r = \frac{1}{S\Omega_P}\left\{-\left(U_0 - \frac{V_E^*}{G}\right) \pm \sqrt{\left(U_0 - \frac{V_E^*}{G}\right)^2 - 4\eta D \sin^2\frac{\theta}{2}}\right\} \tag{5}$$

where θ is the poloidal angle, U_0 is U at the outboard midplane,

$$V_E^* = -\frac{\varphi^{*\prime}}{B_0} = V_E\left(1 + \frac{V_E'}{\Omega}\right), \tag{6}$$

$G = \eta/q$, η is the inverse aspect ratio, q is the safety factor, S is the squeezing factor [11,12]

$$S = 1 - \frac{B_0 V_E^{*\prime}}{\Omega_P B_P}, \tag{7}$$

$\Omega = eB/m$, $\Omega_P = eB_P/m$, B_P is the poloidal magnetic field, B_0 is the magnetic field at the outboard midplane, primes denote radial derivative and D is given by

$$D = \left(\frac{V_E^*}{G}\right)^2 + S\mu B_0 + (S-1)U_0^2. \tag{8}$$

Δr is rewritten as

$$\Delta r = \frac{1}{S\Omega_P}\left\{-\tilde{U}_0 \pm w\sqrt{\kappa^2 - \sin^2\frac{\theta}{2}}\right\} \tag{9}$$

where

$$\tilde{U}_0 = U_0 - \frac{V_E^*}{G}, \quad w = 4\eta D, \quad \kappa^2 = \frac{\tilde{U}_0^2}{4\eta D}. \tag{10}$$

Particles are trapped if $\kappa^2 < 1$ and passing if $\kappa^2 > 1$. Hence, particles with $U_0 \sim V_E^*/G$ are trapped, while particles with small parallel velocity are trapped in the case without large flow. When U_0 is comparable to the thermal speed, the above condition is satisfied for the subsonic $\mathbf{E}\times\mathbf{B}$ flow due to the factor G. It is seen that the effect of the V_E^2 correction to the electrostatic potential on the trapping condition is small when $V_E'/\Omega \ll 1$.

The radial deivative of V_E^* is written as

$$V_E^{*\prime} = V_E' + \frac{(V_E^2)''}{2\Omega}. \tag{11}$$

We consider a radial electric field with a single extremum which is observed in an H-mode tokamak edge. Then $(V_E^2)''$ is negative around the radius of maximum of V_E^2 where $V_E' \simeq 0$. Hence, even in the region $V_E' \simeq 0$, $V_E^{*\prime} \simeq (V_E^2)''/2\Omega$ is finite and this yields $S > 1$. Now we estimate the contribution of $(V_E^2)''/2\Omega$ to S by using data of recent JT-60U H-mode edge measurement [13]. In the JT-60U H-mode edge with B ∼3.9 T and $q_{95} \sim 4.2$, the maximum radial gradient of V_E is $V_E' \sim 10^6$ V/m^2 which gives roughly ±0.7 contribution to S. On the other hand, the $(V_E^2)''/2\Omega$ contribution to S is $\sim 10^{-2}$ in the $V_E' \simeq 0$ region. Thus the V_E^2 effect on S is negligible in the JT-60U H-mode edge. Besides, its effect on the trapping condition is also negligible because of $V_E'/\Omega \sim 10^{-2}$. $(V_E^2)''/2\Omega$ may give $O(1)$ contribution to S if $V_E' \sim 10^7$ V/m^2, which is relevant to the DIII-D H-mode edge [14].

