Excitation of Alfvénic Instabilities in Spherical Tokamaks

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Abstract. Understanding energetic particle confinement in spherical tokamaks (STs) is important for optimising the design of ST power plants, and provides a testbed for theoretical modelling under conditions of strong toroidicity and shaping, and high beta. MHD analysis of some recent beam-heated discharges in the MAST ST indicates that high frequency modes observed in these discharges can be identified as toroidal Alfvén eigenmodes (TAEs) and elliptical Alfvén eigenmodes (EAEs). It is possible that such modes could strongly enhance fusion alpha-particle transport in an ST power plant. Computations of TAE growth rates for one particular MAST discharge, made using the HAGIS guiding centre code and benchmarked against analytical estimates, indicate strong drive by sub-Alfvénic neutral beam ions. HAGIS computations using higher mode amplitudes than those observed indicate that whereas co-passing beam ions provide the bulk of the TAE drive, counter-passing ions provide the dominant component of TAE-induced particle losses. Axisymmetric Alfvénic mode activity has been detected during ohmic discharges in MAST. These observations are shown by computational modelling to be consistent with the excitation of global Alfvén eigenmodes (GAEs) with n=0 and low m, driven impulsively by low frequency MHD.

1. Introduction

The low toroidal fields in spherical tokamaks such as MAST [1] make it possible for Alfvén eigenmodes (AEs) to be excited by fast particles with relatively modest energies. STs thus provide opportunities to achieve an improved understanding of AE instabilities, which may affect the confinement of fusion alpha-particles in ITER [2]. Moreover, the ST is a possible candidate for a fusion power plant after ITER [3]. In this paper we present recent theoretical modelling of Alfvénic modes and their interactions with both energetic and thermal particles in ST geometry, with particular reference to experimental results from MAST.

2. Alfvénic Modes in Beam-Heated MAST Discharges

Modes with frequencies in the TAE and EAE ranges have been clearly observed in a number of MAST discharges with both H and D neutral beam heating [4,5]. The beam injection energy in these discharges was about 40keV, with the beam line approximately tangential to the magnetic axis, along the plasma current. Because the toroidal magnetic field in MAST is typically around 0.4T, 40keV beam ions initially have speeds of the order of the Alfvén speed c_A, and can thus drive Alfvén eigenmodes. In MAST discharge 4936, for example, discrete activity occurred in two distinct frequency bands from around 15ms after the start of 1MW H neutral beam injection, and continued until just after an internal reconnection event (IRE) approximately 20ms later (see FIG. 7 in [4]).

Modelling of beam-heated MAST discharges with high frequency mode activity has been carried out using the CSCAS [6] and MISHKA-1 [7] codes. Results from CSCAS (FIG. 1) for one particular timeslice in discharge 4936 indicate a wide toroidicity-induced (TAE) gap and a narrower ellipticity-induced (EAE) gap in the Alfvén spectrum for toroidal mode number

n=3. A discrete mode with eigenfrequency in the EAE gap was found using MISHKA-1 (FIG. 2): this is the first theoretical demonstration of the existence of an EAE in ST geometry. For this discharge the parallel velocity of the beam ions at birth was about $1.7c_A$: under these circumstances, mode drive by passing beam ions at the fundamental resonance is expected.

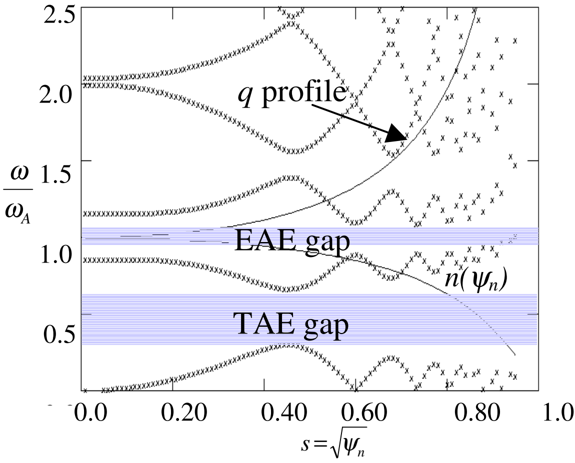

FIG.1. Alfvén spectra for n=3 computed using CSCAS for MAST discharge 4936. Frequency ω *is normalised to* $\omega_A=c_{A0}/R_0$ *where* c_{A0}, R_0 *are the Alfvén speed and major radius at the magnetic axis, and* ψ_n *is poloidal flux normalised to its plasma edge value.*

