Profile Relaxation and Tilt Instability in a Field-Reversed Configuration

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Abstract. The profile relaxation from a magnetichydrodynamic (MHD) profile to a kinetic equilibrium in field-reversed configurations (FRCs) is investigated by two-dimensional electromagnetic particle simulation. The radial oscillation takes place in order to relax an excess energy in the MHD profile, and the system spontaneously relaxes toward a kinetic equilibrium. In this kinetic equilibrium, the hollow electron current profile is realized as a result of the combined effects of the single particle orbits and the ion finite Larmor radius, and the ion current profile becomes peaked due to the effect of the ion meandering motion. Three-dimensional full electromagnetic particle simulation is also performed to study the stability of these kinetic equilibrium against the tilt mode. The growth rate of the tilt instability is reduced by the kinetic effects. It is found that the stabilization effect of tilt mode becomes much distinct when the current density changes from the peaked profile to the hollow one.

1. Introduction

The tilt instability in the field-reversed configuration (FRC) plasma is predicted by the magnetichydrodynamic (MHD) theory, but it has not been observed in the experiments[1]. It is also reported that most experimental equilibrium states tend to take a hollow current profile[2]. This tilt instability has been studied by the extended MHD models, but they could not give the satisfactory explanation as yet. An MHD equilibrium was used as the initial condition for the three-dimensional (3D) electromagnetic particle simulation[3,4,5]. However, the influence of the kinetic effect on the tilt mode was not clarified, because the MHD equilibrium relaxes to the kinetic one simultaneously with the evolution of the tilt instability. It is important to investigate this problem by the full particle simulation, because most experimental FRC plasmas are so kinetic that the MHD theory can not deal with them.

2. Simulation Method

Taking it into account that the tilt mode is 3D instability, we first perform two-dimensional (2D) electromagnetic particle simulation to get the kinetic equilibrium without exciting the tilt instability, and clarify the property of the kinetic FRC plasma. An initial profile for 2D simulation is given by a one-fluid MHD equilibrium which is controlled by the hollowness parameter D, the plasma beta value $\beta_{\rm sp}$ at separatrix and the finite Larmor radius (FLR) parameter $\overline{s}$[6]. We choose two-types of initial particle distribution. The first one is the shifted-Maxwellian under the zero $E(0)$ condition, where the average flow velocity is equal to the diamagnetic velocity. The second one is the shifted-Maxwellian under the required $E(0)$ condition, where the electric current is carried only by the electron and the average ion flow velocity is zero. In these distribution, the ion temperature and the electron temperature are the same and spatially constant. Next we examine the feature of the tilt instability by means of 3D full electromagnetic particle simulation in which the kinetic equilibrium obtained from 2D simulation is used as the initial condition.

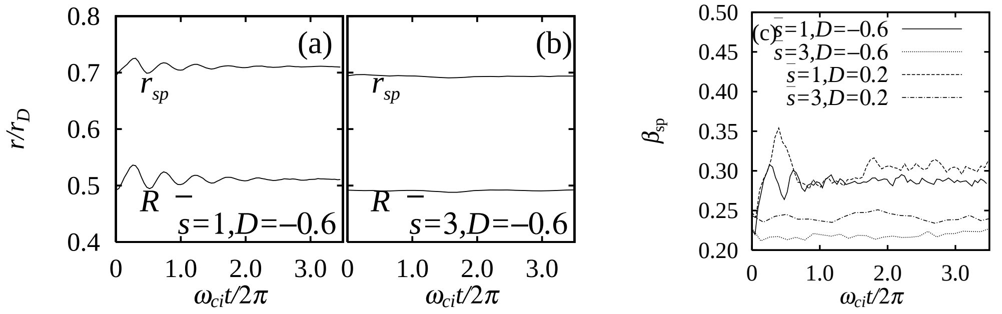

FIG. 1: *Time evolution of the field-null line and separatrix radii (R and* $r_{\rm sp}$*) in (a)*$\overline{s} = 1$*, (b)*$\overline{s} = 3$*, and (c)time evolution of the plasma beta value at the separatrix (*$\beta_{\rm sp}$*).*

