Nonlinear Drift Wave Instability due to Nonlinear Structures

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Abstract. A nonlinear instability due to zonal flows and magnetic islands has been found. The instability has the character of a dissipative drift instability due to an anomalous resistivity. The anomalous resistivity is typically two orders of magnitude larger than the classical at the edge.

1. Introduction

Although fluid models now give drift wave transport coefficients that grow towards the edge in the bulk plasma region [1, 2], linearly unstable drift waves can still not explain the continued growth of transport coefficients close to the edge in L mode. Because of this the interest is now focused on nonlinear drift wave type instabilities [3, 4]. These may be due to nonlinear electromagnetic effects on the electron motion along the field lines [3] or due to streamers that twist the density gradient into the poloidal direction [4]. Very far out in the edge, where the plasma is very strongly collisional, linear excitation may again be important due to resistive ballooning modes. The nonlinear electromagnetic effects considered in the present work can, in fact, widen this regime somewhat. We here consider the destabilizing effect of zonal flows and nonlinear magnetic islands, which may be stronger than the stabilizing shearing effects of zonal flows.

2. Formulation

We will here consider the equation of electron motion along a magnetic field

$$m_e \left(\frac{\partial}{\partial t} + \mathbf{v_E} \cdot \nabla + \nu_{ei} \right) v_{e||} = e \left[\left(\hat{\mathbf{e}}_{||} \cdot \nabla \right) \phi + \frac{\partial A_{||}}{\partial t} \right] - \hat{\mathbf{e}}_{||} \cdot (\mathbf{v} \times \delta \mathbf{B}_{\perp}) - \frac{1}{n} \nabla_{||} P, \qquad (1)$$

just in order to see how different nonlinear effects can be added to the resistive friction term. We then consider examples of such terms and finally derive an anomalous resistive instability in a simple case as an illustration.

Perturbations of the magnetic field that are parallel to the background field are ignored throughout the paper. Thus $\mathbf{A} = A_{||}\hat{\mathbf{e}}_{||}$ and $\delta\mathbf{B}_{\perp} = \nabla \times \mathbf{A} = \nabla A_{||} \times \hat{\mathbf{e}}_{||}$. Using Ampere's law $\mu_0 j_{||} = \Delta A_{||}$, ignoring the parallel ion current and also using that electron inertia is small and will be included only for electron ion collisions, equation (1) can be written

$$e\frac{\partial A_{||}}{\partial t} + m_e \nu_{ei} v_A^2 \rho_s^2 \Delta \frac{eA_{||}}{T_e} = -e \left(\hat{\mathbf{e}}_{||} \cdot \nabla \right) \phi - e \left(\mathbf{v_E} + \mathbf{v}_{*e} \right) \cdot \nabla A_{||} + \frac{1}{n} \nabla_{||} P, \qquad (2)$$

where $\mathbf{v_E} = \frac{1}{B} \left(\hat{\mathbf{e}}_{||} \times \nabla \phi \right)$, $\mathbf{v}_{*e} = \frac{1}{neB} \left(\hat{\mathbf{e}}_{||} \times \nabla P \right)$ are the $\mathbf{E} \times \mathbf{B}$ and diamagnetic drift velocities, v_A is the Alfvén velocity and ρ_s is the ion gyroradius at the electron temperature. We have also here neglected the effect of background current velocity v_0 which gives a perpendicular drift $v_0 \delta \mathbf{B}_\perp / B$.

The parts of main interest of the term $e\left(\mathbf{v_E}+\mathbf{v}_{*e}\right)\cdot\nabla A_{||}$ are $\mathbf{v_E}^{(0)}\cdot\nabla A_{||}$ and $-ik_y\phi/B_0\cdot\partial A_{||}^{(0)}/\partial x$ where $\mathbf{v_E}^{(0)}$ and $A_{||}^{(0)}$ are generated nonlinearly. We here consider zonal flows of the form $\mathbf{v_E}^{(0)}=\rho_s c_s\frac{\partial\hat{\phi}_0^{(2)}}{\partial x}\hat{\mathbf{y}}$ and magnetic islands $A_{||0}^{(2)}$.

