MHD Equilibrium and Pressure Driven Instability in L=1 Heliotron Plasmas

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Abstract. Free boundary MHD equilibrium properties of Heliotron J are investigated by VMEC, HINT and PIES codes, and ideal MHD stability properties are studied by the Mercier criterion, the ballooning mode equation and the CAS3D global stability code. It is shown by the equilibrium calculations that the change of the plasma boundary shape is substantial in a low shear helical system even if the beta is relatively low. Preliminary comparison between PIES results and HINT results shows that the beta value at which the magnetic island begin to be perceptible is almost the same in both codes, but the island width seems to be different. From the stability analysis, good correlation is found between local and global analyses for the three dimensional(3D) or helical ballooning mode whose mode structure shows strong poloidal and toroidal mode (helical mode) coupling. In the helical ballooning mode, the eigenmode is localized within a flux tube. It is also found that the positive shear of the rotational transform is favorable for the 3D ballooning mode stability in a low shear helical system.

1. Introduction

Heliotron J [1,2] is an L=1/M=4 helical-axis heliotron device whose major radius is 1.2m and average plasma radius is 0.1 - 0.2m. It has been designed to be suitable for experimental optimization studies of the basic properties of helical heliotron plasmas. Especially, It is an important purpose to clarify the role of the bumpy field component (toroidal mirror ratio) on the improvement of the neoclassical transport experimentally. In the design study and theoretical analysis, its equilibrium has been obtained mainly by the VMEC code[3] with fixed boundary constraint for simplicity. However, it is suspected that the Shafranov shift of an entire plasma column is not negligible in Heliotron J even for a low beta plasma because the magnetic shear is weak and the effective toroidicity is not small. Moreover, since the plasma boundary is influenced by the neighboring rational surface in many cases of low shear stellarators, its shape often has wavy structure (see Fig.1, for example) and changes quite easily with plasma beta. To see the change of the plasma boundary due to the finite plasma pressure, free boundary calculations by the VMEC are performed. Since the position of the low order rational surface is important for the island formation in a finite beta plasma of a weak shear configuration, PIES code[4] and HINT code[5], which can calculate a three-dimensional equilibrium without assuming existence of nested flux surfaces, are applied to Heliotron J plasmas. Several problems in applying these codes to a Heliotron J plasma and solutions to resolve them are discussed and some results from the calculation are shown in the next section.

To understand the basic properties of pressure driven instabilities in Heliotron J plasmas, linear ideal MHD stabilities are studied for fixed boundary VMEC equilibria of the "standard" configuration[2] (the STD configuration). The STD configuration has deep magnetic well at every flux surface and the Mercier criterion shows that the equilibrium is stable against the interchange mode. The detailed local analysis using the ballooning mode equation shows that the ballooning mode can be unstable in the region where the field line curvature is unfavorable and the local shear is weak[6]. Since the bumpy field component has an important role on the production of the favorable curvature region on a flux surface, the ballooning mode shows non-axisymmetric feature inherently. The result from the local analysis suggests that the perturbation of the eigenmode is extended along the flux tube and the toroidal mode coupling between Fourier modes of the perturbation are substantial. The characteristics of the global ballooning mode in Heliotron J are verified by the CAS3D[7] global stability code. In the present study, the properties of the three dimensional (3D) ballooning mode[8] (helical ballooning mode) and the importance of the magnetic shear on the ballooning mode stability are discussed in the Section 3.

2. Free boundary MHD equilibrium of a Heliotron J plasma

Magnetic field and flux surfaces of the STD configuration produced by the external coil current (Fig.1) are calculated by the KMAG code[1,9] based on the Biot-Savart law. Since the rotational transform gets close to 4/7 gradually near the edge, flux surfaces have wavy shape there. In the fixed boundary calculation by the VMEC, Fourier coefficients of the plasma boundary shape with optimized poloidal angle[3] are input data. However, it is quite difficult to get them when the boundary shape is wavy. Therefore, a flux surface which has smaller cross section and less wavy shape than the outermost flux surface is chosen as a plasma boundary in the fixed boundary calculation for the STD configuration. Flux surfaces of β_{axis}=1% fixed boundary equilibrium obtained by the VMEC are also shown in Fig.1. The rotational transform $\iota/2\pi$ decreases at the plasma boundary and increases at the plasma center as beta increases in the fixed boundary calculation.

