Rotation Dynamics and Stability of Collisional Edge Layers in Tokamak Plasma

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Abstract: The revisited transport theory for collisional edge layer with steep radial gradients in the toroidal damping time scale is extended to include also the faster poloidal damping time scale. A two-time-scale analysis is applied in order to determine the stability behaviour of both poloidal and toroidal rotations of plasma. It is observed, that the rotational spin-up tendencies are strongly controlled by charge exchange interactions, neutral beam injection, or a radial current.

1. Introduction

As experiments show, transition to the tokamak H-mode from the L-mode, which is connected to a drastic change in confinement characteristics, has a close connection to the sudden onset of a poloidal rotation, and to rise of a radial electric field. Among mechanisms which drive spin-up tendencies of plasma are considered particle, momentum and energy asymmetries among the outer and inner parts of the plasma magnetic surface [1-3]. Another observation indicates that the plasma rotation velocity is also sensitive to the rise of a transport barrier in the main body of plasma.

Steep gradients of density and temperature at transport barriers and collisional plasma edge layers were considered recently in the revisited neoclassical transport theory [4-6], for shorter radial gradient lengths. Included in the new theory were also the Mikhailowskii-Tsypin corrections [7] to the Braginskii's stress tensors. To analyze poloidal or toroidal rotation related phenomena in the edge layer, we use the modified fluid equations including mass and momentum sources derived in the revisited neoclassical theory for collision dominated toroidal plasma with steep gradients near the edge. For example, when the parameter $\Lambda_1 \equiv (\nu_i/\Omega_i)(q^2R^2/rL_\psi)$ is larger than 1/3, the revisited theory introduces additional terms into the parallel momentum equation [4-6]. The resulting ambipolarity equation is,

$$m_i N_i \frac{\partial U^{(1)}_{\varphi,i}}{\partial t} = \frac{\partial}{\partial r}\left[\eta_{2,i}\left(\frac{\partial U^{(1)}_{\varphi,i}}{\partial r} - \frac{0.107\, q^2}{1+Q^2/S^2}\frac{\partial \ln T_i}{\partial r}\frac{B_\varphi}{B_\theta}U^{(2)}_{\theta,i}\right)\right] + J_r B_\theta - m_i \oint \frac{d\vartheta}{2\pi} h^2 S_i^{\,N} U_{i\varphi} + \oint \frac{d\vartheta}{2\pi} h^2 \vec{S}_i^{\,M}\cdot \vec{e}_\varphi \tag{1}$$

where, $h = 1 + (r/R_0)\cos\vartheta$, $Q = [4B_\varphi U^{(2)}_{\theta,i} - 2.5(T_i/e_i)\partial \ln N_i^2 T_i/\partial r]B^{-1}$, $S = (2r\chi_{\|,i}N_i^{-1})/q^2R^2$ and J_r is radial polarization current, and parallel heat diffusion coefficient is $\chi_{\|,i} = 3.9 P_i/m_i \nu_i$.

The equation derived in the revisited theory for the poloidal velocity, however, disregards the proper time evolution of this velocity component, as an essential term involving the time derivative of poloidal velocity was considered small and remained obscure. For tokamak plasmas of circular cross section, adding now also the contribution from the time derivative of the poloidal velocity, (see, for example, [8,9] for such a projection) we have

$$m_i N_i (1+2q^2)\frac{B_\theta}{B_\varphi}\frac{\partial U^{(2)}_{\theta,i}}{\partial t}$$

$$= \frac{3\eta_{0,i}}{2R^2}\left[U^{(2)}_{\theta,i} + 1.833(e_i B_\varphi)^{-1}\frac{\partial T_i}{\partial r}\right] - 0.54\frac{\eta_{2,i}}{1+Q^2/S^2} q^2 \frac{e_i B_\varphi}{T_i}\frac{\partial \ln T_i}{\partial r}\left[\frac{T_i}{e_i B_\theta}\frac{\partial U^{(1)}_{\varphi,i}}{\partial r}\right.$$

