Variations of the Magnetic Moment of Fast Ions in Spherical Tori

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Abstract. Due to the strongly inhomogeneous magnetic field in low-aspect-ratio spherical tori the magnetic moment of fast ions confined therein appears to be non-conserved. We examine the non-adiabatic fast ion motion in toroidal configurations as effected by the flux surface shape and by ripples of the magnetic field *B*. Specific attention is paid to poloidal ripples of axisymmetric spherical tori, which arise from the combined effects of plasma paramagnetism and a large Shafranov shift of noncircular flux surfaces in the case of a high-beta plasma, causing *B* to vary non-monotonically on the outboard of the plasma column in the vicinity of the mid-plane. The strong sensitivity of non-adiabatic variations of the magnetic moment to weak small-scale poloidal perturbations of flux surfaces and of *B* is demonstrated. The investigation presented helps to understand the nature of the enhanced non-adiabaticity observed in the numerical EFIT field of NSTX, and it enables the evaluation of the accuracy of analytical estimates of non-adiabatic changes of the magnetic moment. The effect considered is expected to substantially impact fast ion transport in spherical tokamaks as well as in stellarators.

1. Introduction

The magnetic moment μ of fast ions confined in axisymmetric low-aspect-ratio spherical torus (ST) plasmas is not well conserved due to the pronounced inhomogeneity of the magnetic field **B** [1,2]. In this paper we evaluate, both analytically and numerically, the magnitudes of non-adiabatic [3] variations of the magnetic moment of fast ions in STs as induced by perturbations of the flux surface (FS) shape and/or by small-scale (in poloidal angle χ) modulation of B along the field line. Our consideration is based on a model magnetic field $B_0(\chi)$ with non-circular but poloidally smooth FSs [4], which has been extended to account for small-scale poloidal modulations $\delta B(\chi)$ typical for high beta ST equilibria. These perturbations arise from the combined effects of plasma paramagnetism and a large Shafranov shift of noncircular flux surfaces, causing $|\mathbf{B}|=B$ to vary non-monotonically as a field line is followed from its inner to the outer mid-plane crossing point. The resulting small-scale modulation of the magnetic field we will call poloidal field (PF) ripples in analogy to the toroidal field ripples.

2. Effect of poloidal field ripples on non-adiabatic jumps of μ

2.1 Theory

Evaluating non-adiabatic jumps of the magnetic moment μ, which occur when particles cross the outer mid-plane (χ=0) of the axisymmetric toroidal magnetic field $\mathbf{B}=\mathbf{B_t}+\mathbf{B_p}$, we follow Refs. [1,3] and derive for the magnitude of relative non-adiabatic jumps the expression

$$\frac{\Delta\mu}{\mu}=-\frac{V}{V_{\perp 0}}\mathrm{Re}\left\{\frac{\hat{M}}{\varepsilon^{1/8}}\exp\left(i\vartheta_0+i\zeta_{\perp s}-\frac{\alpha_0}{\varepsilon}+\alpha_1\right)\right\}+O\left(\varepsilon^{1/8}\right) \quad (1)$$

with

$$\frac{\alpha_0}{\varepsilon} = i\int_{t_0}^{t_s} dt\,\omega_B, \quad \alpha_1 = i\int_{t_0}^{t_s} dt\left(V_\parallel \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_0 \cdot \nabla)\boldsymbol{\tau}_2 - \frac{V_\parallel}{2}\boldsymbol{\tau}_0 \cdot \nabla \times \boldsymbol{\tau}_0\right), \tag{2}$$

where $V_\perp$ and $V_\parallel$ are the particle velocities perpendicular and, respectively, parallel to the magnetic field, ε is the adiabaticity parameter (ratio of full ion gyro-radius, $\rho_L=V/\omega_B$, to the curvature radius of a magnetic field line, R_c) and V is the total particle velocity; further, the time moments $t = t_s$ correspond to the stationary gyro-phase points, $\omega_B(t_s)=0$, $\boldsymbol{\tau}_0=\mathbf{B}/B$, $\boldsymbol{\tau}_1 = \nabla r/|\nabla r|$, $\boldsymbol{\tau}_2=[\boldsymbol{\tau}_0\boldsymbol{\tau}_1]$, $\zeta_\perp = arctg(\boldsymbol{\tau}_2 \cdot \nabla B/\boldsymbol{\tau}_1 \cdot \nabla B)$ denotes the phase shift, and $\hat{M}$ is a factor of order unity, which is determined by the FS shape parameters.

