Unannounced Inspection for Integrated Safeguards:
A Theoretical Perspective

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1 Introduction

The application of a safeguards verification regime based on existing agreements under INFCIR/153 [1] and on the Additional Protocol, INFCIRC/540 [2], has the potential to allow the International Atomic Energy Agency (IAEA) to relax its traditional facility and material-oriented inspection procedures. The relaxation will take into account the IAEA’s enhanced access to information as well as complementary access to locations gained through the application of extended measures foreseen under the Additional Protocol. It will reflect the associated confidence achieved at the State level.

It is generally agreed that such a trade-off between the new strengthening measures and the traditional measures is reasonable and desirable, both from the point of view of the inspected State, which would like to receive tangible credit for providing increased openness and transparency in its peaceful nuclear activities, and from the viewpoint of the IAEA, which must apportion its limited inspection resources efficiently.

An often-discussed proposal to reduce routine inspection effort while maintaining safeguards effectiveness is to replace scheduled interim inspections with a smaller number of random, unannounced visits, eventually just one. Intuitively, the unpredictability aspect is appealing, as it places the potential diverter in a permanent state of uncertainty. There are also some disadvantages, however, such as the difficulty of planning and implementing truly random inspections and the burden experienced by facility operators obliged to accommodate them. An objective evaluation of a randomised inspection regime vis-à-vis conventional routine inspections requires an objective measure of effectiveness and a means of optimising that measure – in other words a theoretical framework for analysing verification problems. In our paper we provide such a framework by quantifying the notion of timely detection and by treating the problem consistently a strategic one.

2 Playing for time

Consider a single safeguarded facility and a reference period of one year. In order to “isolate” the timeliness aspect of routine inspection, let us assume that a physical inventory verification (PIV) always takes place at the end of the reference period and that it will detect a diversion of nuclear material with certainty if one has occurred. Then we can interpret interim inspections as serving solely to reduce the time to detection below one
year. Again with simplicity in mind, we will similarly assume that an interim inspection will also detect a preceding diversion with certainty. At this level of abstraction we can compare the effectiveness of conventional, scheduled interim inspections with that of randomised unannounced interim inspections in a rigorous and quantitative fashion.

Suppose there are \( k \) interim inspections. If they are scheduled to occur at regular intervals, then the detection time is obviously \( 1/(k + 1) \) year. That is, the operator knows when the next inspection will occur and will divert immediately after an interim of PIV inspection in order to maximize the time to detection.

Now consider a random, unannounced inspection scheme, where the \( k \) interim inspections occur at any randomly chosen times within the reference period. For convenience we shall label the inspection times according to the number of interim inspections still available prior to that time. Thus prior to an inspection at time \( t_k \) there are \( k \) unused interim inspections available, prior to an inspection at time \( t_1 \) there is one interim inspection left and so on. For convenience we label the beginning of the reference period \( t_0 = 0 \), so we have \( 0 = t_{k+1} < t_k < ... < t_0 = 1 \). Now we have to answer the following two questions:

1. What is the best choice for \( t_i \), \( i = k \ldots 1 \), from the point of view of the inspector?

2. What is the associated detection time?

Following Rothenstein [3], let \( v_i[1-t] \) be the time to detection when time \( t \) has elapsed within the reference period, when no diversion has yet occurred and when there are still \( i \) interim inspections available. Thus the answer to question 2 is simply \( v_i[1-t] = v_i[1-t_{k+1}] \), since this is the detection time at the outset, \( t = 0 \), with 1 year and \( k \) interim inspections to go. We have to calculate it and, in doing so, we will also answer question 1.

First of all, it is obvious that

\[
v_0[1-t] = 1-t,
\]

since, when no interim inspections are left, detection can occur only at the end of the reference period. Our plan is now to work backward, determining the functions \( v_1 \), \( v_2 \) and finally \( v_k \). To do this we must consider the strategic alternatives of inspector and operator. We can characterize the inspector’s interim inspection strategy completely generally as follows: She chooses the time \( t_k \) for her first interim inspection according to some probability density function (abbreviated pdf) on the interval [0, 1]. We write this as \( f_k(t) \).