$$\left. + \frac{1}{2}U^{(1)\,2}_{\varphi,i} - U^{(1)}_{\varphi,i}\frac{B_\varphi}{B_\theta}\left(U^{(2)}_{\theta,i} - \frac{T_i}{e_i B_\varphi}\frac{\partial \ln N_i^2 T_i}{\partial r}\right) + 1.90\frac{B_\varphi^2}{B_\theta^2}\left(U^{(2)}_{\theta,i} - 0.8\frac{T_i}{e_i B_\varphi}\frac{\partial \ln N_i^{1.6} T_i}{\partial r}\right)^2\right] \qquad (2)$$

$$+ J_r B_\varphi$$

The superscripts on rotation velocities above indicate their respective orders in the small parameter μ, which was defined in the revisited theory as the ratio $\mu \sim L_\psi/r \sim r/qR$, where L_ψ is the scale length of radial gradients [4]. The term with the time derivative on l.h.s. was omitted in the revisited theory [4,6] as it seems to be of a much smaller order, ε, namely, decay time of poloidal damping to the mean toroidal damping time, which depends again on the collisionality. However, this term becomes, on a faster time scale, of the zeroth order and thus controls, for example, the poloidal spin-up phenomena. Whereas, the toroidal spin-up time scale is much longer than the poloidal spin-up time scale [9]. Thus, we have above a coupled nonlinear system of partial differential equations which governs the both rotation velocities near plasma edge with steep radial gradients. The Pfirsch-Schlüter factor for inertia enhancement on the left depends on the collisional regime and should be modified in the plateau regime [10]. Transferring to the faster time scale, $\tau = t/\varepsilon$ which is characteristic for the poloidal relaxation, and transforming to a stretched distance from the separatrix as radial position variable, $\xi = (r - r_{sep})/L_\psi$ these two equations can be rewritten shortly as,

$$m_i\, N_i \frac{\partial U_\varphi}{\partial \tau} = \varepsilon\left[F(\xi, U_\varphi) + G(\xi, U_\theta)\right] \qquad (3)$$

$$(1+2q^2)\, m_i N_i \frac{B_\theta}{B_\varphi}\frac{\partial U_\theta}{\partial \tau} = H(\xi, U_\theta, \partial U_\varphi/\partial\xi; U_\theta)$$

$$= H_1(\xi, U_\theta) - H_2(\xi, U_\theta)\, U_\varphi + H_3(\xi, U_\theta)\, U_\varphi^2 + H_4(\xi, U_\theta)\frac{\partial U_\varphi}{\partial \xi} \qquad (4)$$

For a uniform asymptotic solution of Eqs. (3,4) we assume the following two time scale expansions [11]:

$$U_\varphi(\xi,\tau,t) = \sum_{n=1}^{N}\varepsilon^n U_\varphi^{(n)}(\xi,\tau,t) + O(\varepsilon^N),\; U_\theta(\xi,\tau,t) = \sum_{n=1}^{N}\varepsilon^n U_\theta^{(n)}(\xi,\tau,t) + O(\varepsilon^N) \qquad (5)$$

Expanding also the time derivatives in Eqs.(3,4) we find

$$\frac{\partial U_\varphi}{\partial \tau} + \varepsilon\frac{\partial U_\varphi}{\partial t} = \varepsilon\left[F(\xi, U_\varphi) + G(\xi, U_\theta)\right], \qquad (6)$$

$$\frac{\partial U_\theta}{\partial \tau} + \varepsilon \frac{\partial U_\theta}{\partial t} = H(\xi, U_\varphi, \partial U_\varphi / \partial \xi, U_\theta) \tag{7}$$

and substituting (5) in (6,7), we find from (6)

$$\partial U\varphi^{(0)}/\partial\tau = 0. \tag{8}$$

In other words, $U_\varphi^{(0)}$ can depend only on ξ, and the slow time t. Using this result, and expanding also the r.h.s. of (4) an equation for $U_\theta^{(0)}(\xi,\tau,t)$ is found:

$$\frac{\partial U_\theta^{(0)}(\xi,\tau,t)}{\partial \tau} = H^{(0)}\left(\xi, U_\varphi^{(0)}(\xi,t), \partial U_\varphi^{(0)}(\xi,t)/\partial\xi\ ;\ U_\theta^{(0)}(\xi,\tau,t)\right) \tag{9}$$

where ξ, $U_\varphi^{(0)}(\xi,t)$, $\partial U_\varphi^{(0)}(\xi,t)/\partial\xi$ can be considered as parameters. An implicit solution of (9) can be expressed as