To introduce a magnetic field perturbation to $B_0(\chi)$ in accordance with the FS representation $\{R_0(r,\chi), Z_0(r,\chi)\}$ of [4] we use

$$R=R_0 \text{ and } Z= Z_0+\delta Z = Z_0[1- \delta(r)(1-M(r)\cos^2\chi)^2\cos\chi] , \tag{3}$$

where r denotes the FS radius and the functions $\delta(r)<<1$ and $M(r)>1$ define the magnitude and the poloidal scale of FS perturbation, respectively. Taking $\delta(r)=\delta_0(r/a)^2$ and $M(r) = 2.7$ with $\delta_0\equiv\delta(a)=0.04\div0.06$ and a being the plasma radius, the chosen perturbation δZ conforms qualitatively with the axisymmetric perturbation $\delta B \sim 0.01\ B_0(\chi)\ \cos(6\chi)$ that fits the equilibrium FSs and $B(\chi)$ in NSTX (with $\beta = 40\%$) better than $B_0(\chi)$. This is demonstrated in

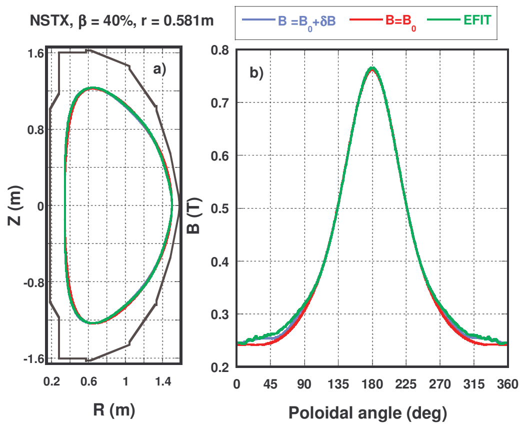

Fig. 1 FSs and poloidal dependences of model and EFIT magnetic fields in NSTX (β=40%). FS radius is r=0.581 m.

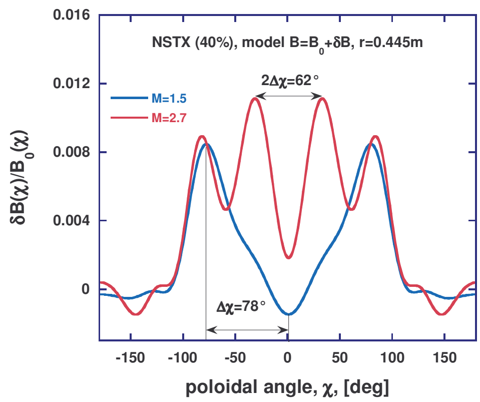

Fig. 2 Poloidal dependences of δB of model magnetic filed (Eq.(3)) for δ_0 = 0.06 and M= 2.7 and M = 1.5. FS radius is r=0.455 m.

Fig.1 where the FSs and $B(\chi)$ are displayed for a given flux surface radius $r = 0.581\ m$ for the unperturbed ($\delta = 0$), the perturbed ($\delta = 0.03$) and the EFIT magnetic field, respectively. The main discrepancy between B_0 and the EFIT field is observed at the outboard part of the plasma column. It is seen that the above δB induces an additional modulation of $B_0(\chi)$ such that the modeled **B** is congruous with the EFIT field. The typical poloidal dependence of $\delta B(\chi)$ at $r = 0.445\ m$) is illustrated in Fig.2 for $M = 2.7$ and 1.5, and for $\delta = 0.026$ ($\delta_0 = 0.06$). As seen, reducing M from $M = 2.7$ to $M = 1.5$ results in an increase of the typical poloidal scale $\Delta\chi$ from 31° to 79°. Roughly, this corresponds to a decrease of the poloidal mode number of PF ripples from $m\approx6$ to $m\approx2$. Next we examine the effect of poloidal field ripples on the value of α_0/ε by using the Taylor expansion of $B(t) = B_0(t) + \delta B(t)$ about the mid-plane, i.e. $B(t)=B_0\left[1+\omega_0^2\left(t-t_0\right)^2+\mathrm{O}\left(t-t_0\right)^3\right]$ with $\omega_0^2=\ddot{B}_0/\left(2B_0\right)$. Following Ref. [3], the value α_0/ε is given by