Using the reductive perturbation method [5], where $\phi_1^{(1)}=\phi\left(x,\xi,\tau\right)e^{i(k_y y-\omega t)}$, $\xi=\epsilon\left(y-\lambda t\right)$, $\tau=\epsilon^2 t$ and ϵ is a small parameter and $\hat{\phi}=\frac{e\phi}{T_e}\sim\epsilon$, gives

$$\left(\mathbf{v}_\perp\cdot\nabla\right)A_{||}=ik_y\frac{1}{B}\left[\frac{\partial\phi_0^{(2)}}{\partial x}-\frac{\partial A_{||0}^{(2)}}{\partial x}f\right]A_{||1}^{(1)}. \tag{3}$$

Here $\phi_1^{(1)}=fA_{||1}^{(1)}$ was used. From the low frequency relation $\nabla\cdot\mathbf{j}=0$, Ampere's law, using $\mathbf{v}_\perp=\mathbf{v}_E+\mathbf{v}_p+v_0\delta\mathbf{B}_\perp/B$ and $\nabla\cdot\mathbf{v}_E=0$ we obtain a relation between $\phi_1^{(1)}$ and $A_{||1}^{(1)}$ as:

$$f \;=\; \frac{k_y v_0\left(\kappa+\eta\right)\Omega_{ci}+k_\perp^2\rho_s^2 k_{||}v_A}{k^2\omega_2}, \qquad \omega_2=\omega_1-k_y v_{0y}, \tag{4}$$

where $\kappa=-\left(d/dx\right)ln n_0$, v_0 is an electron background velocity along B and $\eta=-\left(d/dx\right)ln v_0$. For reference we note that for $k_{||}=0$, the magnetic perturbations lead to the magnetic drift mode with dispersion relation [6, 7].

$$\omega_1=\frac{1}{1+\frac{k^2c^2}{\omega_{pe}}}\left[\omega_{*e}+k_y v_0\left(\frac{\left(\kappa+2\eta\right)v_0}{\Omega_{ce}}-\frac{m_e}{m_i}\frac{k_y v_0\left(\kappa+\eta\right)^2}{k^2\omega_2}\right)\right]. \tag{5}$$

The linear effects of v_0 are generally small as shown by the dispersion relation (5).

Equation (2) can now be written in the form:

$$e\frac{\partial A_{||}}{\partial t}+m_e\nu_{eff}v_A^2k_\perp^2\rho_s^2\frac{eA_{||}}{T_e}=-eik_{||}\phi+eik_y\frac{\tilde{n}}{n}\rho_s^2\frac{\partial A_{||0}^{(2)}}{\partial x}+\frac{ik_{||}}{n}\delta P, \tag{6}$$

where

$$\nu_{eff}=\nu_{ei}+i\frac{T_e}{m_e}\frac{\left(\omega_{E\times B}+\omega_B\right)}{v_A^2k_\perp^2\rho_s^2}, \quad \omega_{E\times B}=k_y\rho_s c_s\frac{\partial\hat{\phi}_0^{(2)}}{\partial x}, \quad \omega_B=-fc_s\rho_s k_y\frac{\partial\hat{A}_{||0}^{(2)}}{\partial x}. \tag{7}$$

We note that this $\omega_{E\times B}$ corresponds to a homogeneous rotation with mode number k_y and is thus usually larger than the shearing rate.

The flow potential $\phi_0^{(2)}$ can be calculated by the reductive perturbation method as was done in [8, 9]. The solution for $\hat{\phi}_0^{(2)}$ as a driven mode in the edge region $(\epsilon\ll1)$ is:

$$\hat{\phi}_0^{(2)}=k_mL_nk_y^2\rho_s^2T\left|\hat{\phi}_1^{(1)}\right|^2\sin 2k_mx, \tag{8}$$

where λ is and

$$T=\frac{2\lambda v_{*e}}{N}, \qquad N=4k_m^2\rho_s^2\lambda\left(\lambda+\frac{V_{*e}}{\tau}\right)+V_dV_{*e}\Gamma, \qquad \Gamma=1+\eta_e+\frac{1}{\tau}\left(1+\eta_i\right). \tag{9}$$

For the magnetic islands including electron inertia we get from [7], which also is based upon the reductive perturbation method, that

$$\hat{A}^{(2)}_{||0} = C\left|\hat{A}^{(1)}_{||1}\right|^2 \sin 2k_m x, \qquad C = 16f\frac{k_y^2\rho_s c_s k_m^3\lambda_2}{K}\left(1 - k_y\kappa\frac{v_0^2+v_{te}^2}{\omega_1\Omega_{ce}}\right), \quad (10)$$

$$K = 4k_m^2\lambda_2 P - \frac{m_e}{m_i}v_0^2\left(\kappa+\eta\right)^2, \qquad P = -\left(1+4\frac{c^2k_m^2}{\omega_{pe}^2}\right)\lambda_1 + v_{*e} + \frac{v_0^2}{\Omega_{ce}}\left(\kappa+2\eta\right), \quad (11)$$

where $\lambda_2 = \partial\omega_2/\partial k_y$.