$$\tau = \int dX \left[\frac{1 + K(X+L)^2}{\alpha X^3 + \beta X^2 + \gamma X + \delta} \right] + \text{Const.} \equiv \int dX \frac{g_2(X)}{f_3(X)} + \text{Const.}, \tag{10}$$

where g_2 and f_3 are polynomials of $X \equiv U_\theta^{(0)}$, and K, L, α, β, γ, δ are functions of parameters ξ, $U_\varphi^{(0)}(\xi,t)$, $\partial U_\varphi^{(0)}/\partial\xi$. Hence, integration yields

$$\tau = A \ln\left|\frac{U_\theta^{(0)} - a}{U_\theta^{(0)}(0) - a}\right| + B \ln\left|\frac{U_\theta^{(0)} - b}{U_\theta^{(0)}(0) - b}\right| + C \ln\left|\frac{U_\theta^{(0)} - c}{U_\theta^{(0)}(0) - c}\right|, \tag{11}$$

where a, b, c are three real zeros of f_3, and coefficients $A = g_2(a)/f_3'(a)$, $B = g_2(b)/f_3'(b)$ and $C = g_2(c)/f_3'(c)$ are functions of slow time t. If there is only one real zero, a, and two conjugate zeros $b = b_{re} \pm i b_{im}$, then the solution becomes

$$\tau = A \ln\left|\frac{U_\theta^{(0)} - a}{U_\theta^{(0)}(0) - a}\right| + B_{re} \ln\left|\frac{(U_\theta^{(0)} - b_{re})^2 + b_{im}^{\ 2}}{(U_\theta^{(0)}(0) - b_{re})^2 + b_{im}^{\ 2}}\right|$$
$$-2B_{im}\left[\tan^{-1}\left(\frac{U_\theta^{(0)} - b_{re}}{b_{im}}\right) - \tan^{-1}\left(\frac{U_\theta^{(0)}(0) - b_{re}}{b_{im}}\right)\right]. \tag{12}$$

2. Stability Analysis

Now, we can summarize the behaviour of $U_\theta^{(0)}$ for increasing τ by means of coefficients A, B, and C, as follows:

1) Case of f_3 having three real zeros:
   a) if $A+B+C < 0$, then for all initial values $U_\theta^{(0)}$ remains stable.
   b) if $A+B+C \geq 0$, then some initial values will choose an unstable branch, and $U_\theta^{(0)}$ will move along this unstable branch. To decide about the stability of a chosen branch, one must either look at the branching map, or use a logical classification based on the critical points of the branches, as follows: Supposing that the zeros of f_3 are ordered as $a<b<c$, then $U_\theta^{(0)}$ is always stable for $a < U_\theta^{(0)}(0) < c$. If, however, $c < U_\theta^{(0)}(0)$, or $U_\theta^{(0)}(0) < a$, then one

must also look at the zeros of g_2, i.e., s_1 and s_2. Supposing these are ordered as $s_1 < s_2$, then if

i) $(c, s_2) < U_\theta^{(0)}(0)$, then $U_\theta^{(0)}$ is unstable;
ii) $c < U_\theta^{(0)}(0) < s_2$, then $U_\theta^{(0)}$ is stable;
iii) $U_\theta^{(0)}(0) < (a, s_1)$, then $U_\theta^{(0)}$ is unstable;
iv) $s_1 < U_\theta^{(0)}(0) < a$, then $U_\theta^{(0)}$ is stable.

2) Case of f_3 having only one real zero:
a) if $A+2B_{re}<0$, then for all initial values, $U_\theta^{(0)}$ moves on some stable branches.
b) if $A+2B_{re}\geq 0$, then stability of $U_\theta^{(0)}$ depends on the chosen initial value. In this case we can again determine stability using an analogous scheme as in case 1-b).

