Non-local transport modeling of heat transport in the LHD

D.del-Castillo-Negrete[1], N. Tamura[2], and S. Inagaki[3]

[1]Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-8071
[2]National Institute for Fusion Science, Gifu 509-5292, Japan
[3]Kyushu University, Fukuoka 816-8580, Japan

email: delcastillod@ornl.gov

Abstract. A non-local model is proposed to describe heat transport experiments conducted at the Large Helical Device (LHD). Previous successful applications of the model include the description of perturbative experiments in JET. Here we focus on recent cold pulse experiments in LHD where edge cooling was produced by pellet injection. As in previous experiments of this type, the cold pulses in LHD exhibit a time delay of the order of ~4 *ms* which is much faster than the time expected from diffusive transport. However, the key unique signature of these experiments is that the core cooling happens in the absence of significant cooling of the intermediate regions of the plasmas. This non-monotonic cooling of the edge perturbation is accompanied by the existence of a long-range flux that extends throughout the whole plasma domain. Describing these phenomena using a diffusive model is problematic because the measured perturbed fluxes and temperature gradients show regions of up-hill transport (implying a negative effective diffusivity) and regions where the flux changes without a significant local change in the temperature gradient (implying a diverging effective diffusivity). In addition, the measurements indicate the lack of a single valued flux-gradient functional relation needed to justify a diffusive model with a physically meaningful diffusivity. We show that all this phenomenology can be described using a non-local transport model in which the total flux, q, has a component, q_{nl}, that depends on the temperature gradient throughout the whole plasma domain.

1. Introduction

Cold pulse experiments in tokamaks [1] and stellarators [2] have shown that cold pulse perturbations applied at the edge can travel to the center much faster than expected on the basis of diffusive time scales compatible with plasma confinement. An open issue in describing these experiments is whether the observed fast propagation is due to non-local transport mechanisms, or if it could be explained on the basis of non linear but local transport models. Determining the role of non-locality in cold pulse propagation is difficult because both non-linear local and non-local models can exhibit fast propagation for properly chosen parameters and conditions. To make progress on this challenging problem it is productive to look for unique signatures of non-locality, beyond the fast propagation of the pulse.

To discriminate between local and non-local transport in JET, Ref.[3] considered, in addition to cold pulses, heat waves resulting from RF power modulation perturbations. The key issue is that heat waves exhibit clear propagation asymmetries compared to pulses. In particular, cold pulses can propagate fast even in regions where heat waves slowdown. Previous attempts to account for this type of experiments using local diffusive models found problematic to reconcile the fast propagation of the cold pulses with the comparatively slower propagation of the heat waves generated by power modulation. Reference [4] showed that the non-local model proposed in Ref.[5] is able to describe these experiments.

The recent experiments conducted in the Large Helical Device (LHD) [6,7] provide another case study with unique non-local transport signatures. In particular, these experiments exhibit, in addition to the fast propagation, other phenomena including non-monotonic cooling and multi-valued flux-gradient relations that seem to be incompatible with the standard, local diffusion paradigm. The goal of this paper is to explore the application of the non-local transport model in Ref.[5] to describe these experiments.

2. Non-local perturbative transport experiments in LHD

In this section we describe the perturbative transport experiments conducted at the Large Helical Device (LHD) (major radius at the magnetic axis, R_{ax}= 3.5 m, averaged minor radius, a =0.58 m, and magnetic field at the axis, B_{ax}=2.829 T) [2,6-8]. We focus on discharge #49719 that shows experimental evidence of non-monotonic cooling, multi-valued flux-gradient relations, and long-range spatial correlations on the heat flux.