Free boundary equilibrium calculations is done by the VMEC with the constraint of fixed total toroidal flux for the STD configuration. Since the VMEC is an inverse solver based on the energy principle assuming existence of nested flux surfaces, magnetic islands and wavy flux surfaces surrounding them inside a plasma cannot be seen in the VMEC equilibrium even if they exist in the vacuum configuration. Maybe, because of the similar reason, we could not get

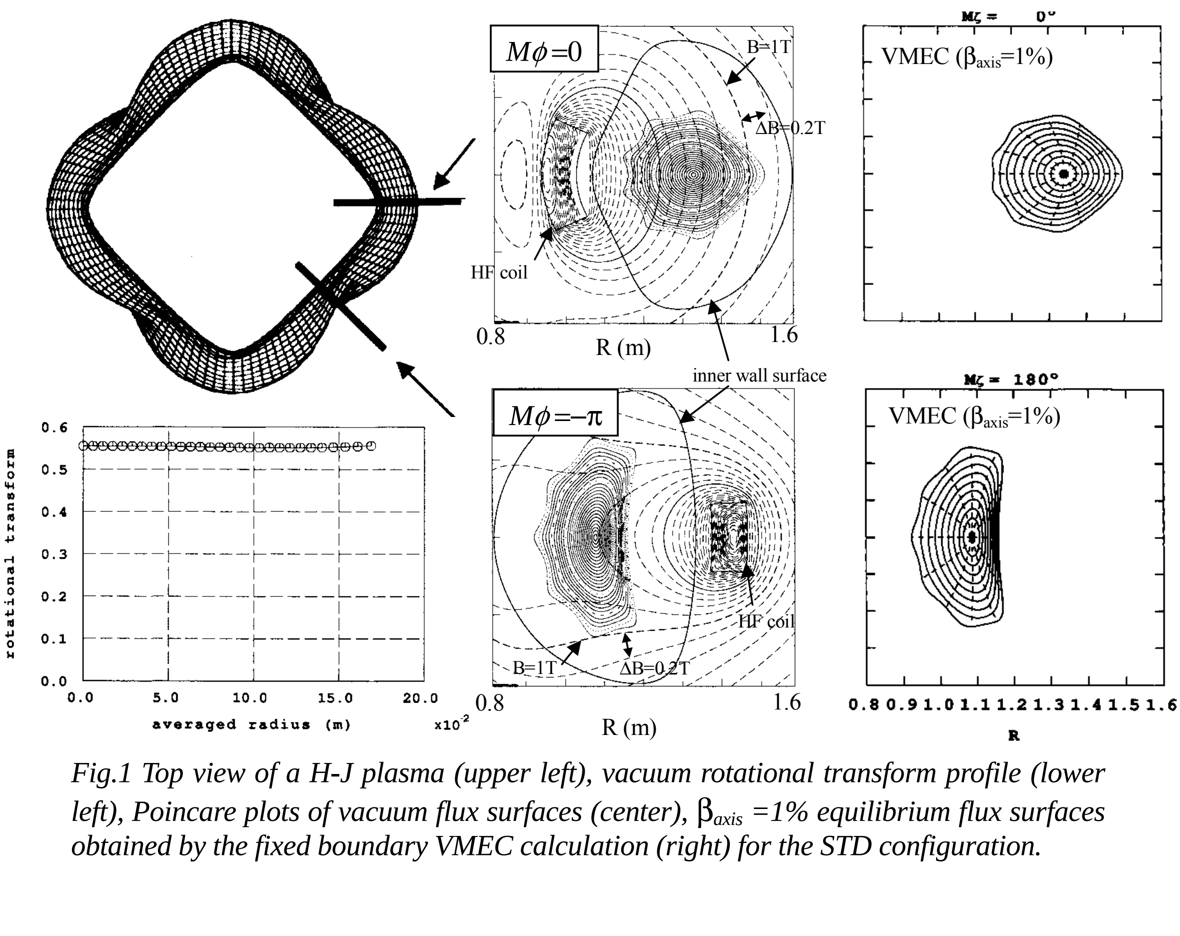

Fig.1 Top view of a H-J plasma (upper left), vacuum rotational transform profile (lower left), Poincare plots of vacuum flux surfaces (center), β_{axis} =1% equilibrium flux surfaces obtained by the fixed boundary VMEC calculation (right) for the STD configuration.

wavy plasma boundary in the VMEC free boundary calculation for the STD configuration though we use sufficient number of Fourier modes to reproduce a wavy shape. In the free boundary case, $\iota/2\pi$ increases at the plasma center but does not change at the edge as beta increases. The difference of $\iota/2\pi$ profile in the fixed and free boundary equilibria gives different position of the low order rational surface ($\iota/2\pi$=4/7) inside the plasma.

To see the effect of the boundary condition on the island formation in a finite beta plasma, PIES and HINT codes, which can calculate a 3D equilibrium without assuming existence of nested flux surfaces, are applied to Heliotron J STD configuration. The PIES code is an iterative MHD equilibrium solver[4] and can calculate free boundary equilibria. In the PIES code, (quasi) flux coordinates are used