Two exemplary branching maps are seen in Fig.1:

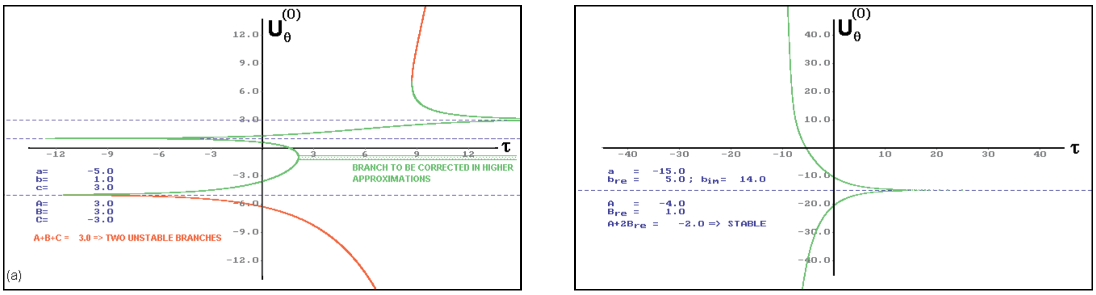

Fig. 1. A model calculation for $U_\theta^{(0)}(\xi,\tau,t)$ for some fixed ξ and t. Solution branches and their stability depend on the initial values taken $U_\theta^{(0)}(\xi,0,t)$. a) case of three real zeros; b) case of one real and 2 complex conjugates zeros. Red branches are unstable; green ones are stable.

In the above calculation of $U_\theta^{(0)}$, we assumed that $U_\varphi^{(0)}(\xi,t)$ and its radial derivative were just parameters. We must, however, now also find the slow time evolution of $U_\varphi^{(0)}(\xi,t)$. For this purpose we take in Eq.(6) all terms of the first order:

$$\frac{\partial U_\varphi^{(1)}(\xi,\tau,t)}{\partial \tau} + \frac{\partial U_\varphi^{(0)}(\xi,t)}{\partial t} = F^{(0)}\left[\xi, U_\varphi^{(0)}(\xi,t)\right] + G^{(0)}\left[\xi, U_\theta^{(0)}(\xi,\tau,t)\right] \tag{13}$$

Now, we note in (13) that a consistent solution for $U_\varphi^{(1)}(\xi,\tau,t)$ is possible if only

$$\frac{\partial U_\varphi^{(0)}(\xi,t)}{\partial t} = F^{(0)}\left[\xi, U_\varphi^{(0)}(\xi,t)\right] \tag{14}$$

holds. This equation can be written explicitly with the effects of charge exchange, neutral beam injection and a possible radial current (see Eq. (1)) as

$$\frac{\partial U_\varphi^{(0)}(\xi,t)}{\partial t} = \frac{1}{m_i N_i}\frac{\partial}{\partial \xi}\left[\eta_{2,i}\left(\frac{\partial U_\varphi^{(0)}(\xi,t)}{\partial \xi}\right)\right] - \nu_{cx} U_\varphi^{(0)}(\xi,t) + \dot{m} + J_r B_\theta / m_i N_i , \tag{15}$$

for $t > 0, \quad -\infty < \xi < +\infty$

For arbitrary $\eta_{2,i}(\xi)$, $N_i(\xi)$, and initial condition $U_\varphi^{(0)}(\xi,0) = \Phi(\xi)$ $(-\infty < \xi < +\infty)$, parabolic equation (15) can be solved by numerical methods. It is seen that, even for a zero initial velocity distribution along ξ, $U_\varphi^{(0)}(\xi,t)$ can be driven in time by the inhomogeneous terms in (15) due to charge exchange, neutral beam injection etc.

A representative case of the steady state solutions of the Eqns. (1,2), calculated by numerical means, are seen in Fig. 2. These solutions can now be tested for stability.

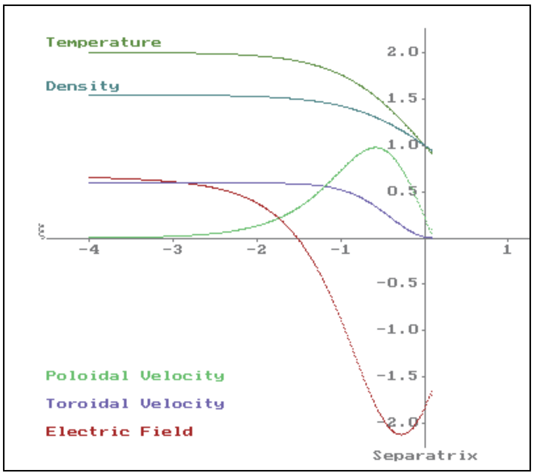

Fig. 2. Steady state solutions of the toroidal and parallel momentum equations yielding normalized toroidal and poloidal velocities, U_φ , U_θ (For details, see [12-14]).

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