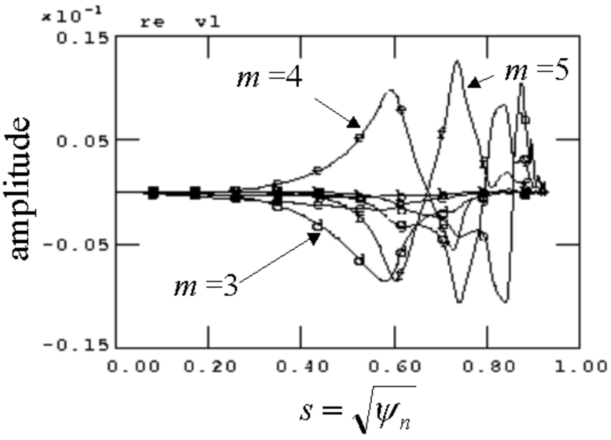

FIG. 2. Radial velocity eigenfunction of EAE with n=3 computed for MAST discharge 4936 using MISHKA-1. Note that more than two poloidal harmonics contribute significantly to the mode structure.

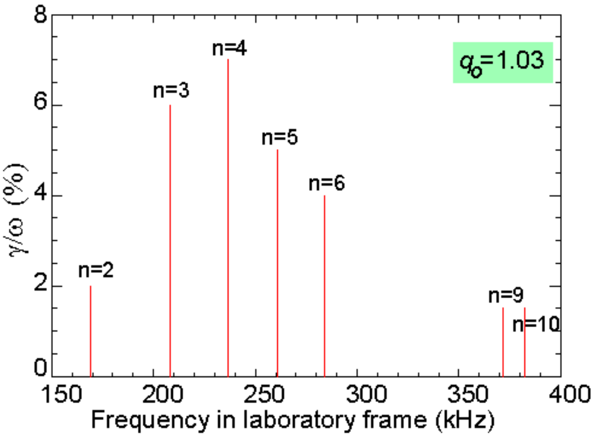

FIG.3. TAE growth rates computed using HAGIS for MAST discharge 5586 (thermal ion Landau damping has been taken into account). The spectrum is sensitive to the assumed value of the safety factor at the magnetic axis, q_0.

We have computed the fast particle drive of TAEs in MAST [5], both numerically, using the guiding centre HAGIS code [8], and analytically, using the approach described in [9]. The fast particle distribution used as input to HAGIS was obtained with the LOCUST code [10], which solves the full orbit equations for neutral beam-injected particles, taking into account both collisions and charge exchange losses. FIG. 3 shows growth rates of TAEs with a range of n values computed for discharge 5586, in which several discrete modes were observed with time-evolving frequencies scaling with the centre of the TAE gap. These observed modes may thus be TAEs with different n. The fact that several such modes are observed simultaneously sets constraints on q_0, the safety factor in the plasma centre, which cannot be measured directly on MAST. Most of the frequencies in FIG. 3 are lower than the measured values, but this discrepancy is partly attributable to experimental uncertainties. Our analytical estimates of the TAE drive are based on a large aspect ratio expansion with circular flux surfaces [9]. For this purpose we also take the fast particle radial excursions Δ_b to be finite but smaller than k_θ^{-1}, where k_θ is the poloidal wavenumber, and the TAE eigenfunction is taken to be localised in the radial direction, with radial width $\Delta_m < k_\theta^{-1}$. Finite Larmor radius effects and the contribution of fast particle energy gradients to the wave drive or damping are neglected. Under these assumptions, the linear power transfer from the energetic particles to the wave reduces to a one-dimensional integral that can be readily evaluated for different wave-particle resonances [9]. We have evaluated the contributions of these resonances for a mode with n=3 and dominant poloidal mode number m=4 in MAST discharge 5586, taking the limits $\Delta_b \gg \Delta_m$ and $\Delta_b \ll \Delta_m$. The results (TABLE I), which are insensitive to Δ_b/Δ_m, indicate that most of the drive is due to co-passing ions at the $\mathrm{v}_{||}=c_A/3$ resonance (the model beam distribution used in this analysis was sub-Alfvénic). This illustrates an important general result, that strong Alfvén eigenmode drive, is possible even in the absence of super-Alfvénic particles.