Multiplying equations (7) by $\sin k_m x$ and integrating over one period in x we arrive at

$$\omega_{E\times B} = -k_m L_n \omega_{*e}\hat{\phi}_0^{(2)}, \qquad \omega_B = k_y c_s \rho_s k_m f A^{(2)}_{||0} = \frac{\omega_{*e}C}{k_y^2\rho_s^2 T f}\hat{\phi}_0^{(2)}, \quad (12)$$

Thus we may rewrite (7) as

$$\nu_{eff} = \nu_{ei} + \nu_{an}, \qquad \nu_{an} = -i\frac{T_e}{m_e v_A^2}\frac{\omega_{*e}}{k_\perp^2\rho_s^2}\left(k_m L_n - \frac{C}{k_y^2\rho_s^2 T f}\right)\hat{\phi}_0^{(2)} \quad (13)$$

The renormalized collision frequency ν_{an} is now independent of background parallel velocity v_0, although the amplitude of nonlinear magnetic islands $A^{(2)}_{||0}$ is. We note that the two contributions usually are comparable in magnitude except for the case $\lambda \approx \omega_{*e}$, i.e. $\omega \approx \omega_{*e}$ where C has a resonance. Note that $\left(k_m L_n - \frac{C}{k_y^2\rho_s^2 T f}\right)\hat{\phi}_0^{(2)}$ in general is complex and ν_{an} will have both real and imaginary parts. A real part requires an imaginary part of the group velocity λ. Thus, we now need to consider a particular dispersion relation.

We will here just pick a simple case of a resistive drift interchange mode as an example and for simplicity take the limit $\omega_{De} \ll \omega_{*e}$ and ignore temperature perturbations and gradients. Combining continuity and parallel momentum equations and ignoring parallel ion motion gives

$$\frac{\delta n_e}{n_e} = \left[\frac{\omega_{*e} - \omega_{De} + ik_{||}^2 De}{\omega - \omega_{De} + ik_{||}^2 De}\right]\frac{e\phi}{T_e} = \frac{\delta n_i}{n_i} = \left[\frac{\omega_{*e} - \omega_{De} - \omega k_\perp^2\rho_s^2}{\omega - \omega_{Di}}\right]\frac{e\phi}{T_e} \quad (14)$$

where $De = \frac{T_e}{m_e \nu_{eff}}$. We here introduce the orderings

$$k_\perp^2\rho_s^2 \sim \epsilon_n \sim \frac{k_{||}^2 c_s^2}{\omega^2} \sim \frac{\omega}{k_{||}^2 De} \sim \epsilon \quad (15)$$

Using the Ballooning mode formalism $k_{||}^2 \rightarrow -\frac{1}{q^2R^2}\frac{\partial^2}{\partial\theta^2}$, and $k_\perp^2 = k_\theta^2\left(1+s^2\theta^2\right)$, we may write the eigenvalue problem as:

$$\frac{\partial^2\phi}{\partial\theta^2} = -\kappa\left[\omega + \frac{A/B}{1+\hat{s}^2\theta^2}\right]\phi, \quad (16)$$

where

$$\kappa = iq^2R^2/D_e, \qquad A = \omega_{*e}\omega_{De}\left(1+\frac{1}{\tau}\right) - \omega\Omega, \qquad B = \omega k_\theta^2\rho_s^2 + \Omega, \tag{17}$$

$$\Omega = \omega - \omega_{*e} + \omega_{De}\left(1+\frac{1}{\tau}\right), \qquad \hat{s}^2 = s^2\frac{k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2 + \epsilon_n\left(1+\frac{1}{\tau}\right)}, \tag{18}$$

where we used $\omega \approx \omega_{*e}$ in equation (18).

Equation (16) has the asymptotic solution $\phi = e^{i\sqrt{\kappa\omega}(\theta-\theta_b)}$, where θ_b is the θ where this solution starts to be valid. Equation (16) has the same form as part of the averaged equation for MHD ballooning modes in [10]. There it was found that the inner solution could be approximated by a constant and that $\theta_b = \hat{s}/4$ was a good choice. We now multiply equation (16) by ϕ^* and integrate from 0 to ∞. Then

$$\int_0^{\hat{s}+\epsilon} \phi^*\frac{\partial^2\phi}{\partial\theta^2}d\theta \approx \left[\phi^*\frac{\partial\phi}{\partial\theta}\right]_{\hat{s}/4} = i\sqrt{\kappa\omega}. \tag{19}$$

