Role of Flow Shear Layer and Edge Plasma Turbulence in Density Limit Physics

R. Singh 1), P. K. Kaw 1), M. Tokar 2), P. H. Diamond 3)

1) Institute for Plasma Research, Bhat, Gandhinagar – 382 428, India
2) Institute for Plasma Physics, Forschungszentrum Juelich, D-52425, Germany
3) University of California, San Diego, La Jolla, California 92093-0319

Abstract. The fusion power scales as square of the plasma density and an achievement of a possibly highest density is a topic of great interest for economically profitable fusion reactors [1]. It is well known, however, that in tokamaks in the high confinement H-mode, there is a dramatic deterioration of plasma particle and energy confinement and back transition to the low confinement L-mode when one approaches the Greenwald limit ($n_{Gr} \sim I_p / \pi a^2$, with n_{Gr} measured in $10^{20} m^{-3}$, the plasma current I_p in MA and the separatrix minor radius a in m) [2]. The physical mechanism and theoretical model for the density limit in H-mode plasmas are presented. It is demonstrated that sheared flows, which develop during L-H transition and suppress anomalous transport at the plasma edge in the H-mode state, start to decay by generating tertiary modes like Kelvin-Helmholtz instabilities when the plasma density exceeds a critical level n_{cr}. The dependence of n_{cr} on the plasma current I_p, $n_{cr} \sim I_p^{1.25}$, implies that for not to high currents n_{cr} is lower than the Greenwald density limit $n_{Gr} \sim I_p$. This offers an explanation for the experimental observations on the H-mode density limit often happening noticeably lower than n_{Gr} [2].

1. Introduction

The fusion power scales as square of the plasma density and an achievement of a possibly highest density is a topic of great interest for economically profitable future fusion reactors [1]. It is well known, however, that in tokamaks in the high confinement H-mode, being the basic scenario for plasma performance in ITER, there is a dramatic deterioration of plasma particle and energy confinement and back transition to the low confinement L-mode by approaching to the Greenwald limit $n_{Gr} \sim I_p / \pi a^2$, with n_{Gr} measured in $10^{20} m^{-3}$, the plasma current I_p in MA and the separatrix minor radius a in m [2]. Although this is a limit for the edge density, the average density cannot be significantly higher by a typically flat density profile in the H-mode. Therefore the physics of the density limit is of great importance and diverse mechanisms have been proposed.

It is widely accepted that a final plasma disruption at the density limit in the L-mode happens because MHD instabilities develop due to shrinkage of the current channel [3]. The latter is triggered normally by a collapse of the edge thermal equilibrium caused by impurity radiation. In order to set off this channel of energy losses the edge plasma temperature has to drop to low enough level. This can happen either by the development of MARFE [4] or when the character of edge anomalous transport changes principally through drift resistive ballooning modes [4-7]. In the H-mode the mechanisms considered above are most probably not operative because of high temperature at the plasma edge. In the edge transport barrier (ETB) instabilities, leading in the L-mode to anomalous transport, are robustly suppressed by the plasma-sheared flow in the radial electric field and neoclassical transport processes are dominant. As it has been demonstrated recently [4], in this case the Greenwald density limit corresponds perfectly to the transition from plateau to Pfirsch-Schlüter neoclassical transport regime and is explained by the fact that in the latter one the edge temperature drops extremely fast with increasing density, as n^{-30}. However, these findings do not make clear why most often the H – L transition happens at a density lower than n_{Gr} and the latter is actually the ultimate limit. Here we explain this by demonstrating that shear flows, providing suppression of anomalous transport in the ETB, start to decay by generating tertiary waves like Kelvin-Helmholtz (K-H) instabilities if the plasma density exceeds a critical level n_{cr} and $n_{cr} < n_{Gr}$ at not very high plasma current.

In this paper, we first describe the edge turbulence in L-mode discharge is dominant by high – m drift resistive ballooning mode (DRBM) and later explore the modulational stability of the short scale DRBM turbulence to a long scale zonal flows (ZFs). Since the long scale ZFs are well separated from the small-scale modes driving them therefore the wave kinetic equation and adiabatic theory is used to study the interaction between them. By using a "predator and prey" [8] model we estimate the saturation level of turbulence in a state of stable L-mode and the minimum amplitude of shear flow layer required for stable H-mode. It is observed [9] that when the collision damping is weak, the model predicts unrealistically low value of saturated primary turbulence, thus we study the tertiary instability of ZFs, which take the place of collisional damping of ZFs. Finally, a self-consistent model for interaction of shear layer, tertiary instability [i.e., Kelvin Helmholtz (K-H) instability], edge transport barrier (ETB) heat balance and the physics of penetration dynamics of recycling hydrogen neutral into ETB is constructed and derived the density scaling closed to Greenwald limit.