$$\frac{\alpha_0}{\varepsilon}=\frac{2}{3}\sqrt{\frac{2B^3}{B_{\chi\chi}}}\frac{\omega_{B0}}{\left|V_{//}B\nabla\chi\right|}\Bigg|_0\cong\frac{\alpha_{00}}{\varepsilon}\frac{1}{\sqrt{1+\delta B_{\chi\chi}/B_{0\chi\chi}}}\Bigg|_0, \tag{4}$$

where $\alpha_{00}=\alpha_0(\delta=0)$. Using the FS representation, Eq. (3), for the poloidal ripple induced enhancement of the non-adiabatic change of the magnetic moment, we thus obtain

$$\frac{\Delta\mu(\delta)}{\Delta\mu(0)}\cong\exp\left\{\frac{\alpha_{00}}{\varepsilon}\frac{\delta B_{\chi\chi}}{2B_{0\chi\chi}}\Bigg|_0\right\}=\exp\left\{\frac{\alpha_{00}}{\varepsilon}\frac{\delta(M-1)^2}{1-\delta(M-1)^2}\frac{1+\varepsilon_t}{2\varepsilon_t\left(1+d\Delta/dr\right)}\left(\frac{B_p}{B_t}\right)^2\Bigg|_0\right\}, \tag{5}$$

where $\Delta(r)$ represents the Shafranov shift. Taking into account that, in NSTX, the ratio B_p/B_t ~1 at the outer mid-plane, it is apparent from Eq. (5) that poloidal ripples with δ of the order of several percents may enhance the non-adiabatic variations of the magnetic moment of fast ions more than 10-30 times (if $M\approx3$ and $\alpha_{00}/\varepsilon>5$). This conclusion is confirmed by numerical calculations presented in the following section.

2.2 Results of numerical calculations

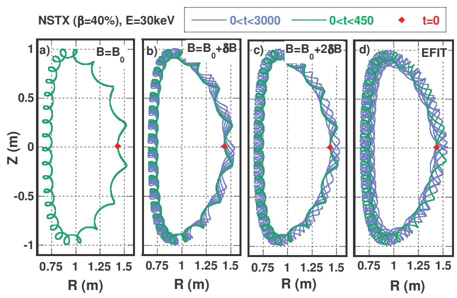

Fig. 3 Full gyro-orbits of 30 keV co-moving deuterons in NSTX during 7 bounce periods.

Figures 3 and 4 display the full gyro-orbits and the time variations of the magnetic moment of 30 *keV* co-moving deuterons in the unperturbed (Figs. 3a, 4a), the perturbed (Figs. 3b, 3c, 4b, 4c) and the numerical EFIT magnetic field (Figs. 3d, 4d) of NSTX with β = 40%. All orbits start at R = 1.433 *m* in the equatorial plane with an initial velocity satisfying the resonance $l\equiv\omega_B/\omega_b=25$, where ω_B and ω_b are the values of the averaged gyro- and the bounce frequency, respectively. Fig. 4 demonstrates the crucial effect of small-scale poloidal perturbations of the magnetic field on the particle motion. While in the unperturbed magnetic field the non-adiabatic variation of μ during 7 bounce periods is of order 1% (Fig.4a), the perturbation $\delta B\sim0.01\,B_0(\chi)\cos(6\chi)$ effects a non-adiabatic change of μ of order 10% (Fig.4b). Doubling of the perturbation yields non-adiabatic $\Delta\mu$ of order 30% (Fig.4c), which is comparable to the 45% variation of the magnetic moment observed in the numerical field (Fig. 4d). However, we note that the rather strong non-adiabaticity observed in the EFIT field may be possibly caused also by small-scale numerical errors. From Fig.1b we see small-scale poloidal modulations of B remaining in discrepancy with the perturbed