to calculate parallel current density, and "background" coordinates are used to solve Poisson equation when the magnetic field $\boldsymbol{B}$ is obtained from the current density. There are several ways to construct the background coordinates. The most convenient way when the configuration is not so simple is to use a coordinate system obtained by the VMEC equilibrium calculation. In the construction of background coordinates for the free boundary calculation, the VMEC coordinates should be extrapolated to the vacuum region. In this study, because the shape of the last closed flux surface is complicated in Heliotron J, we use zero beta equilibrium obtained by the VMEC with reduced Fourier harmonics, and expand or extrapolate it to the vacuum region in preparing background coordinates for a free boundary PIES calculation. The puncture plots of the magnetic field lines in the vacuum field and in the equilibrium with $\beta_{axis} \sim$ 1% calculated by the PIES code are shown for the STD configuration of Heliotron J in Fig.2. The dotted lines around puncture plots denote the control surface (computational boundary of background coordinates) which is the interface between the inner solution and the outer solution. It can be seen in Fig.2 that the shape of the last closed flux surface is clearly changed and the magnetic islands corresponding to $\iota/2\pi$ = 4/7 begin to be

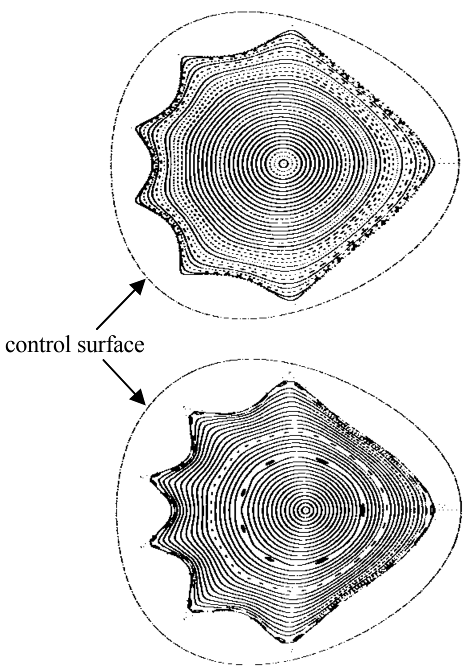

Fig.2 Puncture plots of magnetic field lines in the vacuum field (above) and in the equilibrium of β_{axis}~1% (below) calculated by the PIES code for the "standard" configuration of Heliotron J.