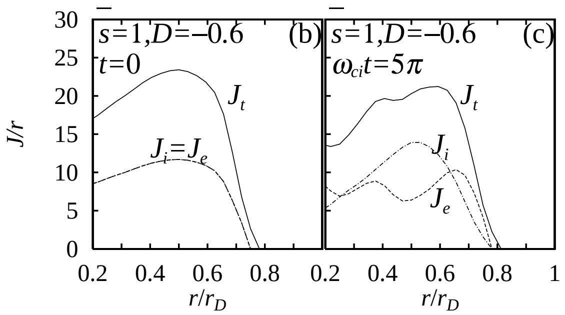

FIG. 2. *The radial profile of toroidal current density in* $\overline{s}$*=1 at (a)*$\omega_{ci}t$*=0 and (b)*$\omega_{ci}t = 5\pi$*.*

3. Profile relaxation

Let us examine how the plasma profile changes from an MHD profile to a kinetic equilibrium based on 2D simulation results. Figure 1(a) 1 (b) shows the time evolutions of the field-null line and separatrix radii (R and $r_{\rm sp}$) on the midplane in (a) $\overline{s} = 1$ and (b) $\overline{s} = 3$, respectively. In the full kinetic case ($\overline{s} = 1$), both R and $r_{\rm sp}$ oscillate with frequency $\omega \sim 2\omega_{ci}$ in the early period and damp gradually until $\omega_{ci}t \sim 5\pi$ shown in Fig. 1(a). The plasma beta value $\beta_{\rm sp}$ at the separatrix jumps from an initial small value to about 0.3 in an initial moment, and keeps this value after that (Fig. 1(c)). This phenomena indicates that the profile oscillates in the radial direction to relax an excess energy in an MHD profile. In the moderate kinetic case ($\overline{s} = 3$), on the other hand, no oscillation appears (Fig. 1(b)) and $\beta_{\rm sp}$ keeps an initial value (Fig. 1(c)). When a plasma is fully kinetic, the energy difference between an initial MHD profile and an obtained kinetic equilibrium is so large that a relaxation oscillation is excited.

Figure 2 shows the radial profiles of toroidal current density in $\overline{s} = 1$ at (a)$\omega_{ci}t = 0$ and (b)$\omega_{ci}t = 5\pi$, respectively. After the relaxation oscillation, the electron current density J_e increases near the separatrix, and decreases near the field-null line. An initial peaked profile ($D < 0$) changes to a hollow profile ($\tilde{D}_e > 0$). On the other hand, the ion current density J_i becomes more peaked ($\tilde{D}_i < D < 0$). So the total current J_t changes to the hollow profile near the field-null line. Both the decrease of J_e and the increase of J_i near the field-null line can be explained by the character of the single particle orbit (Fig. 3). The dominant electron motion near the field-null line is the gradient-B drift. Because the gradient-B drift has the opposite sign to the electron diamagnetic drift, J_e decreases

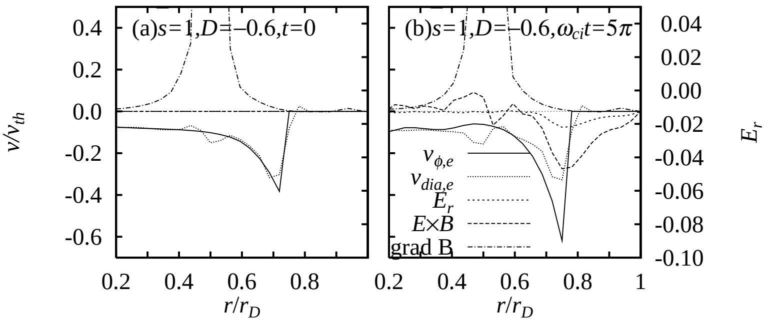

FIG. 3. *The radial profile of toroidal electron flow velocity in* $\overline{s}$*=1 at (a)*$\omega_{ci}t$*=0 and (b)*$\omega_{ci}t = 5\pi$*.*

near the field-null line. On the other hand, when the spatial scale of magnetic field is almost the same as the ion orbit scale, ions execute meandering motions along the field-null line. The average toroidal velocity is so large due to this meandering motion that J_i increases near the field-null line. Next, we consider why J_e increases near the separatrix (Fig. 3). Since the density profile becomes steep locally in the narrow periphery region near the separatrix, the ion FLR effect generates the strong radial electric field E_r there. Because the generated $E \times B$ drift has the same sign as the electron diamagnetic drift, J_e increases in the periphery. On the other hand, E_r acts on ions less effectively since the ion Larmor radius is larger than the spatial size of a strong electric field region. That is, the modification of ion current profile becomes relatively smaller.