3 Push-forward representation of particle fluid moments in subsonic case

3.1 Perturbative expansion of the exact representation

The modification of the guiding-centre transformation also affects the push-forward representation of fluid moments. A general particle fluid moment is defined by

$$m_{kl}(\mathbf{r}) \equiv \int \left(\frac{mw^2}{2B}\right)^k v_\parallel^l f \delta^3(\mathbf{x}-\mathbf{r}) d^3\mathbf{x} d^3\mathbf{v}, \tag{12}$$

where f is the particle distribution function, $w = |\mathbf{v}_\perp - \mathbf{V}_E|$ is the perpendicular particle velocity in a frame moving with the $\mathbf{E}\times\mathbf{B}$ velocity, $v_\parallel$ is the parallel particle velocity. The particle fluid moment can be written in terms of the guiding-centre distribution function F and the push-forward transformation associated with the guiding-centre transformation $\mathsf{T}_{\rm GC}^{-1*}$ as

$$m_{kl}(\mathbf{r}) = \int d^6\mathbf{Z}\mathcal{J}(\mathbf{Z})\left[\mathsf{T}_{\rm GC}^{-1*}\left\{\left(\frac{mw^2}{2B}\right)^k v_\parallel^l\right\}\right](\mathbf{Z})F(\mathbf{Z})\delta^3(\mathsf{T}_{\rm GC}^{-1}\mathbf{x}-\mathbf{r}), \tag{13}$$

where $\mathsf{T}_{\rm GC}^{-1}\mathbf{x} = \mathbf{X} + \boldsymbol{\rho} + \boldsymbol{\rho}_E + \cdots$ denotes the particle position in the guiding-centre phase space with $\boldsymbol{\rho} = \mathbf{b}\times\mathbf{w}/\Omega$ and $\boldsymbol{\rho}_E = \mathbf{b}\times\mathbf{V}_E/\Omega$. The difference from the standard guiding-centre transformation is $\boldsymbol{\rho}_E$ which corresponds to the gyroaverage of the gyro-centre displacement vector [15]. The velocity variables are related with the guiding-centre variables as

$$\frac{mw^2}{2B} = \mu - G_1^\mu + \cdots, \quad v_\parallel = U - G_1^U + \cdots, \tag{14}$$

where G_1^μ and G_1^U are μ and U components of the vector field generating the guiding-centre transformation at first order in ϵ, respectively. Equation (13) is the formal exact representation. Assuming that the $\mathbf{E}\times\mathbf{B}$ velocity is subsonic $V_E \sim \epsilon^{1/2} v_{ti}$ (v_{ti} the ion thermal speed) and expanding the above exact representation perturbatively, we have the push-forward representation of m_{kl} up to $O(\epsilon^2)$ [16],

$$m_{kl} = M_{kl} + \frac{1}{2}\nabla\cdot\left[\frac{\nabla_\perp M_{k+1l}}{e\Omega}\right] + (k+1)\nabla\cdot\left[\frac{M_{kl}}{B\Omega}\nabla_\perp\varphi\right] - k\mathbf{V}_E\cdot\frac{\mathbf{b}\times\nabla M_{kl}}{\Omega}, \tag{15}$$

where M_{kl} is a guiding-centre fluid moment defined by

$$M_{kl} \equiv \int \mu^k U^l F \mathcal{J} dU d\mu d\xi. \tag{16}$$

The last term on the right hand side of Eq. (15) does not appear in the one obtained from the standard gyrokinetic theory in which $v_\perp$ is used for the magnetic moment [17]. For $k = l = 0$, we have the push-forward representation of particle density,

$$n = N + \frac{1}{2}\nabla\cdot\left[\frac{\nabla_\perp P_\perp}{e\Omega B}\right] + \nabla\cdot\left[\frac{N}{B\Omega}\nabla_\perp\varphi\right], \tag{17}$$

where $n \equiv m_{00}$, $N \equiv M_{00}$ and $P_\perp \equiv BM_{10}$ are particle density, guiding-center density and guiding-centre pependicular pressure, respectively. This representation is the same as the standard one formally. The push-forward representation of particle density can be regarded as the quasi-neutrality condition for electrons and singly charged ions in the reduced model.

