## Tera-scale computation of Wave-Plasma Interactions in Multidimensional Fusion Plasmas

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**Abstract:** There is great potential for electromagnetic wave techniques to provide control of fusion plasmas by means of localized heating, current drive, flow drive and energetic particle production. With support from the Scientific Discovery Through Advanced Computation (SciDAC) program, we have established a multiinstitutional partnership between plasma physicists and computational scientists, the overarching goal being to obtain quantitatively accurate predictive understanding of electromagnetic wave processes important for heating, current drive, stability and transport applications in fusion-relevant plasmas. Activities during the first year of the project have focused on massive parallelization and acceleration of computer-intensive full-wave rf field solver codes, extension of all-orders methods to two- and three-dimensional plasmas, increasing the physics detail contained in the rf conductivity operator, benchmarking and code comparison, and application to wave propagation problems in Alcator C-Mod, NSTX and LHD.

#### 1. Advances in Solution of the Full-Wave Equation

Full-wave models are needed, particularly in the ion cyclotron range of frequencies, because ray tracing does not optimally treat realistic antenna spectra, plasma cutoffs, or mode conversion layers, nor does it include large-scale coherent effects such as diffraction, interference, and wave focussing. Full-wave models such as PICES [1] and TORIC [2], use a superposition of Fourier modes in the poloidal direction to represent the wave field, thus reducing the parallel gradient operator to an algebraic quantity. These codes can accurately calculate electron and ion damping, but still rely on FLR expansions to express the plasma current as a differential operator.

Recently, we have made great progress in improving the numerical performance of the fullwave spectral ICRF solver TORIC [1]. Implementation of a serial out-of-core algorithm for matrix inversion and adaptation of the code to a massively parallel platform (MPP) have made it possible to accurately study ion Bernstein wave (IBW) mode conversion scenarios in tokamaks where k <sub>i</sub>< 1. Improved numerical resolution is achieved in the poloidal mode number and manifests itself as a reduction in spurious absorption of the mode-converted wave at the (<sup>3</sup>He) cyclotron resonance layer in a D (<sup>3</sup>He) discharge. A dramatic example of this reduction in cyclotron absorption is shown in Fig. 1. Parameters typical of D (<sup>3</sup>He) mode conversion in Alcator C-Mod were used [B<sub>0</sub>=7.9 T, 24% -(<sup>3</sup>He), 5%-H, 47%-D, n<sub>e</sub>(0) = 2.4 ×  $10^{20}$  m<sup>-3</sup>]. The spurious damping at 0.4 r/a 1.0 accounts for about 60% of the total power absorbed for N<sub>m</sub>=63, whereas for N<sub>m</sub>=511, this damping is almost completely eliminated. With improved resolution, the mode-converted wave power is then absorbed via electron Landau damping, in good agreement with experiment [3] and with 1-D all-orders kinetic model predictions.

FIG. 1. Reduction in spurious cyclotron damping with improved poloidal mode resolution in TORIC for D ( $^{3}$ He) mode conversion in Alcator C-Mod

In practice, however, FLR codes are limited to relatively long wavelengths and to cyclotron harmonics of two or less (because each cyclotron harmonic raises the radial



order of the differential equation by 2). In addition, the magnetic flux coordinates required for the poloidal mode expansion are singular at the magnetic axis, the origin of the coordinate system, and often lead to numerical problems. Recently, alternate full-wave models have been developed that eliminate these difficulties by using a fully spectral representation for the wave field, a Cartesian coordinate system, and a collocation method of discretization. Allorders spectral algorithms (AORSA) allow the solution of the integral wave equation without any restriction on wavelength relative to orbit size, and with no limit on the number of cyclotron harmonics retained. With these methods, it is possible to model very shortwavelength structures such as ion Bernstein waves, as well as wave-particle interactions at high harmonics of the ion cyclotron frequency. However, because they require large amounts of computer memory, these methods have previously been applied in 1-D only [4]. Now these calculations have been extended to 2-D and 3-D [5,6] by taking advantage of the massively parallel architecture of today's super computers. With this approach, the limit on attainable resolution comes, not from the theory itself, but from the size and speed of the available computer.