In the LHD experiments studied here, the plasma was heated continuously by a negative-ion-based neutral beam injection in the co-direction with a deposited power of 1.2 MW before and after the edge cooling. A large fraction (~ 70 %) of the negative-ion-based neutral beam power went into the electrons due to the high acceleration energy (~ 150 keV). For the discharge #49719, ECH with 82.7 GHz and 84 GHz gyrotrons for the fundamental resonance heating (total injected power ~ 1 MW) was additionally applied. The ECH power was absorbed inside x ~ 0.2. Edge cooling in LHD was produced by injecting a tracer-encapsulated solid pellet (TESPEL) into the plasma from the outboard side of the torus. For shot #49719, the diameter of the pellet was 0.5 mm, and it typically ablated within ~ 1 millisecond. The line-averaged electron density just before the edge cooling was 1.3 x 10^{19} m^{-3} for #49719 and it only increased about 10 % due to the TESPEL injection, inducing little change in the electron heat transport. Further details on the experiments and diagnostics can be found in Refs.[2] and [6-8].

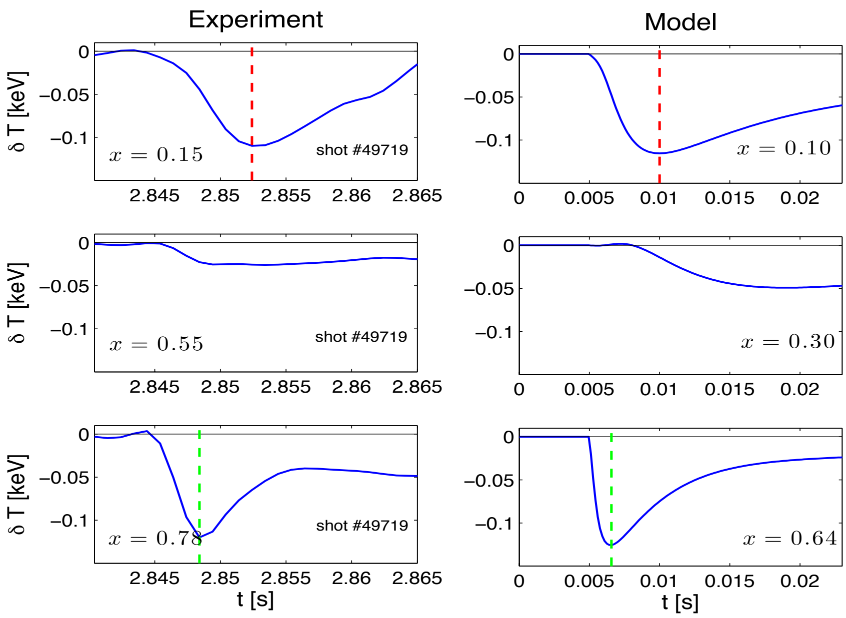

FIG. 1 Temperature perturbation time traces at three different spatial locations. The experiment (left column) and the model (right column) show a comparable pulse delay of ~4ms and, most importantly, both exhibit absence of intermediate cooling as the traces at x=0.55 and x=0.30 respectively show.

The first column in Fig.1 shows the experimentally measured temperature time traces at different spatial location. The cold pulse exhibits fast propagation with a time delay of the order of ~4 *ms* which is significantly smaller than the time scales expected from diffusive transport. Similar cold pulse fast propagation phenomena have also been observed in tokamaks. However, what is interesting and unique about discharge #49719 in LHD is that, as observed in the temperature trace at x=0.55, the cooling of the core happens without significant intermediate cooling. Further evidence of this "non-monotonic cooling" is observed in Fig. 2 that shows the cold pulse propagating from the edge to the core as two detached "blue blobs". As the bottom of Fig. 2 indicates, the anomalous propagation of the pulse is accompanied by the presence of a long-range flux "tongue" that extends from the edge to the core.