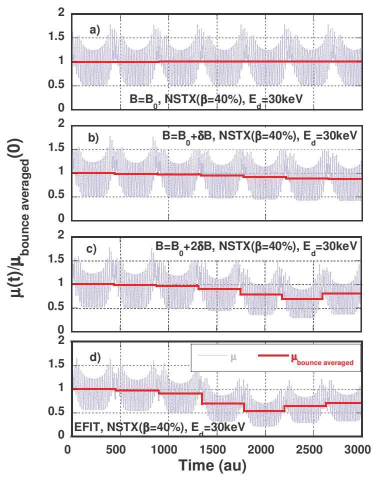

Fig.4 Variations of μ of 30 keV co-going deuterons in NSTX during 7 bounce periods

field, which are apparently responsible for the strong non-conservation of μ even at moderate (E<30 keV) energies. The numerically calculated non-adiabatic jumps of μ of 40 keV co-moving deuterons in NSTX (β=40%, δ_0=0.06) are depicted in Fig. 5 as a function of the gyro-phase θ when the particle has crossed the mid-plane. The observed $\Delta\mu(\theta)$ are seen to be in satisfactory agreement with analytical predictions, thus confirming the resonant nature of the non-adiabatic jumps of μ. Finally we consider the contribution of poloidal field ripples to $\Delta\mu_{max}$. In Fig. 6 we display the numerically obtained dependence of $\Delta\mu_{max}$ on the magnitude of the FS shape perturbation, δ, for the parameters of Fig. 5. The dependence $\Delta\mu_{max}(\delta)$ is essentially nonlinear. For $\delta\leq 0.008$ the contribution of FP ripples is weak and

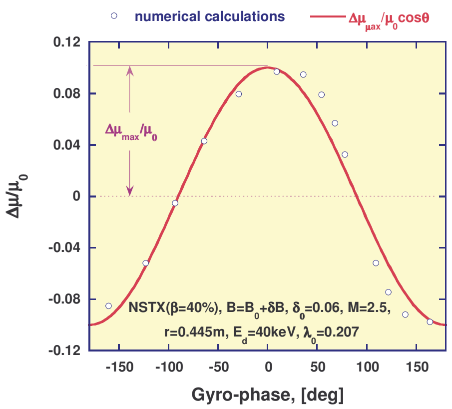

Fig. 5: Non-adiabatic jumps of the magnetic moment as a function the gyro-phase when the particle has crossed the mid-plane.

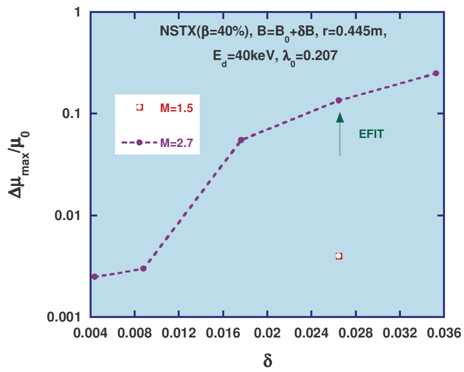

Fig. 6: Magnitude of non-adiabatic jumps of the magnetic moment vs the magnitude of FS shape perturbation

$\Delta\mu_{max}(\delta) \approx \Delta\mu_{max}(0)$, whereas for δ>0.008 we obtained $\Delta\mu_{max}(\delta_0) \sim \delta^2$. For δ = 0.027 corresponding to the EFIT poloidal modulation of B_0 for a flux surface radius r = 0.445 m = 0.66a, the PF ripples enhance the non-adiabatic changes of the magnetic moment of 40 keV deuterons more than 10 times. Fig. 6 demonstrates also the essential reduction of non-adiabatic jumps of μ when the poloidal scale of field perturbation $\Delta\chi$ is increased from 31° (M = 2.7) to 78° (M = 1.5). The red point in Fig. 6 gives the value of $\Delta\mu_{max}(\delta = 0.027, M = 1.5)$ that is about 35 times less than $\Delta\mu_{max}(\delta = 0.027, M = 2.7)$. Note that the value $\Delta\mu_{max}(\delta)$ presented in Fig. 6 is in satisfactory agreement with formula of Eq. (5).