In this way, an initial MHD equilibrium with a peaked current profile relaxes to a kinetic equilibrium with a hollow current profile through the effects of the single particle orbit and FLR. We find the tendency for the electron current to become a hollow profile and for the ion current to become a peaked profile independently of the initial condition, such as the initial hollowness parameter D, the FLR effect $\overline{s}$, the initial electric field $E(0)$.

4. Tilt instability

We clarify from 2D simulation that the kinetic equilibrium with the hollow current profile is spontaneously generated in the FRC plasma. In this section, the feature of the tilt mode in the kinetic equilibrium is also investigated based on 3D simulation results. In 3D simulation, the kinetic equilibrium solution obtained after the profile relaxation in 2D simulation is adopted as the initial condition.

There are several parameters which are related to the stabilization of the tilt mode. That is, the plasma beta value β, the Alfvén Mach number M_A, the hollowness parameter D, and the FLR parameter $\overline{s}$. We discuss the relationship between the growth rate of the tilt mode and these parameters. Figure 4 shows the dependence of the tilt growth rate γ_{tilt} on (a) the plasma beta value β_{sp} at the separatrix, (b) the Alfvén Mach number M_A, and (c) the electron hollowness parameter $\tilde{D}_e$, where γ_{tilt} is normalized by those obtained from MHD simulation γ_{MHD}, and M_A is the associated with the ion toroidal flow velocity, the plasma density at the field-null line, and the magnetic field at the wall. A glance at Fig. 4 reveals that γ_{tilt} reduces to about 5% to 25% of γ_{MHD} because of the kinetic effect.

Figure 4(a) shows that γ_{tilt} tends to decrease as β_{sp} increases. This result means that the separatrix beta value is relevant to the tilt stabilization. Nishimura *et al* suggest from this

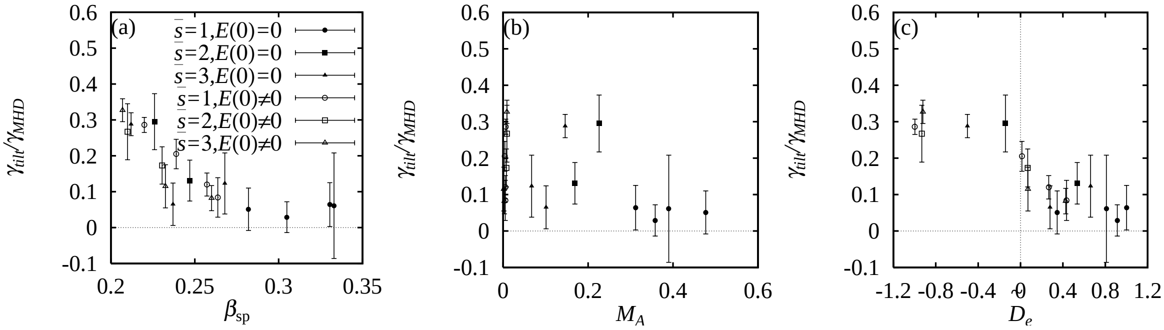

FIG. 4: *Dependence of the tilt growth rate* γ_{tilt} *on (a)plasma beta value* $\beta_{\rm sp}$ *at separatrix, (b)Alfvén Mach number* M_A*, and (c)electron hollowness parameter* $\tilde{D}_e$*.*

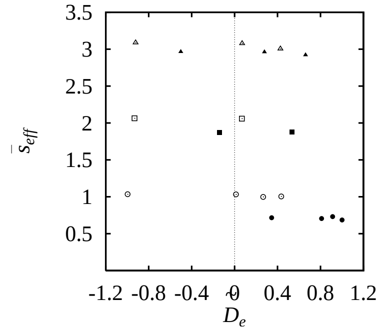

FIG. 5. *Relationship between effective kinetic parameter* $\overline{s}_{eff}$ *and electron hollowness parameter* $\tilde{D}_e$*.*

tendency that the anchoring ions may play a role to connect the unstable internal plasmas with stable external plasmas and keep the system stable against the tilt instability[3].