3.2 Correspondence to the standard gyrokinetic model

The modern standard gyrokinetic model is formulated through the two-step phase space transformation which consists of the guiding-centre transformation T_{GC} and the transformation from the guiding-centre phase space to the gyro-centre phase space T_{Gy} [18]. The exact representation usually used in the standard gyrokinetic theory is given by [19]

$$m_{kl}(\mathbf{r}) = \int d^6\bar{\mathbf{Z}}\mathcal{J}(\bar{\mathbf{Z}})\left[\mathsf{T}_{\mathrm{GC}}^{-1*}\left\{\left(\frac{mv_\perp^2}{2B}\right)^k v_\parallel^l\right\}\right](\bar{\mathbf{Z}})[\mathsf{T}_{\mathrm{Gy}}^*\bar{F}](\bar{\mathbf{Z}})\delta^3([\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{x}](\bar{\mathbf{Z}}) - \mathbf{r}), \tag{18}$$

where $\bar{\mathbf{Z}}$ denotes the gyro-centre coordinates and $\mathsf{T}_{\mathrm{Gy}}^*$ is the pull-back transformation associated with T_{Gy}. Note that $[\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{x}](\bar{\mathbf{Z}}) \simeq \bar{\mathbf{X}} + \boldsymbol{\rho}(\bar{\mathbf{Z}})$ does not denote the particle position in the gyro-centre phase space. Moreover, effects of the electrostatic potential is contained in the pull-back of $\bar{F}$, $\mathsf{T}_{\mathrm{Gy}}^*\bar{F}$, in this representation:

$$\mathsf{T}_{\mathrm{Gy}}^*\bar{F} \simeq \bar{F} + \epsilon_\delta\{S_1, \bar{F}\} \simeq \bar{F} + \epsilon_\delta\frac{e\tilde{\varphi}}{B}\frac{\partial\bar{F}}{\partial\bar{\mu}}, \tag{19}$$

where $S_1 = (e/\Omega)\int \tilde{\varphi} d\bar{\xi}$, $\tilde{\varphi} = \varphi(\bar{\mathbf{X}}+\bar{\boldsymbol{\rho}}) - \langle \varphi(\bar{\mathbf{X}}+\bar{\boldsymbol{\rho}})\rangle$ is the gyrophase dependent part of the electrostatic potential, $\bar{\boldsymbol{\rho}} = \boldsymbol{\rho}(\bar{\mathbf{Z}})$, $\langle\cdot\rangle$ denotes the gyrophase average, ϵ_δ is the small parameter for the amplitude of φ. Although the exact representation (18) seems to be different from Eq. (13), direct correspondence between the standard gyrokinetics and our model is found by considering the alternative exact representation for the standard gyrokinetics [20],

$$m_{kl}(\mathbf{r}) = \int d^6\bar{\mathbf{Z}}\mathcal{J}(\bar{\mathbf{Z}})\left[\mathsf{T}_{\mathrm{Gy}}^{-1*}\mathsf{T}_{\mathrm{GC}}^{-1*}\left\{\left(\frac{mv_\perp^2}{2B}\right)^k v_\parallel^l\right\}\right](\bar{\mathbf{Z}})\bar{F}(\bar{\mathbf{Z}})\delta^3(\mathsf{T}_{\mathrm{Gy}}^{-1}\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{x}-\mathbf{r}), \quad (20)$$

where $\mathsf{T}_{\mathrm{Gy}}^{-1}\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{x}$ denotes the particle position in the gyro-centre phase space. Similarity between Eqs. (13) and (20) is apparent. $\mathsf{T}_{\mathrm{Gy}}^{-1}\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{x}$ is written explicitly as

$$\mathsf{T}_{\mathrm{Gy}}^{-1}\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{x} = \bar{\mathbf{X}} + \epsilon\bar{\boldsymbol{\rho}} + \epsilon_\delta\epsilon\bar{\boldsymbol{\rho}}_{\mathrm{gy}} + \cdots, \quad (21)$$

where $\bar{\boldsymbol{\rho}}_{\mathrm{gy}} = -\{S_1, \bar{\mathbf{X}}+\bar{\boldsymbol{\rho}}\}$ is the gyro-centre displacement vector. The gyroaverage of $\bar{\boldsymbol{\rho}}_{\mathrm{gy}}$ corresponds to $\boldsymbol{\rho}_E$ in our model as mentioned before.