Since we take the constant inner value equal to 1. In the other region $\frac{\partial^2\phi}{\partial\theta^2}$ and $-\kappa\omega\phi$ cancel exactly. Since we consider small growthrates $(\gamma \ll \omega_r)$ the asymptotic solution is varying more slowly than the last part of equation equation (16). We thus approximate

$$\int_0^\infty \frac{|\phi|^2}{1+\hat{s}^2\theta^2}d\theta \approx |\phi|^2\int_0^\infty \frac{d\theta}{1+\hat{s}^2\theta^2} = \frac{\pi}{2\hat{s}}. \tag{20}$$

The dispersion relation can then be written

$$\omega\left(1+k_\theta^2\rho_s^2\right) = \omega_{*e} - \omega_{De}\left(1+\frac{1}{\tau}\right) + i\frac{qR}{v_A}\frac{\sqrt{\left(k_mL_n - \frac{C}{k_\theta^2\rho_s^2Tf}\right)\hat{\phi}_0^{(2)}}}{k_\theta\rho_s}\left(\omega B\frac{\hat{s}}{4} + \frac{\pi}{2\hat{s}}A\right). \tag{21}$$

The growthrate can be simplified by averaging equation (8) over x and using $\gamma \ll \omega_r \approx \omega_{*e}$ and $\lambda \approx V_{*e}$, which gives

$$\gamma \approx \frac{\pi}{2s}\frac{\omega_{*e}^2}{\omega_A}\sqrt{k_mL_n\left(k_mL_n - \frac{C}{k_\theta^2\rho_s^2Tf}\right)}G|\phi_1^{(1)}|, \quad G = \frac{\left(k_\theta^2\rho_s^2 + \epsilon_n\left(1+\frac{1}{\tau}\right)\right)^{3/2}}{k_\theta\rho_s\sqrt{4k_m^2\rho_s^2\left(1+\frac{1}{\tau}\right)+\epsilon_n\Gamma}}, \tag{22}$$

where $\omega_A = \frac{v_A}{qR}$. Although G is formally of the order of the root of our small parameter, it is typically of order 1. Thus at the edge where $|\phi_1^{(1)}|$ is not much smaller than 1, γ can become of the order of ω_{*e}.

The saturation of the instability will most likely be outside of the present ordering. However, one can make an estimate from the dispersion relation (equation (14)). For a usual resistive drift instability with $k_{||}^2D_e > \omega$, the dispersion relation can be expanded with an imaginary part proportional to the resistivity. The anomalous resistivity then grows with amplitude and the dispersion relation can eventually be expanded for $k_{||}^2D_e \leq \omega$ resulting in an imaginary part, which decreases with the amplitude. We would thus get saturation roughly when

$$|\langle k_{||}^2D_e\rangle| = \omega_{*e} = \left|\frac{\sqrt{\kappa\omega}}{q^2R^2}D_e\right|, \tag{23}$$

since in this case the averaged $k_{||}^2$ enters only at the matching point between the interior and exterior solutions according to equation (19). Combining equations (23), (8),(13),(17) and (18) one gets

$$|\hat{\phi}_1^{(1)}|^2 = \frac{N}{V_{*e}^2}\frac{\omega_A^2}{\omega_{*e}^2}\frac{1}{k_m L_n\left(k_m L_n - \frac{C}{k_\theta^2\rho_s^2 T f}\right)}. \tag{24}$$

Although N is formally of order ϵ, it is numerically rather large. (ω_A/ω_{*e}) is of order ϵ_n/β_e, which is of the order 10. Thus $|\phi_1^{(1)}|$ can be of order 1 as has been observed in the tokamak edge.

3. Conclusions

We have in this work shown that although nonlinear flows can have a stabilizing effect on turbulence due to their shearing rate, they can be destabilizing due to the generation of anomalous resistivity under certain conditions. We expect this to be important near the edge where the zonal flow amplitude is large, due to small ϵ_n. Thus at the edge the resonance from the ion temperature dynamics [9] is less important. In this case we also expect to be close to the resonance for magnetic islands, i.e. $\omega \approx \omega_{*e}$. The growthrate, of the nonlinearly driven modes, which here has been assumed to be in the regular drift wave regime, could become at least of the order of the diamagnetic drift frequency. This makes the present instability a strong candidate for explaining the transport in the edge outer region where linear driftwaves are not sufficient to explain the transport while the temperature is still too high for resistive ballooning modes. Since saturation occurs when $k_{||}^2 D_e \sim \omega_{*e}$ we can not get a stronger instability by a further increase in the amplitudes of the nonlinear structures.

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