TABLE I: CALCULATED TAE DRIVE IN MAST DISCHARGE 5586.

$\mathrm{v}_{\|\|}$	γ/ω (%)	
	$\Delta_b \gg \Delta_m$	$\Delta_b \ll \Delta_m$
$-c_A/5$	0.2	0
$-c_A/3$	1.0	1.5
$-c_A$	0	0
c_A	0	0
$c_A/3$	9.9	9.2
$c_A/5$	0.7	0

Analysis of HAGIS computations for the same mode confirms that the drive is provided mainly by co-passing ions at the $\mathrm{v}_{||}=c_A/3$ resonance. Although co-current beam injection was used in this discharge, a significant number of beam ions were pitch angle scattered into the counter direction. Counter-passing ions have only a subsidiary role in contributing to the drive, but they are much more likely than co-passing ions to be lost from the plasma through resonant interactions with TAEs. FIG. 4 shows HAGIS computations of the power associated with lost particles crossing the separatrix versus initial TAE amplitude, $\delta B/B_{\mathrm{mag}}$ (B_{mag} is the field at the magnetic axis). TAEs are only predicted to cause particle losses when the assumed amplitude exceeds a threshold $\delta B/B_{\mathrm{mag}} \approx 10^{-3}$ that is much higher than TAE amplitudes measured so far on MAST, and indeed there is no evidence that the observed modes produce such losses. Above the threshold, losses increase quadratically with mode amplitude. FIG. 4 indicates that if the TAEs in MAST were more strongly-driven, counter-passing beam ions would dominate the resulting fast particle losses. Thus, instabilities driven by fast particles in one region of phase space ($\mathrm{v}_{||}=c_A/3$) can strongly affect the confinement of fast particles in a different region of phase space.

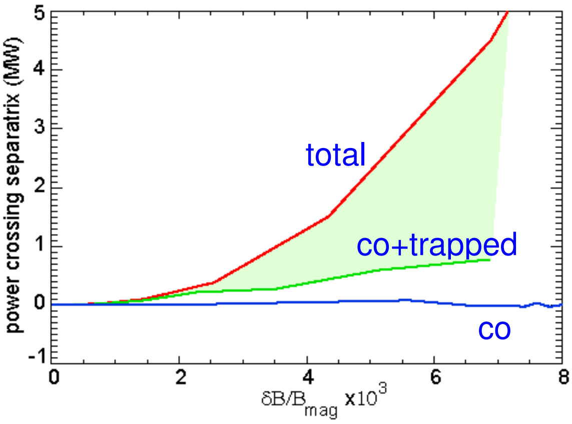

FIG.4. Results from HAGIS simulations of MAST-like plasmas with large-amplitude TAEs, showing the power in particles crossing the separatrix. The green area indicates the contribution of counter-passing ions. TAE-induced losses only occur when the amplitude is much higher than measured amplitudes in MAST.

3. Alfvénic Modes in Ohmic Discharges

Mode activity in the Alfvén range has also been detected during ohmic discharges in MAST [11]; similar activity was observed in TFTR [12]. In TFTR and MAST discharges for which mode number information is available the dominant toroidal mode number was found to be zero: these observations are consistent with the excitation of global Alfvén eigenmodes (GAEs) with n=0 and low m. Such modes are often observed in MAST ohmic discharges, but have low amplitudes and do not cause any degradation of plasma performance. Ideal MHD modelling [11] indicates that the frequencies of undamped n=0 GAEs extend up to about the minimum of c_A/qR, where q is safety factor and R is major radius. Observed frequencies could thus be used in principle to obtain information on plasma equilibria. A mechanism for GAE (or TAE) excitation in the absence of fast ions is suggested by two-fluid simulations carried out for the conventional aspect ratio COMPASS-D tokamak using the CUTIE code [13], in which high frequency activity is correlated with long-timescale MHD events such as tearing modes and edge localised modes. In the case of MAST, an example of such a long-timescale event is an IRE. FIG. 5 shows time series (left) and spectra (right) of fluctuations in a CUTIE simulation of a discharge in COMPASS-D. The upper and lower plots correspond to radial distances $r \approx 0.8a$ and $r \approx a$, where a is minor radius. There are spectral peaks at frequency $f \sim c_A/qR$. These results, combined with the data from ohmic MAST discharges, suggest that the evolution of AEs in ohmic plasmas can be represented by