3. Discussion

The poloidal ripple effect considered here can essentially impact on fast ion transport in spherical tori. The reason for the strong effect of small-scale poloidal B-perturbations on non-adiabatic jumps of the fast ion's magnetic moment is their significant influence on the local 'resonance' ω_B=0 in the vicinity of the ST mid-plane. Note that this resonance can be significantly affected also by small-scale toroidal perturbations of non-axisymmetric magnetic configurations, e.g of a stellarator field [5]. This is confirmed by Fig. 7 depicting the time variations of the magnetic moment and of the full gyro-orbit of a 25 keV deuteron in the poloidal cross section of the Compact Helical System (CHS, Japan) during more than 13 bounce periods τ_b. The orbit starts at R = 1.00 m in the equatorial plane with an initial velocity V_R = 0, V_φ = 0.896V. It can be seen that μ executes a non-adiabatic superbanana oscillation with a period about (10÷11)τ_b featuring rather strong non-adiabaticity, $\Delta\mu/\mu \geq$

10%, contrary to the expected exponentially small $\Delta\mu$ ~ exp(-α_{00}/ε) ~ 10^{-8}-10^{-9} for an adiabaticity parameter $\varepsilon \sim \rho_L/R_c$ = (3÷5)%.

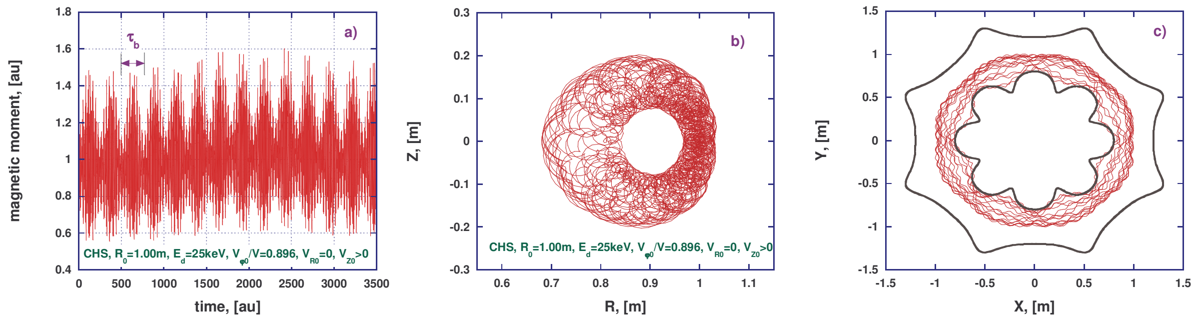

Fig. 7 Time variations of μ and full gyro-orbit of a marginally confined 25 keV deuteron in CHS during 14τ_b.

Evidently, TF ripples in conventional tokamaks may also result in an enhancement of non-adiabatic jumps of the magnetic moment of fast ions, if

$$\frac{\alpha_{00}}{\varepsilon}\sqrt{\frac{\varepsilon_t}{\delta_{TF}}}\frac{1}{Nq} \propto \frac{R_a}{\rho_L}\sqrt{\frac{\varepsilon_t}{\delta_{TF}}}\frac{(1+d\Delta/dr)(1-\Lambda)}{N} \leq 5 \div 10\,,$$

where δ_{TF} denotes the TF ripple magnitude, N the TF coil number and Λ is the FS triangularity.

Acknowledgement

This work has been partially carried out within the Association EURATOM-OEAW project P4. The content of the publication is the sole responsibility of its authors and does not necessarily represent the views of the European Commission or its services.

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