If no diversion occurred up till then, she chooses the time \( t_{k+1} \) for the second interim inspection according to some other pdf \( f_{k+1}(t) \) on the remaining interval \([t_k,1] \), etc. The facility operator’s strategy can likewise be characterized completely generally: He decides to divert at the beginning of the reference period (time \( t_{k+1} \)) with some probability \( q_{k+1} \) and to wait until the first interim inspection with probability \( 1 - q_{k+1} \). If he waits, he then decides to divert immediately after the first interim inspection (time \( t_k \)) with probability \( q_k \).
and to wait for the next inspection with probability $1 - q_k$. If he waits till the bitter end, then he will divert immediately after the $k$th interim inspection (time $t_k$) with probability $q_k = 1$, since he knows that no further interim inspections will take place.

Suppose the operator diverts immediately after the last interim inspection but one, i.e. at time $t_2$. Then detection will occur at the last interim inspection, at time $t_1$. The average detection time is thus

$$\int_{t_2}^{t_1} (t - t_2)f_1^*(t)dt.$$ 

If the operator decides to wait, his average detection time at the instant he makes his decision is

$$v_1[1 - t_2] = \int_{t_2}^{1-t_2} f_1^*(t)dt.$$ 

The right hand side is just the time to detection with no interim inspections remaining weighted with the pdf for the last interim inspection. The best that the inspector can do is to choose her optimal pdf, call it $f_1^*(t)$, so as to make the operator indifferent to his two alternatives. Thus

$$\int_{t_2}^{1-t_2} (t - t_2)f_1^*(t)dt = \int_{t_2}^{1-t_2} v_1[1 - t]f_1^*(t)dt = \int_{t_2}^{1-t_2} (1 - t)f_1^*(t)dt$$

(2)

where the second equality form (1). It is easy to see that equation (2) is satisfied by the constant probability distribution function $f_1^*(t) = 1/(1 - t_2)$. Therefore,

$$v_1[1 - t_2] = \int_{t_2}^{1-t_2} (t - t_2)f_1^*(t)dt = \frac{1-t_2}{2}.$$ 

Arguing similarly for the interim inspection at time $t_3$, we arrive at the equation

$$\int_{t_3}^{1-t_3} (t - t_3)f_2^*(t)dt = \int_{t_3}^{1-t_3} v_2[1 - t]f_2^*(t)dt = \int_{t_3}^{1-t_3} (1 - t)f_2^*(t)dt$$

(3)

for the pdf $f_2^*(t)$. Its solution is $f_2^*(t) = 2(1-t)/(1-t_3)^2$ and hence

$$v_2[1 - t_3] = \int_{t_3}^{1-t_3} (t - t_3)f_2^*(t)dt = \frac{1-t_3}{3}.$$ 

3
Continuing to work backwards in this way, we conclude that the inspector’s best strategy is always to choose his next random inspection time according to the probability distribution function

$$f_i^*(t) = i \cdot \frac{(1-t)^{i-1}}{(1-t_{i+1})}$$  \hspace{1cm} (4)

on the interval $[t_{i+1}, 1]$ when he has $i$ interim inspections remaining and has just inspected at time $t_{i+1}$. This answers question 1 above. (Although we won’t bother to demonstrate it, the operator’s optimal diversion strategy is easily shown to be $q_i^* = 1/i$, $i = k + 1...1$.)

We now observe that the inspector’s pdf at the beginning of the reference period is $f_k^*(t) = k(1-t)^{k-1}/(1-t_{k+1})$, and so determine the average time to detection to be

$$v_k[1-t_{k+1}] = \int_{t_{k+1}}^{1} (t-t_{k+1})f_k^*(t)dt = \frac{1-t_{k+1}}{k+1} = \frac{1}{k+1},$$

answering question 2. We see that this is precisely the detection time for regular, announced inspections.