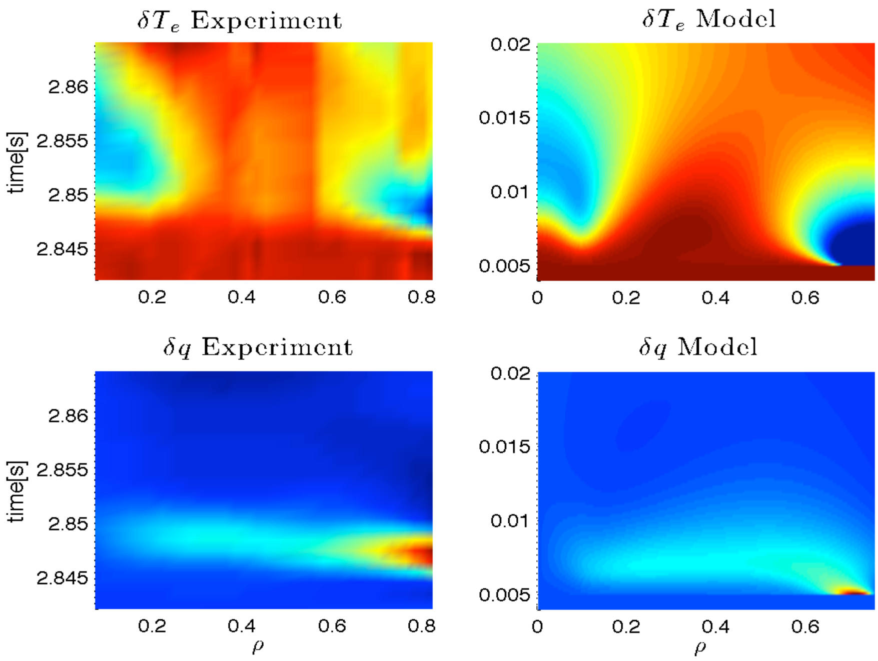

FIG. 2 Spatio-temporal evolution of temperature perturbation, δT, with dark blue (dark red) denoting large negative (zero) values; and flux perturbation, δq, with dark red (dark blue) denoting large positive (zero) values. The left column shows the experimental result and the right column shows the non-local model.

The top panel of Fig.3 shows the experimentally measured spatial temperature profiles at three different times. The green curve corresponds to the time when the pulse reached the minimum value at the edge (as indicated with the green dashed line in Fig.1). The red curve corresponds to the time when the pulse reached the minimum value at the core (as indicated with the green dashed line in Fig.1). The blue line gives the profile at an intermediate time. The spatial temperature traces exhibit clear evidence of non-monotonic cooling: although the core rapidly responds to the edge cooling, the intermediate temperature around the interval (0.4,0.55) remains practically constant. The plots at the bottom of Figure 3 show the flux-gradient relation at two spatial locations in the LHD #49719 discharge. The black dot denotes the time when the pulse is introduced, the green dot denotes the time when the pulse reaches the minimum value at the edge, and the red dot denotes the time when the pulse reaches the minimum at the core. Contrary to the local, diffusive Fourier-Fick's prescription, the paths in the flux-gradient space do not trace straight lines.

3. Application of non-local transport model to LHD experiments

In this section we discuss the application of the non-local transport model proposed in Ref.[5], to describe the LHD perturbative transport experiments. We consider one-dimensional radial heat transport in a plasma with constant density, n, in the slab approximation

$$\partial_t\left[3/2\, nT\right] = -\partial_x q + S \tag{1}$$

where x denotes the radial coordinate, and S is a source. The flux consists of a local diffusive component

$$q_d = -\chi_d\, n \partial_x T \tag{2}$$

and a non-local component of the form

$$q_{nl} = -\chi_{nl}\, n\left[l\ {}_aD_x^{\alpha-1} - r\ {}_xD_b^{\alpha-1}\right]T \tag{3}$$

where l and r are parameters determining the asymmetry of the non-local flux. ${}_aD_x^{\alpha-1}$ and ${}_xD_b^{\alpha-1}$ denote the integro-differential non-local operators