4 Reduced quasi-neutrality condition with sonic flow

4.1 Variational derivation of push-forward representation of particle density

Although the perturbative expansion of the exact representation is straightforward and only information of the vector field generating phase space transformation is needed, higher order calculations are so complicated. The push-forward representation of particle density is also obtained from the single particle Lagrangian (1) by a variational method. To this end, we consider a functional derivative of the action functional $I = \int_{t_1}^{t_2} L dt$ with a Lagrangian for the Vlasov-Poisson system [10],

$$L = \sum_s \int d^6\mathbf{Z}_0\mathcal{J}_s(\mathbf{Z}_0)F_s(\mathbf{Z}_0,t_0)L_{ps}[\mathbf{Z}_s(\mathbf{Z}_0,t_0;t),\dot{\mathbf{Z}}_s(\mathbf{Z}_0,t_0;t),t] - \int d^3\mathbf{x}\frac{1}{4\mu_0}\mathsf{F}:\mathsf{F}, \quad (22)$$

where $\sum$ denotes a sum over species, $\mathbf{Z}_s(\mathbf{Z}_0,t_0;t)$ denotes the guiding-centre coordinates of the particle at t with the initial condition $\mathbf{Z}_s(\mathbf{Z}_0,t_0;t_0) = \mathbf{Z}_0$, μ_0 is permeability of vacuum and the electromagnetic field tensor F is defined by $\mathsf{F}_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ with the covariant four vector potential $A_\mu = (-\varphi/c, \mathbf{A})$ and the four gradient operator $\partial_\mu = ((1/c)\partial_t, \nabla)$. $\delta I/\delta\varphi(\mathbf{r}) = 0$ yields a reduced Poisson equation in which charge density is expressed in terms of the guiding-centre quantities. The requirement that the charge density part in the equation should agree with the one in the particle phase space $\sum e_s n_s$ yields the push-forward representation of particle density,

$$n(\mathbf{r}) = -\frac{1}{e}\int d^6\mathbf{Z}\mathcal{J}(\mathbf{Z})F(\mathbf{Z})\frac{\delta L_p(\mathbf{Z})}{\delta\varphi(\mathbf{r})}, \quad (23)$$

where the subscript s is suppressed. This representation is general and valid even when the symplectic part of L_p contains φ as in the conventional formulations with large flow [3]. In our case, φ appears in the Hamiltonian only. Then the above representation is reduced to

$$n(\mathbf{r}) = \frac{1}{e}\int d^6\mathbf{Z}\mathcal{J}(\mathbf{Z})F(\mathbf{Z})\frac{\delta H(\mathbf{Z})}{\delta\varphi(\mathbf{r})}. \quad (24)$$

We can obtain an explicit push-forward representation from this equation if we know the guiding-centre Hamiltonian which is derived naturally in the modern Lie-transform perturbation analysis. For example, we consider

$$H = \underline{e\varphi} + \frac{m}{2}U^2 + \mu B \underline{- \frac{m}{2}V_E^2} + \underline{\frac{m}{2e}\mu \mathbf{b}\cdot\nabla\times\mathbf{V}_E} \tag{25}$$

which is valid in well localised transport barrier regions with subsonic flow [1]. It is similar to the standard gyrokinetic Hamiltonian in the long wavelength limit. Since we need $\delta H/\delta\varphi$ for the push-forward representation of particle density, it is sufficient to keep in mind underlined terms which include φ. Substituting the above Hamiltonian into Eq. (24) and integrating by parts yield Eq. (17). The first underlined term leads to the first term in Eq. (17), the second leads to the term including φ and finally the third leads to the term with $P_\perp$. Thus the variational method is more transparent and useful than the perturbative expansion of the exact representation if the guiding-centre Hamiltonian is known.