$$\frac{da}{dt} + i\omega_0 a = \alpha \frac{df}{dt}, \tag{1}$$

where $a(t)$ is the mode magnetic field, ω_0 is the eigenfrequency, $f(t)$ is a field perturbation arising from an impulsive MHD event, and α is a dimensionless parameter measuring the coupling between the impulsive event and the AE (either a GAE or a TAE). Loop voltage time traces during IREs in MAST typically rise more rapidly than they decay, implying that a suitable choice for $f(t)$ might be

$$f(t) = e^{t/\tau_1}, t < 0 \qquad f(t) = e^{-t/\tau_2}, t > 0 \tag{2}$$

where $\tau_1 < \tau_2$. If $1/\omega_0 << \tau_1$, τ_2 the real part of the solution of Eq. (1) for $t > 0$ is

$$\mathrm{Re}[a(t)] \approx -\frac{\alpha}{\omega_0}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)\sin\omega_0 t. \tag{3}$$

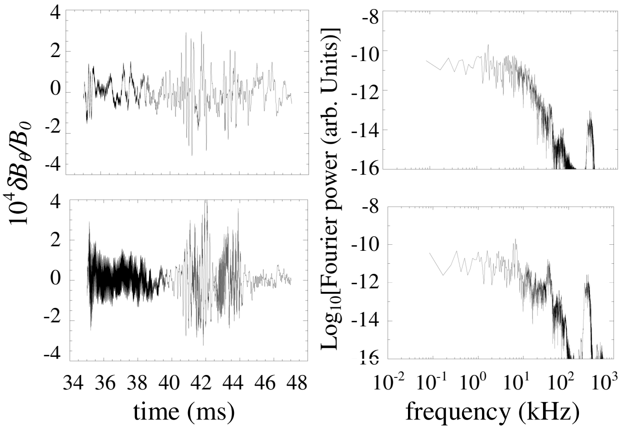

FIG. 5. Time series (left) and Fourier power spectra (right) of magnetic fluctuations in a CUTIE simulation of a discharge in COMPASS-D. Upper plots correspond to $r \approx 0.8a$, while lower plots correspond to $r \approx a$.

The solution is thus an oscillation whose amplitude is essentially the Fourier transform of the driving term. Since, for this particular $f(t)$, the Fourier transform falls off only slowly with frequency, the AE mode amplitude is substantial. Estimates of the parameters in Eq. (3) [11] yield high frequency mode amplitudes that are comparable to or greater than the measured amplitudes, suggesting that the modes may indeed be excited via such a mechanism. If the modes were highly localised and peaked at a radial location r_0 distant from that of the MHD event represented by $f(t)$, one would expect α, and hence the amplitude, to be extremely small. The mode width of a GAE with n=0 is approximately r_0/m: the dominant contribution to GAEs computed for MAST equilibria comes from $|m|$ =1 [11], and so such modes extend over the entire plasma. One would then expect a relatively strong coupling between high and low frequency MHD, i.e. high α. Moreover, the n=0 GAE frequency increases with m while the predicted mode amplitude scales as *1/ω*. The mechanism proposed here thus favours the excitation of low mode numbers.

4. Summary

Discrete high frequency MHD activity during beam heating in MAST has been shown to be consistent with the excitation of TAEs and EAEs. Analytical and numerical computations of TAE growth rates for a MAST discharge indicate that it is possible for such modes to be strongly-driven by sub-Alfvénic neutral beam ions, and simulations carried out using higher mode amplitudes than those observed indicate that counter-passing ions constitute the dominant component of TAE-induced particle losses. High frequency axisymmetric MHD activity observed in ohmic MAST plasmas is consistent with the excitation of GAEs. We have shown that it is possible to generate GAEs or TAEs via impulsive MHD events in the absence of energetic particles. The proposed mechanism favours the excitation of radially-extended GAEs or TAEs with low mode numbers.

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