$${}_aD_x^{\alpha-1}T = \frac{1}{\Gamma(2-\alpha)}\int_a^x \frac{T'(y)-T'(a)}{\left(x-y\right)^{\alpha-1}}dy\ , \qquad {}_xD_b^{\alpha-1}T = -\frac{1}{\Gamma(2-\alpha)}\int_x^b \frac{T'(y)-T'(b)}{\left(y-x\right)^{\alpha-1}}dy\,, \tag{4}$$

where Γ is the gamma function, and (a,b) is the domain of the system. In an unbounded domain, the Fourier representation of the non-local flux in Eqs.(3)-(4) is

$$F\left[q_{nl}/(\chi_{nl}\, n)\right] = -\left[l\left(-ik\right)^{\alpha-1} - r\left(ik\right)^{\alpha-1}\right]F\left[T\right]\ . \tag{5}$$

The parameter $1<\alpha<2$ determines the degree of non-locality. When $\alpha=2$, Eq.(5) reduces to the diffusive flux and in the limit $\alpha \to 1$, the flux reduces to the non-local free streaming case. For a discussion on the physics behind the flux in Eq.(3), the mathematical aspects of the non-local operators in Eq.(4), and the numerical method use to solve the non-local transport Eq.(1) see Ref.[5].

In the calculations presented here the integration domain, normalized to the minor radius, is the interval, $x \in (0,1)$, and the boundary conditions are

$$q(x=0,t) = [q_d + q_{nl}](x=0,t) = 0, \qquad T(x=1,t) = 0\,, \tag{5}$$

where x=0 denotes the magnetic axis and x=1 denotes the plasma edge. Based on the assumption that in magnetically confined plasmas there is a qualitative difference between core transport and edge transport, we assume a non-local diffusivity of the form

$$\chi_{nl}(x) = \frac{\chi_{nl0}}{2}\left[\tanh\left(\frac{x-x_c}{L}\right) + \tanh\left(\frac{x_c}{L}\right)\right]\ . \tag{6}$$

The local diffusivity is assumed constant $\chi_d = \chi_{d0}$. According to Eq.(6), in the core region, x~0, $\chi_d >> \chi_{nl}$, and transport is dominated by diffusive processes. The transition to non-diffusive transport occurs in a boundary layer at $x = x_c$ of width ~L in which χ_{nl} changes from zero to the edge value χ_{nl0}.

Given an equilibrium temperature profile $T_0(x)$ maintained by an on-axis, time-independent source, $S(x)$, we consider the spatio-temporal evolution of the perturbed temperature, $\delta T(x,t) = T(x,t) - T_0(x)$, and the perturbed flux, $\delta q = q(x,t) - q_0(x)$, where $q_0(x)$ is the equilibrium steady state flux. The edge cooling is modeled using an initial condition consisting of a narrow Gaussian perturbation centered at x=0.75. For the modeling of the LHD experiments, we use n=1.3 x 10^{19} m^{-3}, $\chi_d = \chi_{d0} = 0.75\,\mathrm{m}^2/\mathrm{sec}$, $\chi_{nl0} = 6.31\,\mathrm{m}^\alpha/\mathrm{sec}$, $x_c = 0.1$, and $L = 0.025$. Like in the modeling of the JET experiments in Ref.[4], the non-local operator is assumed symmetric, $l = r$, with $\alpha = 1.25$. However, note that for the JET experiments, the amplitude of the fractional diffusivity ($\chi_{nl0} = 2\,\mathrm{m}^\alpha/\mathrm{sec}$) was smaller.