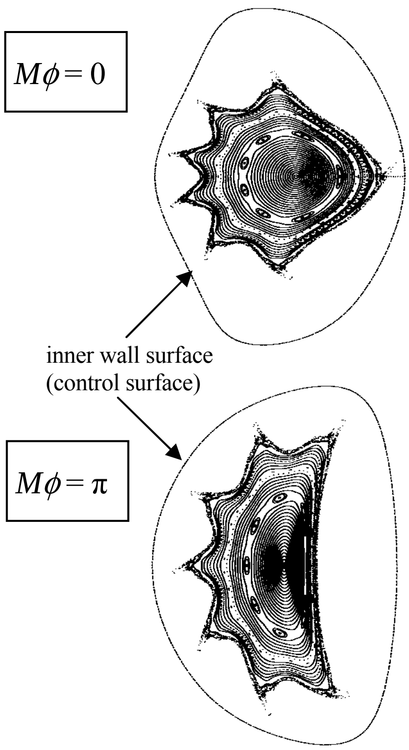

Fig.3 Puncture plots of magnetic field lines in the equilibrium of β_{axis}~1.5% calculated by the PIES code for the STD configuration of Heliotron J.

formed at $\beta_{axis} \sim 1\%$. The rotational transform profile is comparable to the free boundary VMEC results though the wavy structure cannot reproduced in the VMEC.

Though the above method to construct background coordinates is useful, extrapolation widely to the vacuum region is sometimes difficult for the bean shaped poloidal cross section and the physical plasma boundary might touch the control surface. Furthermore, touch of the plasma boundary with a material wall is not taken into account in the calculation. So, we tried to make the inner wall surface of the vacuum chamber the control surface. In this case, Fourier coefficients of the wall surface is used as a "plasma boundary" in the VMEC calculation, and the obtained "virtual equilibrium" is used for the background coordinates in the PIES calculation. Puncture plots of flux surfaces in the $\beta_{axis} \sim 1.5\%$ equilibrium of the STD configuration calculated by the PIES with this background coordinates are shown in Fig.3. Advantages of this method are; (1) touch of the plasma boundary with the wall can be considered, (2) magnetic field in the entire region inside the wall can be obtained and can be used for an analysis of field line structure in the divertor region or particle orbits to the wall, for example, (3) it is possible to do the equilibrium calculation in the case that the wall can be considered as a conducting wall. In the $\beta_{axis} \sim 1.5\%$ equilibrium, the plasma boundary shape changes further and the width of $\iota/2\pi = 4/7$ magnetic island becomes wider than in the $\beta_{axis} \sim 1\%$ equilibrium.

In the HINT code, an MHD equilibrium is calculated by the relaxation method which solves time evolution of dissipative MHD equations properly[5]. The calculation is composed of two steps. First step is a pressure relaxation step and the second is a magnetic field relaxation step. In the first step, pressure is allocated on the computational grid points by the field line tracing. In a low shear stellarator, however, the field line should be traced for a very long distance when the magnetic islands exist, because the local rotational transform of flux surfaces inside the island is very small and the field line cannot cover the flux surface unless its length is sufficiently long. So we used an improved method[10] as a pressure relaxation step to save computational time in this study. Puncture plots obtained by the HINT for the $\beta_{axis} = 1\%$ and 1.5% cases in the STD configuration are shown in Fig.4. The result obtained by the HINT also suggests that the magnetic islands corresponding to $\iota/2\pi = 4/7$ are appeared above $\beta_{axis} = 0.8 \sim 1\%$. However, island width is wider than that by the PIES. It is possible that equilibrium state and pressure distribution in the real space are different from each other because of the different numerical algorithm though the specified initial pressure profile as a function of toroidal flux is the same. Further detailed comparison between PIES results and HINT results is necessary.