In the MHD simulation[7], the tilt mode is stabilized due to the spin stabilization effect when $M_A > 1$. From Fig. 4(b), on the other hand, the tilt stabilization becomes visible in the region of $M_A \approx 0.5$. Therefore this stabilization is not explained directly by the spin stabilization effect. Because the relationship between γ_{tilt} and M_A is complex, it is suggested that the ion toroidal motion partially contributes to the tilt stabilization.

It is worthly of notice from Fig. 4(c) that the tilt growth rate is remarkably reduced when the electron current profile is hollow ($\tilde{D}_e > 0$). Furthermore, it is important to point out that there is a clear correlation between the growth rate and the electron hollowness parameter, although all sorts of the simulation results obtained from various initial conditions are demonstrated in Fig. 4(c). These results indicate that the electron hollowness parameter has much to do with the tilt stabilization. This tendency coincides with the analysis of the experiments by Steinhauer and Ishida[2].

Let us consider the relationship between the reduction of the tilt growth rate and the electron hollowness parameter by introducing the effective kinetic parameter $\overline{s}_{eff}$ instead of $\overline{s}$, where the effective kinetic parameter $\overline{s}_{eff}$ is given by replacing the thermal velocity in the definition of $\overline{s}$ by the average ion velocity $\sqrt{\langle v_i^2 \rangle}$ as $\overline{s}_{eff} = \frac{m_i c}{r_{\rm sp} q_i} \int_R^{r_{\rm sp}} \frac{r dr}{\sqrt{\langle v_i^2 \rangle} B_z(r)}$. The relation between $\overline{s}_{eff}$ and the electron hollowness parameter $\tilde{D}_e$ is presented in Fig. 5. We see from this figure that $\overline{s}_{eff}$ in the initial zero electric field cases (closed symbols) is smaller than $\overline{s}_{eff}$ in the initial finite electric field cases (open symbols). This tendency is clearly seen in $\overline{s} = 1$, where the electron current profile becomes more hollow ($\tilde{D}_e > 0$)

and γ_{tilt} becomes smaller (Fig. 4(c)). From these results, it is indicated that the ions in the vicinity of the field-null line are not magnetized so much by the effect of the electric current shielding since the large electric current flows near the separatrix in the electron hollow current profile. This result suggests that the stabilization of the tilt mode is deeply related to this unmagnetization effect of ions.

The analysis of the results of 3D simulation leads us to the conclusion that the tilt mode tends to be stabilized in the cases of the hollow current profile.

5. Summary

The two-dimensional electromagnetic particle simulation is performed to investigate the profile relaxation from an MHD profile to a kinetic equilibrium and to clarify the property of the kinetic equilibrium of the field-reversed configurations independently of the tilt instability. And then we perform the three-dimensional full electromagnetic particle simulation using the kinetic profile obtained from the two-dimensional simulation as the initial condition to examine the stability of the kinetic equilibrium against the tilt mode.

The relaxation oscillation takes place when the profile relaxes from an MHD profile to a kinetic equilibrium. After this profile relaxation, the electron current profile changes to a hollow profile around the filed-null line as a result of the combined effects of the gradient-B drift near the field-null line and the $E \times B$ drift generated by the ion finite Larmor radius effect near the magnetic separatrix. On the other hand, the ion current profile becomes a peaked profile because of the effect of the ion meandering motion along the field-null line.

The growth rate of the tilt instability in all cases reduces to a small value because of the kinetic effect. In the system where the hollow current profile is realized after the profile relaxation, the growth of the tilt instability is suppressed, while in the system with peaked current profile, the tilt instability grows. From the investigation into the relationship between the tilt growth rate and several parameters, we find that the electron hollowness parameter and the separatrix beta value are important keys to solve the problem of the tilt stabilization.

Acknowledgments

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