The new calculations have produced high-definition solutions for mode conversion in 2-D toroidal geometry. In multiple dimensions, mode conversion is found to be more complicated than 1-D models suggest. Results show that for high minority ion concentrations, mode conversion is dominated in off-midplane locations by a transition from the fast magnetosonic wave to the slow ion cyclotron wave (ICW). Unlike the mode-converted ion Bernstein wave, the ICW is a cold plasma wave that propagates on the low field side of the mode conversion layer, and can be damped by ions, as well as electrons. The resulting ion interaction provides bulk ion heating as well as sheared poloidal flow at the mode conversion layer. The new calculations also allow self-consistent modeling of high-harmonic fast-wave heating in low-aspect-ratio tokamaks. Ion heating at high harmonics can now be calculated self-consistently with wave propagation in 2-D. A flux surface average of heating contours for NSTX shot #105830 shows that electron heating is peaked near the magnetic axis.

Finally, the AORSA technique has been extended to give fully 3-D solutions for minority ion heating in stellarator geometry. Using an equilibrium magnetic field calculated from the VMEC code, a 50 X 50 X 160 mode solution in (x, y, ) has been obtained for excitation by a single antenna located in one helical field period. This problem is too large to be solved by direct solution. Instead, the underlying symmetry of the ten field-period system has been used to construct the 160-mode solution from ten individual solutions of the 16-mode problem.

FIG. 2. All-orders spectral calculation of minority ion cyclotron heating for all ten field periods of LHD with a single antenna located at the extreme right-hand side. Individual cross sections show the logarithm of the minority ion power absorption at various toroidal angles. Antenna is in one field period only.

# 2. Advances in Physics Contained in the RF Conductivity Operator



A number or restrictive assumptions are conventionally made in the computation of the perturbed plasma current induced by the rf waves. Though most models assume that the equilibrium magnetic field is uniform, the analysis for the rf conductivity operator has been carried out for cyclotron resonance including the effects of changing cyclotron frequency due to gradient of the magnetic field strength parallel to the equilibrium magnetic field direction [7]. However, application of this analysis to multidimensional wave solutions has been hampered in the past by the difficulty of numerically computing the more complicated resonant phase integrals (Z-function). A tabulation method has been developed to improve the speed and performance of computing this quantity, thus opening this type of analysis to 2-D and 3-D study in the future. The more complex analysis requires tabulation of a function of two variables, the traditional resonance parameter,  $z=(w-nW)/(|k_{\parallel}|v_{th}|),$ and  $a = nW/(k_{\parallel}|k_{\parallel}|v_{th}L_{\parallel})$ , the parallel gradient parameter. The tabulation was accomplished with a "local" - "non-local" partition, which derives from underlying physical processes. For one sign of  $k_{\parallel}$ , the resonant particles have sign of  $v_{\parallel}$  that causes them to travel through the resonance to the other side, where they add to locally resonant particles that have opposite sign in  $v_{\parallel}$ . The two different particle populations may have different cyclotron phasing, opening up the possibility of a wide range of effects that have not previously been studied.

Because nonthermal ion populations are frequently encountered in experiments, the models for the rf conductivity must be extended to allow for the presence of non-Maxwellian equilibrium distributions. In this case, the terms of the dielectric kernel do not have analytical expressions and the 2-D integrals over velocity space must be performed numerically. Obviously, this implies a tremendous increase of the computational requirements and one has to resort to massively parallel computers to solve the problem within acceptable time and memory limits. Modules to calculate the rf conductivity, in the locally uniform plasma approximation, have been developed for arbitrary, non-Maxwellian, non-isotropic distributions defined on a 2-D velocity space grid.