4.2 Sonic flow case

While perturbative expansion of the exact representation is very complicated in the sonic flow case, the variational method is relatively simple. When the flow speed is comparable to the thermal speed, we have to keep the cubic term of V_E in the Hamiltonian (2). In this case, the variational method yields the push-forward representaion with additional terms,

$$n = N + \frac{1}{2}\nabla\cdot\left[\frac{1}{e\Omega B}\nabla_\perp\left(P_\perp + \frac{NmV_E^2}{2}\right)\right] + \nabla\cdot\left[\left(1 - \frac{\mathbf{b}\cdot\nabla\times\mathbf{V}_E}{2\Omega}\right)\frac{N}{B\Omega}\nabla_\perp\varphi\right]. \tag{26}$$

This is valid in well localised transport barrier regions with sonic flow. The additional terms appear as corrections to the polarisation density. The first one is the flow correction to $P_\perp$. The second one is the correction by the vorticity which gives a term proportional to enstrophy density. They are nonlinear to φ because they come from the cubic term of V_E in the Hamiltonian. More terms at higher order are derived from the guiding-centre Hamiltonian with higher order terms by this method.

5 Push-forward representation of particle flux

The push-forward representations of the scalar fluid moments have been discussed in the previous sections. In this section, we consider push-forward representation of a vector fluid moment, a particle flux, by following Refs. [15, 21]. The particle flux is defined in the particle phase space as

$$\mathbf{\Gamma}(\mathbf{r}) \equiv \int f\mathbf{v}\delta^3(\mathbf{x}-\mathbf{r})d^3\mathbf{x}d^3\mathbf{v}. \tag{27}$$

Similar to the scalar fluid moments, $\mathbf{\Gamma}(\mathbf{r})$ can be expressed in terms of an integral in the guiding-centre phase space as

$$\mathbf{\Gamma}(\mathbf{r}) = \int d^6\mathbf{Z}\mathcal{J}F\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{v}\delta^3(\mathsf{T}_{\mathrm{GC}}^{-1}\mathbf{x}-\mathbf{r}), \tag{28}$$

where $\mathsf{T}_{\rm GC}^{-1}\mathbf{v} = \dot{\mathbf{X}} + \dot{\boldsymbol{\rho}}_{\rm gc}$ is push-forward of the particle velocity and $\boldsymbol{\rho}_{\rm gc} \equiv \mathsf{T}_{\rm GC}^{-1}\mathbf{x} - \mathbf{X}$ is the displacement between the guiding-centre position and the particle position.

Expanding the delta function in powers of $\boldsymbol{\rho}_{\rm gc}$ and integrating by parts, we have

$$\boldsymbol{\Gamma} = \boldsymbol{\Gamma}_{\rm gc} + \boldsymbol{\Gamma}_{\rm pol} + \boldsymbol{\Gamma}_{\rm mag}, \tag{29}$$

where

$$\boldsymbol{\Gamma}_{\rm gc} = \int d^3v \dot{\mathbf{X}} F \tag{30}$$

is the guiding-centre flux,

$$\boldsymbol{\Gamma}_{\rm pol} = \frac{\partial}{\partial t}\int d^3v \boldsymbol{\rho}_{\rm gc} F \tag{31}$$

is the polarisation flux,

$$\boldsymbol{\Gamma}_{\rm mag} = \nabla \times \left\{ \int d^3v \dot{\mathbf{X}} F \left[\boldsymbol{\rho}_{\rm gc} \times \left(\frac{1}{2}\dot{\boldsymbol{\rho}}_{\rm gc} + \dot{\mathbf{X}} \right) \right] \right\} \tag{32}$$