1. Linear dispersion relation of High –m DRBM

We write the linear dispersion relation of high-m DRBM derived in ref [6]

$$\begin{aligned}\varepsilon_0(\omega,\vec{k}) &= k_\perp^2\omega(\omega+\alpha_i k_y-\varepsilon_n k_y)+\varepsilon_n k_y^2[(1-\varepsilon_n)(1+\tau_i)-\alpha_i k_\perp^2)] \\ &+i\hat{\chi}_e k_{//}^2[\omega-k_y+\varepsilon_n k_y+k_\perp^2(\omega+\alpha_i k_y)\left[1-\frac{i\hat{\beta}\hat{\chi}_e(\omega-k_y)}{2k_\perp^2}\right]^{-1}=0\end{aligned} \tag{1}$$

This equation contains simple drift mode, a standard drift resistive ballooning mode and also the high - m DRBM [7]. The high-m DRBM is known to be responsible for edge turbulence in L-mode and it has the growth rate similar to the ideal MHD growth rate, even when β is less than the $\beta_{critical}$ [$\beta_{critical} \approx (1+\tau_i)\beta q^2 R/L_n) < 1$]. The DRBM instability dominates over drift Alfven mode, if,

$$k_\perp^2 > [\sqrt{2}\ \lambda_f / q^2 R\sqrt{1+\tau_i}](L_n m_i / R m_e)^{1/2} > \sqrt{2(1+\tau_i)}\ \beta(\lambda_f / R)(R m_i / L_n m_e)^{1/2}.$$

In the limit $\varepsilon_n < 1$ and $k_\perp L_0 < 1$, the real frequency and growth rate of DRBM are given by

$$\omega_r \approx -\frac{\alpha_i}{2} k_y + \omega_1(k_\parallel); \quad \omega_1(k_\parallel) \approx \frac{\hat{\chi}_e k_\parallel^2}{4k_\perp^2 \gamma_0} k_y [2 + \alpha_i (1 - k_\perp^2)]$$
$$\gamma \approx \gamma_0 + \gamma_1(k_\parallel); \quad \gamma_0 = [\varepsilon_n (1+\tau_i) - \alpha_i^2 k_y^2 / 4]^{1/2}, \quad \gamma_1(k_\parallel) \approx -(1+k_\perp^2) \frac{\hat{\chi}_e k_\parallel^2}{2k_\perp^2} \tag{2}$$

Here γ_0 is the ideal mode growth rate. Note that the growth rate of DRBM is close to ideal growth when the plasma is collisional and poloidal 'm' number of the instability is large.

2. **Zonal flows**

Our main interest here lies is in studying the saturation of high-m DRBM through back reaction of zonal flows. In the long wavelength limit, the equations of zonal flow of interest is

$$\left(\partial_t + \nu_i^{neo}\right)\tilde{\phi}_q = \left\langle \partial_x \tilde{\phi}_k \partial_y \tilde{\phi}_k \right\rangle \approx \sum_k k_x k_y \,|\, \tilde{\phi}_k \,|^2 \approx \sum_k k_x k_y \,|\, \omega_{rk} \,|\, / \Lambda_k) \delta N_q \tag{3}$$

Where $\Lambda_k^* \equiv \left(1 + k_y^2 \big/ |\, \omega_k \,|^2 - 2k_y \omega_{rk} \big/ |\, \omega_k \,|^2\right)$. Here the slow variation of $|\, \phi_k \,|^2 \approx |\, \omega_{rk} \,|\, \delta N_q \big/ \Lambda_k$ and δN_q is determined by wave kinetic equation. Thus, the growth rates of zonal flow then is

$$\gamma_q^\phi = q_x^2 \sum_k (1 - 0.5\beta\alpha_{A_\parallel})(\frac{k_x k_y^2 \,|\, \omega_{rk} \,|}{\Lambda_k \gamma_k})(-\frac{\partial N_{0k}}{\partial k_x}) - \nu_i^{neo} \tag{4}$$

Eq. (4) can be rewritten by using the normalization of refs [6].

$$\gamma_q \approx \left(q_x / k_y\right)^2 \left(\alpha_d^2 \hat{m}^4\right) \gamma_k \,|\, e\delta\phi_k L_n / L_0 T_e \,|^2 - \mu \tag{5}$$

3. **Saturation level**

We write the "predator and prey" model equations similar to Ref [9] where the flow shear is equivalent to predator species whereas DRBM fluctuation level to pray species. Equations for fluctuations energy density E_k and shear flow velocity (zonal flows), are:

$$\partial_t \mathrm{E}_k \approx \gamma_k \mathrm{E}_k - a_1 {\mathrm{E}_k}^2 - a_2 \mathrm{E}_k U_q \tag{6}$$

$$\partial_t U_q \approx a_3 \mathrm{E} U_q - \mu U_q \tag{7}$$

Here $E_k \equiv | e\delta\phi_k L_n / L_0 T_e |^2$, $<V_q>' = (c/B) < \partial_x \phi_q >'$, $U_q = <V_q>'^{\,2}$, γ_k is the linear growth rate of fluctuations, $\gamma_k \approx a_1$ describes non-linear damping of fluctuations due to anomalous diffusion, $a_2 \approx (\alpha_d^2 \hat{m}^4 / \gamma_k)(q_x^2 / k_y^2)$ and $a_3 \approx (\alpha_d^2 \hat{m}^4 \gamma_k)(q_x^2 / k_y^2)$ characterize Reynolds stress, driving flow shear that suppresses perturbations.