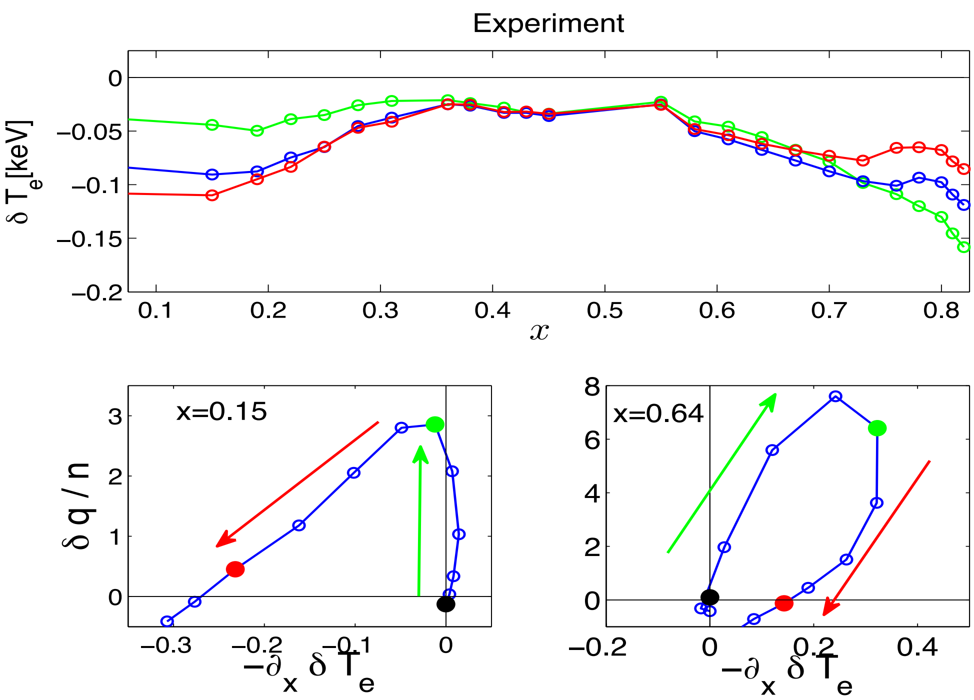

Fig 3. Top panel: Spatial profiles of temperature perturbation at different times t_1(green)<t_2(blue)<t_3(red). Bottom panel: values of flux and gradient for successive times at x=0.15 and x=0.64. Green and red colors denote the time when the pulse minimum is at the edge and the core respectively as indicated by the dashed lines in Fig.1.

Figure 1 shows the time traces of the temperature perturbation at three different spatial locations. Like in the experiment, the model exhibits fast cold-pulse propagation with a time delay of the order of ~4 ms. Most importantly, the model is able to reproduce the observed absence of intermediate cooling. As observed in Fig. 2, the model also captures the spatio temporal evolution of the perturbed temperature and the perturbed flux. The presence of long range flux "tongues" extending from the edge to the core is typically observed in the nonlocal transport model [4,5]. Figure 4 isthe analogue of Fig.3 but for the model. The top panel shows the spatial profiles of the temperature perturbation for three successive times. Like in the experiment, the model shows non-monotonic cooling: while the edge cools the core, the temperature in the middle remains practically unchanged.