3 Critical times and deterrence

Our working assumption in the preceding section was that the operator diverts with certainty within the reference period, merely waiting for the best opportunity to do so. On that basis we were able to make a rather strong statement regarding the effectiveness of unannounced random inspections relative to a conventional interim inspection regime. Empirical observation tells us, of course, that operators in general do not divert nuclear material at all. Therefore it would be useful to have a model which allows for this possibility explicitly. In order to treat the option of legal behavior mathematically it is necessary to work with parameters, called utilities, which reflect the subjective preferences of the operator as a potential diverter. These utilities take the place of the average time to detection, which was the sole concern of the protagonists in Section 2. Moreover, since the problem is still a strategic one, we must parameterize the subjective preferences of the inspector as well.

Once again we have to agree on some appropriate level of abstraction. We will take the IAEA timeliness goals at face value and say that the inspector fails to fulfil her obligations if she does not detect a diversion of nuclear material within a critical time interval $T$ after its occurrence. Unlike the preceding model, we will also allow for imperfect inspections based on physical measurement and associate with each inspection a probability of non-detection $\beta$ and probability of false alarm $\alpha$. The utilities for (inspector, operator) are then

- $(0, 0)$ for legal behavior and no false alarm,
- $(-e, -f)$ for legal behavior and false alarm,
- $(-a, -b)$ for timely detection of a diversion, and
- $(-c, d)$ for non-detection of a diversion within the critical time.
These utilities are determined only up to linear transformations, and have been accordingly normalized to zero for the case of legal behavior without a false alarm. We assume

\[ 0 < e < a < c, \quad 0 < f < b, \quad 0 < d. \]

This means that, for the inspector, the most desirable outcome is legal behavior on the part of the operator (utility 0 or \(-e\) for a false alarm), thereafter timely detection (utility \(-a\)) and finally no detection within the required critical time \(T\) (utility \(-c\)). Thus the inspector’s highest priority is deterrence. For the operator the most desirable outcome is diversion without timely detection (utility \(+d\)). Although hard to accept for the honest operator (or State), without this formal assumption any verification regime would of course be pointless. The cost to the operator of a false alarm is \(-f\) and the sanctions in the event of timely detection \(-b\). Taking the non-detection and false alarm probabilities into account, the expected or average utilities are

\[
\begin{align*}
&(-e\alpha, -f\alpha) \quad \text{for legal action} \\
&(-\alpha(1 - \beta) - c\beta, -b(1 - \beta) + d\beta) \quad \text{for diversion of nuclear material,}
\end{align*}
\]

or, if we introduce quantities

\[ A = (b + d) \cdot (1 - \beta) > 0, \quad B = (c - a) \cdot (1 - \beta) > 0 \]

more suitable for our subsequent considerations,

\[
\begin{align*}
&(-e\alpha, -f\alpha) \quad \text{for legal action} \\
&(B - c, d - A) \quad \text{for diversion.}
\end{align*}
\]

Returning to the reference period of Section 2, let us assume that it consists of \(\lambda\) critical time intervals, i.e., \(\lambda \cdot T = 1\). Complete coverage of the reference period could obviously be achieved with \(\lambda\) inspections, one at the end of each critical time interval. But since we are discussing reduced inspection effort, we will assume that there are only \(k < \lambda\) inspections. The inspector can, as in Section 2, distribute them over the reference period any way she chooses. If the operator decides to behave illegally he does so, again as in Section 2, precisely once during the reference period. If an inspection precedes a diversion, a false alarm is raised with probability \(\alpha\), if an inspection follows a diversion the diversion is detected with probability \(1 - \beta\) and a false alarm is excluded. The detection may or may not be timely, i.e. within the critical time \(T\). Under the additional, rather strong assumption that, before the reference time period begins, the operator decides whether or not to act illegally and if so, when to divert, a general solution can be determined.