Equations (6-7) have two stable points: (i) $U_q = 0$ and $\mathrm{E}_k = \gamma_k / a_1$, and (ii) $U_q = (a_3 \gamma_k - a_1 \mu_i) / a_2 a_3$ and $\mathrm{E} = \mu_i / a_2$. The former could be thought as a state of stable L-mode where Reynolds drive is weak; $\gamma_k a_2 < a_1 \mu_i$, the fluctuation level is restricted to

$$\mathrm{E} = \gamma_k / a_1 ; \quad | e\delta\phi_k / T_e | \approx 2\pi(\rho_s / L_n)(R\nu_e / c_e)^{1/2}(m_e / m_i)^{1/4}[(\alpha_e + \alpha_i)/\varepsilon_n]^{1/4} \tag{8}$$

The latter fixed point is like stable H- mode equilibrium for $\gamma_k a_3 > a_1 \mu_i$ and the transition from L-H occurs when Reynolds drive is stronger enough so that $\gamma_k a_3 = a_1 \mu_i$. For the stable H-mode equilibrium, the minimum amplitude of shear flow required is

$$U_q \approx (\gamma_k^2 / \alpha_d^2 \hat{m}^4)(k_y^2 / q_x^2) \tag{9}$$

In this case, the background level of fluctuations level is typically order of

$$| e\delta\phi_k / T_e | \approx (\mu_i / \gamma_k)^{1/2} (1/\alpha_d \hat{m}^2)(k_y / q_x)(L_0 / L_n) \tag{10}$$

This is much below the mixing length saturation amplitude because collisional damping μ_i is typically less than γ_k. What happens if the plasma density is ramped up in this state? One possibility is a violation the latter condition because of increasing collisional damping coefficient μ_i. Alternatively the flows could themselves be unstable to tertiary waves, which can take the place of additional collisionless damping as well as enhance the turbulent transport. In this scenario, the Greenwald density limit could be hit if the growth rate of tertiary wave $\gamma_T > 0$.

4. Tertiary mode (Kelvin Helmholtz instability)

Next, we determine instability threshold for tertiary mode and demonstrate that it becomes positive significantly before collisional damping start to play role. We consider 1-D flow, which is independent of y coordinate and varies in radial direction, x, $V_q'(x) = V_q' \cos q_x x$. As $V_q'(x)$ is periodic in x, we use Floquet theorem technique. Consider tertiary wave perturbations in the form:

$$\phi_T = \sum_l \phi_{T,l} = \sum_l \phi_T \sin\left((K_x + l q_x)x + K_y y + K_\parallel z\right) e^{\gamma_T t} \tag{11}$$

Here K_x, K_y and $K_\parallel$ are the wave vectors along radial, poloidal and magnetic field, respectively and γ_T is the growth rate of the tertiary mode. For small wave amplitude, we truncate wave equation of tertiary mode by keeping the mode coupling among three adjacent modes (i.e. $n = 0, \pm 1$) [10-11]. This results in:

$$\hat{\gamma}_T \overline{\nabla}_\perp^{\,2} \tilde{\phi}_T - (2\beta^{-1} / \hat{\gamma}_T) L_n^2 \nabla_\parallel^2 \overline{\nabla}_\perp^{\,2} \tilde{\phi}_T = -[\tilde{\phi}_\pm, \overline{\nabla}_\perp^{\,2} \tilde{\phi}_q] - [\tilde{\phi}_q, \overline{\nabla}_\perp^{\,2} \tilde{\phi}_\pm] \tag{12}$$

Where $\hat{\gamma}_T = \gamma_T L_n / c_s$, $\overline{\nabla}_\perp = \rho_s \nabla_\perp$, and $\tilde{\phi} = e\phi L_n / T_e \rho_s$. Linear growth of tertiary mode is:

$$\left(\frac{\gamma_T L_n}{c_s}\right)^2 = \frac{1}{4}\left|\frac{e\phi_{0q} L_n}{T_e \rho_s}\right|^2 q_x^2 K_y^2 \frac{q_x^2 - K_\perp^2}{K_\perp^2}\left(\frac{K_\perp^2 + 2q_x K_x}{K_\perp^2 + q_x^2 + 2q_x K_x} + \frac{K_\perp^2 - 2q_x K_x}{K_\perp^2 + q_x^2 - 2q_x K_x}\right) - \frac{2L_n^2 K_\parallel^2}{\beta} \tag{13}$$