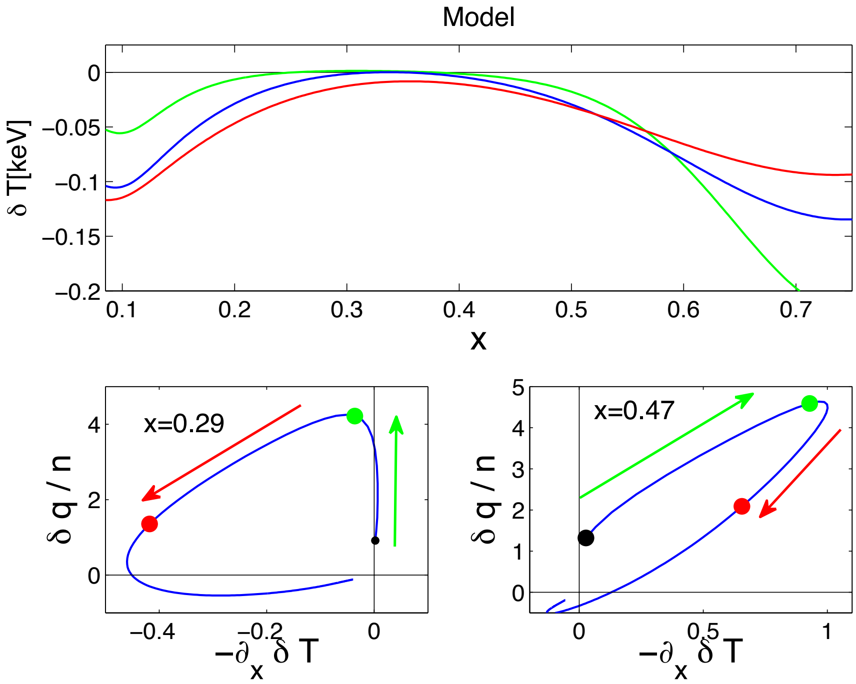

Fig. 4. Same as Fig. 3 but for the model. In both figures the arrows denote the direction of the time evolution in the flux-gradient plane. The model and the experiment exhibit non-monotonic cooling, multi-valued flux-gradient relations and up-hill transport, $\chi_{eff} < 0$*, in the region x~0.29.*

As discussed in Ref.[5], a valuable diagnostic to assess the degree of non-locality in a system is to monitor the evolution of the relationship between the flux and the gradient. In the case of local diffusive transport, the Fourier-Fick's prescription implies a linear relationship between these two quantities. In particular, if the diffusivity does not depend explicitly on time, in the absence of a heat pinch, the time evolution of the flux, $\delta q(x_0 t)/n$, and the gradient, $-\partial_x \delta T(x_0,t)$, at a point x_0, should trace a straight line ($\delta q = -\chi_{eff} n\, \partial_x \delta T$) in the flux-gradient space. However, the plots at the bottom of Figure 3 show that contrary to the local, diffusive Fourier-Fick's prescription, in the LHD experiment the paths in the flux-gradient space do not trace straight lines and exhibit loops. In principle one could argue that by carefully choosing an ad-hoc explicit time dependence in the diffusivity, χ_{eff}, multi-valued flux-gradient loops could be accounted for within the Fourier-Ficks prescription. However, it should be noted that beyond the potential lack of physical justification of such explicit time dependences, there are cases in which enforcing the Fourier-Ficks prescription implies a negative diffusivity. In particular, as the bottom plot in Fig.3 shows, at x=0.15 the flux and the gradient in the LHD experiment have the *same* sign, which within the Fourier-Ficks prescription implies a negative diffusivity. Another clear evidence of non-diffusive transport observed in the LHD experiment is that during the early evolution of the cold pulse, i.e. from the injection of the pellet (black dot in Fig.3) to the time when the pulse reaches the minimum temperature at the edge (green dot in Fig.3), the flux at x=0.15 changes but the gradient remains constant. In this case, enforcing the Fourier-Ficks prescription would yield an infinite diffusivity, unless and ad hoc heat pinch is assumed. Figure 4 shows that the proposed non-local transport model is able to reproduce the observed dynamics in the flux-gradient space. In particular, as shown in the lower left plot, the model exhibits up-hill transport and finite flux in the absence of a local gradient.

To further explore the dependence of the non-monotonic cooling on the model parameters, Fig. 5 shows the normalized temperature perturbation, $\delta\hat{T} = \delta T / |\min[\delta T]|$, as function of position, x, at successive times, for different values of non-locality parameter, α, and different ratios of local and non-local diffusivities, χ_{d0} and χ_{nl0}. For reference, panel (a) shows the evolution of the temperature perturbation in the case of diffusive local transport. As expected, in this case, the single minimum of the cold pulse drifts towards the core and becomes shallower as time advances. Quite differently, in the case of non-local transport a second minimum in the temperature perturbation rapidly develops near the core, while the temperature around x=0.3 remains almost constant. The non-monotonic cooling is more pronounced for smaller values of α and larger values of the χ_{nl0} / χ_{d0} ratio.