We won’t give the details here, since contrary to the previous model they are easily accessible [4]. However the most interesting features will be pointed out. These are best expressed in terms of the quantity

\[ x = 1 - \frac{f\alpha}{A}, \quad 0 < x < 1, \]
and are as follows: When

$$x^k \leq 1 - k(1 - x),$$  \hspace{1cm} (5)$$

which holds for \( k = 1 \) and also \( k > 1 \) when \( f\alpha << A \), and additionally

$$x^k \leq \frac{1}{1 + k \cdot \frac{A}{d} (1 - x)}$$  \hspace{1cm} (6)$$

the inspector should inspect at the end of the \( i \)th critical time interval with probability

$$p^*_i = k \cdot \frac{1 - x_i^{i-1}}{1 - x_i}$$, \hspace{1cm} i = 1 \ldots \lambda.$$  \hspace{1cm} (7)$$

The operator should, according to his preferences as given above, act illegally. When \( x \) satisfies (5) but not (6) the inspector should continue to inspect with the probabilities (7) and the operator should, if he is rational, behave legally.

Condition (5) holds generally for reasonable values of the operator’s utility parameters. As in Section 2 we have not concerned ourselves with the details of the operator’s strategy when he behaves illegally, but they of course form an integral part of the complete solution to the problem.

Under the complement (6) the operator is deterred from illegal behavior. Since the condition is rather complicated, consider the case \( f\alpha << A \). This means that the false alarm costs play a negligible role in the operator’s decision. Then the condition for legal behavior reduces to the very intuitive formula

$$k > \frac{1}{T} \cdot \frac{1}{1 - \beta} \cdot \frac{1}{1 + b/d}.$$  \hspace{1cm} (8)$$

The three factors on the right hand side of the above inequality have an immediate interpretation: The first involves the critical time \( T \), which expresses the operator’s technical capability to construct a nuclear device, the second involves the detection probability \( 1 - \beta \), which quantifies the inspector’s technical capability to detect a diversion, the bird involves the subjective cost assessment of the operator expressed as the ratio \( b/d \) of sanctions to incentive for illegal behavior. Together these three quantities determine the minimum number of inspections \( k \) required to deter the operator from behaving illegally. The larger each one is individually, the smaller the minimum required number of interim inspections.

4 Discussion

Our conclusion form the model of Section 2 is that, as far as timeliness of detection is concerned, unannounced random inspections have no advantage whatsoever over periodic, scheduled inspection. On the other hand randomization also implies no loss of timeliness and, unlike regular inspection, allows for the possibility of short detection times. This may
be perceived by the IAEA to be advantageous irrespective of timeliness considerations. For example, if the operator diverts at the beginning of the reference period there will be an interim inspection within one week with probability

\[
\int_0^{1/52} f_k^*(t) dt = \int_0^{1/52} k(1 - t)^{-1} dt = 1 - \left( \frac{51}{52} \right)^k.
\]

For \( k = 3 \) random unannounced inspections this is about 6 %. Obviously such considerations are of relevance if evidence of diversion can be concealed within a short period of time. The advantage must be weighed against the logistic problems of implementing truly random, unannounced inspection.

With regard to the model of Section 3, we required prior commitment on the part of the operator to a legal or illegal strategy. This is necessary for determining a closed form solution, but might be also justified on practical grounds: Even illegal activitiers must be planned well in advance. As for deterrence, if the operator’s incentive \( d \) to divert is vanishingly small compared to the sanctions \( -b \), i.e. \( d / b \to 0 \), then \( k = 1 \) would suffice to satisfy condition (8). This gives a formal justification for current IAEA considerations of doing away with regular interim inspections at reactors and replacing them with a single, unannounced inspection.

References


[2] IAEA INFCIRC/540, Model Protocol Additional to the Agreement(s) between State(s) and the IAEA for the Application of Safeguards, Vienna, 1998.