Note that the necessary condition for unstable tertiary instability restricted to scale $k_\perp > q_x > K_\perp$. To estimate the threshold of tertiary instability, we simplify by maximizing the driving term in Eq. (13), which requires $K_x \to 0$, $q^2 > K_y^2$. Then we use $K_\parallel \sim 1/L_s$, $L_s = qR/\hat{s}$; $\hat{s}$ is the magnetic shear parameter. The growth of U_q as a function of α_d, α [6] can then be written as:

$$\gamma_T = \sqrt{(K_y^2 / 2q_x^2) U_q - (\hat{s}^2 / \alpha) \gamma_k^2} \tag{14}$$

Note that tertiary wave or Kelvin Helmholtz mode is unstable if

$$\hat{s}\gamma_k \le \sqrt{(K_y^2 / 2q_x^2) U_q \alpha} \tag{15}$$

Thus the threshold of tertiary mode, leading to the decay of zonal, is essentially determined the growth rate of unstable modes which drive edge turbulence [6-7].

5. Density limit scaling

Combining the estimated U_q for H- mode equilibrium, Eq. (9) and Eq. (15), the threshold condition for stable tertiary mode is

$$\hat{s}^2 \alpha_d^2 \ge F(K_y)\, \alpha \tag{16}$$

Here $F(K_y) = 0.5 K_y^2 / q_x^2 \hat{m}^8$, is a constant number for high-m DRBM edge turbulence, $\hat{m} \sim 0.5 - 1$ typically [6-7] and $K_y / q_x < 1$. The threshold condition translates a relation between various plasma parameters sophisticated

$$\hat{s}^2 A_i^{1/2} T_e B^2 L_n^{1/2} \quad \propto \quad n^2 q_a^4 R^{3/2} \tag{17}$$

The density limit in H-mode plasmas is normally related to the pedestal density n_p. Therefore relation (17) has to be considered as relevant for the plasma parameters at top of the edge transport barrier. In order to eliminate the temperature from Eq. (17), adopted henceforth the same temperature for electrons and ions, we consider the edge transport barrier (ETB) heat balance. Following Ref. [12] we assume that the main contribution to the energy losses from the ETB is given by neoclassical contribution κ_{neo} to the ion heat conduction, i.e., $Q_c \approx \kappa_{neo} T_p / \Delta$, where Q_c is the influx of heat into the ETB from the plasma core, T_p the pedestal temperature and Δ the ETB width. It has been shown in Ref. [4] that the Greenwald limit density corresponds well to the transition between the plateau and Pfirsch-Schlüter neoclassical regimes. At slightly lower densities the ETB plasma is in plateau transport regime with $\kappa_{neo} \sim q T^{1.5} A_i^{0.5} n / B^2 R$ [3]. In accordance with findings on diverse tokamaks see ref e.g., [13], we adopt that penetration of recycling hydrogen neutrals defines the ETB width and $\Delta \approx L_n \approx 1/n_p \sigma$, where $\sigma = \sigma_0 \cdot (T_0 / T_p)^{0.4}$, $\sigma_0 = 5.7 \cdot 10^{-19} m^2$ and $T_0 = 0.7 KeV$ [6]. Thus, the ETB power balance results in:

$$Q_c R B^2 \propto T_p^{2.1} n_p^2 q_p A_i^{0.5} \tag{18}$$

By combining relations (17) and (18) we finally get for the critical pedestal density at the threshold of tertiary mode:

$$n_p \propto \left(\frac{I_P}{a^2} \right)^{1.25} \frac{Q_c^{0.157} \hat{s}^{0.55} A_i^{0.06} R}{B^{0.4}} \tag{19}$$

For the DIII-D density limit experiments in Ref. [14] performed at the plasma current $I_p = 1.3 MA$ we get $n_{Gr} \approx 0.92$ and $n_{cr} \approx 0.85.10^{20} m^{-3}$, in good agreement with observations.

5. Conclusion

In this study we have explored the modulational stability of the short scale DRBM turbulence to a long scale Zonal Flows in the edge. We also have estimated the saturation level of turbulence in a state of stable L-mode and the minimum amplitude of shear flow layer required for stable H-mode by using a “predator and prey” model. It is shown that when the collision damping is weak, the model predicts unrealistically low value of saturated primary turbulence, thus we have studied the tertiary instability of ZFs, which take the place of collisional damping of ZFs. Finally, a scaling of critical pedestal density closed to Greenwald limit at the threshold of tertiary mode is derived.

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