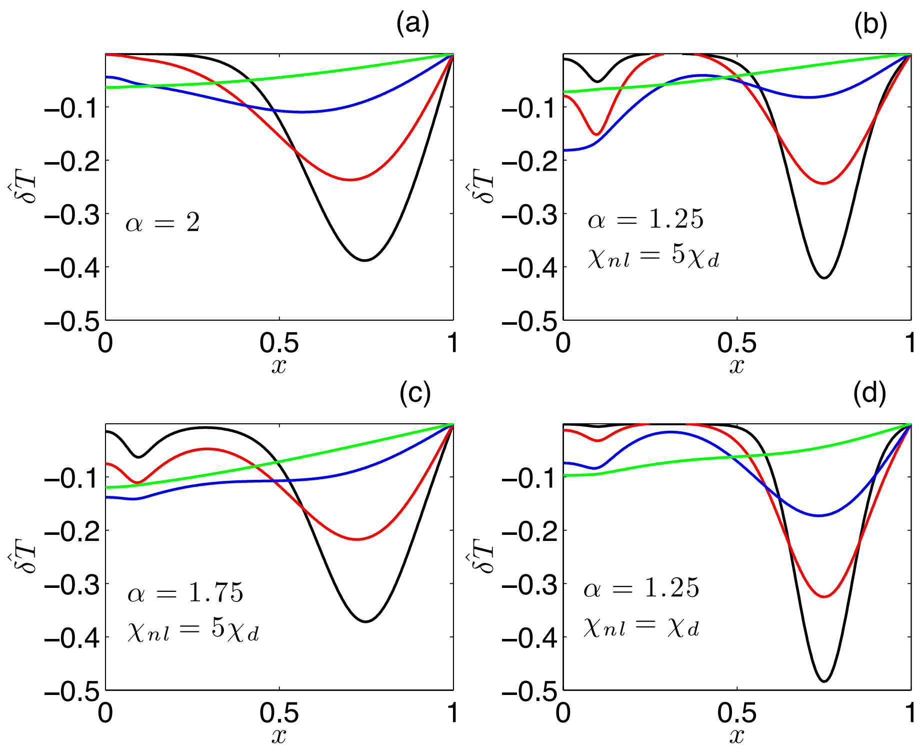

FIG. 5. Dependence of non-monotonic cooling on non-locality. The four panels show the normalized temperature perturbation, $\delta\hat{T} = \delta T / |\min[\delta T]|$, as function of position, x, at four different successive times, t_1(black)<t_2(red)<t_3(blue)<t_4<green. Panel (a) corresponds to local diffusive transport, and panels (b)-(d) correspond to non-local transport with different values of non-locality parameter, α, and different ratios of local and non-local diffusivities, χ_d and χ_{nl}.

5. Conclusions

The non-local heat transport model proposed in Ref.[5] has been applied to describe recent cold pulse perturbative transport experiments in the LHD [6,7]. Like in previous experiments conducted in tokamaks, the cold pulses in the LHD stellarator propagate fast with a time delay of the order of ~4 ms which is significantly shorter than the time delay expected from diffusive transport. However, the LHD experiment exhibits, in addition to the fast propagation, interesting features including non-monotonic cooling, and multi-valued flux-gradient relations including up-hill transport and finite heat fluxes in regions with negligible gradients. These phenomena indicate that non-locality plays a key role and questions the applicability of local diffusive transport models to the experiments in question. We have shown that the non-local model is able to reproduce the spatio-temporal dynamics of the experimentally observed perturbed temperature and perturbed heat flux. In agreement with the experiment, the model exhibits non-monotonic cooling and multivalued flux-gradient relations.

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