To validate these modules and explore the implications for plasma wave dynamics in 1-D, these conductivity modules have been incorporated into the 1-D all-orders code, METS, which has also been adapted for parallel processing[9]. To illustrate the effects of generalized distribution functions on wave propagation and absorption, this new version of METS has been used to simulate a D-T discharge on TFTR, for which ICRH heating was extensively employed. This discharge is characterized by the following parameters:  $B_0=4.7 \text{ T}$ ,  $n_{e0}=4.7 \times 10^{19} \text{ m}^{-3}$ ,  $T_{e0}=6.8 \text{ keV}$ . The temperature of the Deuterium, Hydrogen, and Carbon ions was assumed to be  $T_{i0}=31 \text{ keV}$ . A Tritium beam was injected into the plasma with  $E_b=120 \text{ keV}$ . A comparison was made modeling the beams population as a slowing-down

distribution, and also modeling it as an equivalent Maxwellian with an equivalent temperature computed at each radius. In Figs. 3(a) and 3(b), the power deposition profiles on electrons and tritons, respectively, are shown for both cases.



FIG. 3 Power deposited on (a) Electrons, (b) Tritium when the latter is described by a slowing-down distribution (solid) or by an equivalent Maxwellian (dashed) in TFTR

Although the net absorption is similar in both cases, the absorption profiles differ drastically, especially for the energetic tritium ions. When a slowing-down distribution is used, the deposition profile appears to be much more peaked than for the equivalent Maxwellian.

### 3. Advances in Calculation of the Plasma Response

RF-induced sheared poloidal flows are important both as a practical means of externally controlling turbulence and inducing transport barriers, and also for probing the fundamental physics of the interaction of waves and flows. Damping of either directly-launched IBWs, or mode-converted ion cyclotron waves are capable of driving such flows. The nonlinear forces from the gyrophase *dependent* part of the quasilinear distribution function have been calculated. This calculation generalizes the usual ponderomotive force on a fluid element to include kinetic and dissipative forces and the spatially nonlocal interaction implied by finite  $_{i} \sim 1$ . The results are expressed in terms of a bilinear nonlocal operator Larmor radius k W(k, k') which generalizes the usual plasma conductivity, W(k, k' = k) = s(k). The forcing terms which drive flux-surface-averaged flows (flux surface averaging removes some secular terms associated with heating of the bulk plasma) have been calculated numerically and appear to be of a comparable magnitude to those created by directly-launched IBW. If these results hold up, they would represent, to the best of our knowledge, the first time that significant flows have been predicted from a *mode-converted* slow wave. Previously, it was thought that only the directly-launched IBW would be useful for flow drive. The modeconversion approach is made possible because the 2-D simulations show that the ICW can damp on ions in the presence of a poloidal magnetic field.

High-power rf fields utilized in fusion experiments will produce substantial nonthermal distortions of the velocity - space distribution functions. The analytic calculation of the rf-induced velocity-space diffusion of the ions is particularly challenging, as it may involve finite gyroradius effects with k  $_i > 1$ . At high-ion cyclotron harmonics, several cyclotron harmonic surfaces may be overlapped by the gyro-orbits of the high-energy ions. Two approaches to calculation of the diffusion coefficients are being used. Unapproximated

bounce-averaged ion diffusion coefficients are obtained through appropriate averaging of numerical integration results of the exact Lorentz equation of motion in the realistic toroidal magnetic geometry, using the full-wave rf electromagnetic fields. This approach is computationally tractable, presently requiring ~100 hours of CPU time per flux surface for  $20^4$  orbits in initial parallel and perpendicular velocity phase about the magnetic field and toroidal angle. These results are providing benchmarking for a second diffusion calculation approach based on spatially local Fourier decomposition of the full-wave rf fields and use of the data in the existing quasilinear diffusion calculation in the Fokker-Planck code[8]. The calculated nonthermal distributions are used in the nonthermal dielectric coefficient calculations, with consistency between absorption and distributions obtained by iteration.

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