is the magnetisation flux, and $d^3v = \mathcal{J} dU d\mu d\xi$. In the conventional models, $\boldsymbol{\rho}_{\rm gc}$ is the usual Larmor radius vector $\boldsymbol{\rho}$. Then $\langle \boldsymbol{\rho}_{\rm gc} \rangle = 0$ and $\boldsymbol{\Gamma}_{\rm pol}$ vanishes. Instead the polarisation drift term

$$\mathbf{V}_{\rm pol} = \frac{\mathbf{b}}{\Omega} \times \frac{\partial \mathbf{V}_E}{\partial t} \tag{33}$$

is included in the guiding-centre drift $\dot{\mathbf{X}}$ in the conventional models with the flow and it yields the polarisation flux. On the other hand, $\dot{\mathbf{X}}$ in our model does not include the polarisation drift $\mathbf{V}_{\rm pol}$. This is because of the difference in the symplectic part of the guiding-centre Lagrangian mentioned before. Recall that the purpose of our model is to exclude the time derivative terms from the guiding-centre Hamilton equations. In our model, $\boldsymbol{\rho}_{\rm gc}$ is not purely oscillatory and $\langle \boldsymbol{\rho}_{\rm gc} \rangle = \boldsymbol{\rho}_E$. Then $\boldsymbol{\Gamma}_{\rm pol}$ becomes

$$\boldsymbol{\Gamma}_{\rm pol} = \frac{\mathbf{b}}{\Omega} \times \frac{\partial}{\partial t}(N\mathbf{V}_E). \tag{34}$$

Besides, while the second part of $\boldsymbol{\Gamma}_{\rm mag}$ including $\boldsymbol{\rho}_{\rm gc} \times \dot{\mathbf{X}}$ also vanishes due to $\langle \boldsymbol{\rho}_{\rm gc} \rangle = 0$ in the conventional models, it does not in our model.

6 Summary

We have constructed the guiding-centre model with large flow by modifying the guiding-centre transformation. In contrast to the conventional models with large flow, the symplectic part of the guiding-centre Lagrangian is the same as the standard one with weak flow formally. Therefore our model can be regarded as a natural extension of the standard model and large modifications in theory and simulation are avoided by use of our model. The comparison among the standard model, the present model and the conventional models with large flow is shown in Table 1. The explicit push-forward representations of particle fluid moments have been derived from the exact representation by the perturbative expansion and we have shown the correspondence between our model and the standard gyrokinetic model. The push-forward representation of particle density or the reduced quasi-neutrality condition for singly charged ions and electrons has been derived through the variational method in the sonic flow case.

Table 1: Comparison among the standard model with weak flow, the present model and the conventional models with large flow.

	Standard	Present	Conventional
$\boldsymbol{\rho}_{\rm gc}$	$\boldsymbol{\rho}$	$\boldsymbol{\rho}+\boldsymbol{\rho}_E$	$\boldsymbol{\rho}$
$\mathbf{A}^*$	$\mathbf{A}+(m/e)U\mathbf{b}$	$\leftarrow$	$\mathbf{A}+(m/e)(\mathbf{V}_E+U\mathbf{b})$
H	$e\varphi+\frac{m}{2}U^2+\mu B$	$e\varphi+\frac{m}{2}U^2+\mu B-\frac{m}{2}V_E^2$	$e\varphi+\frac{m}{2}U^2+\mu B+\frac{m}{2}V_E^2$
$B_\parallel^*$	$B+(m/e)U\mathbf{b}\cdot\nabla\times\mathbf{b}$	$\leftarrow$	$B+(m/e)\mathbf{b}\cdot\nabla\times(\mathbf{V}_E+U\mathbf{b})$
p_ζ	$eA_\zeta+mUb_\zeta$	$\leftarrow$	$eA_\zeta+mV_{E\zeta}+mUb_\zeta$

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