Sensitivity Coefficients for Fast Reactor Core Analysis

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1. Introduction

A. Sensitivity Coefficients are utilized in
   ① Evaluation of neutronic characteristics with respect to nuclear data change
   ② Cross section adjustment
   ③ Uncertainty analysis

B. In fast reactors,
   • Core calculation is usually performed in broad-group, not fine groups.
     Sensitivity analysis
   • Transport effect is large, and so for sensitivity coefficient.
Purpose of this work is to

a. Sensitivity reconstruction from broad groups to fine groups.

b. Comparison between sensitivities for transport and diffusion theory calculations.
   - large FBR
   - MONJU type FBR
   - small FBR
2. Theory

**Method A**: Flux reconstruction method

Flow of Neutronic calculation

- Cell calculation (fine group)
  - Cell homogenization \((\sigma^g)\)
    - Spectrum calculation (RZ model) (fine group)
      - Broad-group cross section
        \[
        \sigma^g = \frac{\sum_{g \in G} \sigma^g \varphi^g_{RZ}}{\sum_{g \in G} \varphi^g_{RZ}}
        \]
      - 3-D core calculation using broad-group cross section
Sensitivity in broad-groups
\[ S = \frac{\delta R}{R} \cdot \frac{\delta \sigma}{\sigma} \]

Relative change of \( R \)

Broad-group
\[ \frac{\delta R^G_{3D}}{R^G} = \frac{\langle \varphi^+_G, [\delta L^G - \lambda \delta P^G] \varphi^G_{3D} \rangle}{\langle \varphi^+_P \varphi \rangle_B} \]

Fine group
\[ \frac{\delta R^g_{3D}}{R^g} = \frac{\langle \varphi^+_g, [\delta L^g - \lambda \delta P^g] \varphi^g_{3D} \rangle}{\langle \varphi^+_P \varphi \rangle_F} \]

Fine group flux reconstruction
\[ \varphi^g_{3D} |_{V_i} = \varphi^G_{3D} \frac{\varphi^g_{RZ}}{\varphi^G_{RZ}} |_{V_i} \]

\[ \frac{\delta R^g_{\infty}}{R^g} = \frac{\delta R^G_{\infty} \varphi^+_{RZ}}{R^G \varphi^G_{RZ}} \cdot \sigma^g \frac{\varphi^g_{RZ}}{\varphi^G_{RZ}} \langle \varphi^+_P \varphi \rangle_B \]
\[ = \frac{\delta R^G_{\infty}}{R^G} \cdot \frac{\varphi^+_{RZ}}{\varphi^G_{RZ}} \cdot \sigma^g \frac{\varphi^g_{RZ}}{\varphi^G_{RZ}} \langle \varphi^+_P \varphi \rangle_B \]
\[ = \frac{\delta R^G_{\infty}}{R^G} \cdot \frac{\varphi^+_{RZ}}{\varphi^G_{RZ}} \cdot \sigma^g \frac{\varphi^g_{RZ}}{\varphi^G_{RZ}} \langle \varphi^+_P \varphi \rangle_B \]
Method B: Simple method

\[ \sigma^G = \frac{\sum_{g \in G} \varphi^g \sigma^g}{\sum_{g \in G} \varphi^g} \]

\[ \frac{\delta \sigma^G}{\sigma^G} = \frac{\sum_{g \in G} \delta \sigma^g \varphi^g}{\sum_{g \in G} \sigma^g \varphi^g} - \frac{N \delta \sigma^G}{\sum_t \left( \sum_{g \in G} \frac{\sigma^g \varphi^g}{\sum_{g \in G} \sigma^g \varphi^g} - \frac{\varphi^g}{\sum_{g \in G} \varphi^g} \right)} \]

\[ S^G = \frac{\delta R}{R / \sigma^g} = W^g S^g \]

\[ \sum_{g \in G} S^g = S^G \]
3. Numerical Results

1) $k_{eff}$

**FIG. 1.** Left: Fine group sensitivities (red) and reconstructed sensitivities (green). Right: error between actual fine group sensitivities and reconstruction. Reconstruction performed for a small calculational volume near the core center. Errors are large for low energies, but the sensitivity in these groups (above group 25) is very low.

**U-238 capture cross section change near core center**
FIG. 2. Left: Fine group sensitivities (red) and reconstructed sensitivities (green). Right: error between actual fine group sensitivities and reconstruction. Reconstruction performed the entire inner fuel region according to equation (9). Errors are large for low energies, but the sensitivity in these groups (above group 25) is very low.

U-238 capture cross section change in whole inner fuel region
2) Na void worth

**FIG. 3.** Sensitivity of the void coefficient in the medium-sized reactor. Left: sensitivity coefficient; Right: Error between transport and diffusion calculations. For groups where the sensitivity is large the errors are generally small. Note the rather large errors in groups 12, 13, 18, and 29 sqq. All these groups have very small sensitivities.

**U-238 capture cross section change for 1250 MWe core**
Pu-239 cross section change for Monju-like core

FIG. 4. Sensitivity of the void coefficient in the “Monju-like” reactor. Left: sensitivity coefficient; Right: Error between transport and diffusion calculations. Note in this case a trend of about -5% of the error. Large errors occur in groups 13 and 18, but the corresponding sensitivities are very small.
FIG. 5. Sensitivity of the void coefficient in the small reactor. Left: sensitivity coefficient; Right: Error between transport and diffusion calculations. Note that the shape of the sensitivity curve is opposite to that of the Monju-like reactor. This is caused by the fact that the void coefficient has an opposite sign in the small reactor. In this case a trend of about -20% of the error occurs. In group 11 a large error occurs, but the corresponding sensitivity is small.

Pu-239 cross section change for small core
4. Conclusions

- **Sensitivity reconstruction from broad-group**
  - The reconstruction of fine-group flux, and adjoint flux from broad-group flux and adjoint flux lead the expression of fine-group sensitivities.
  - This formula leads the good results of sensitivities due to cross sections near the core center.

- **Transport effect for Na void coefficient**
  - Transport effect for Na void coefficient.
  - Diffusion calculations yielded erroneous sensitivities for small, high-leakage core.
  - For large cores, the difference between the transport and diffusion sensitivities generally decreases.
  - For Monju-like core, the difference is also small.