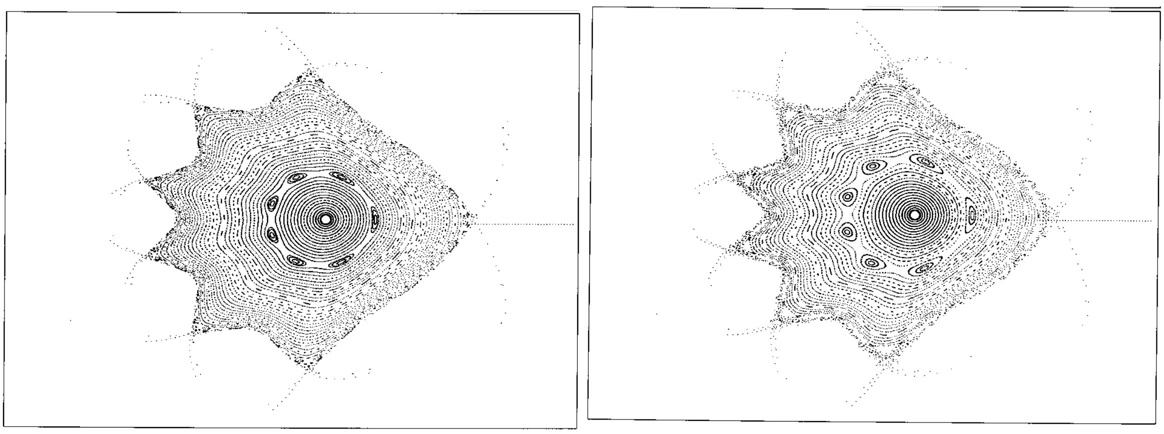

Fig.4 Puncture plots of magnetic field lines in the equilibria of $\beta_{axis} \sim 1\%$ (left) and $\beta_{axis} \sim 1.5\%$ (right) calculated by the HINT for the STD configuration of Heliotron J.

3. Pressure driven instability in a Heliotron J plasma

From the CAS3D calculation for global pressure driven instability in a Heliotron J plasma, it is found that the unstable mode structure shows strong toroidal mode coupling as is expected [11]. In other words, the mode becomes stable unless we use sufficient number of toroidal modes. This property is clearly different from the interchange mode or the tokamak-like ballooning mode and is typical in 3D ballooning mode or helical ballooning mode. If we increase the maximum toroidal mode number used in the CAS3D code, the growth rate approaches to that in the local analysis. Good agreement of the eigenmode structure along the field line in the local and the global analyses can also be seen. These results encourage us to use the local analysis for the optimization study against the ballooning instability. Control of the magnetic shear is found to be important for the helical ballooning mode stability in low shear helical systems (Fig.5). If the ι profile is specified in the equilibrium calculation, positive ι' or smaller magnetic shear with negative ι' improves helical ballooning mode stability. This tendency is similar to the second stability scenario of the ballooning mode in tokamaks.

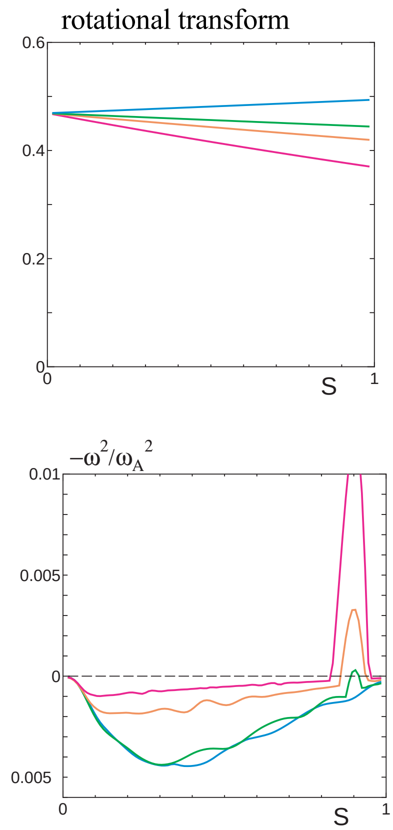

Fig.5 Specified rotational transform profiles and corresponding growth rates of helical ballooning mode.

4. Summary

In the equilibrium study, it is shown that the free boundary calculation is important especially in a low shear stellarator even if the beta is low. Several improvements in calculating free boundary equilibrium are discussed. Though the VMEC free boundary calculation for Heliotron J cannot reproduce the wavy structure of flux surfaces near the edge and island formation inside the plasma, it gives reasonable equilibrium properties like a rotational transform profile.

In the analysis of pressure driven instabilities, close relationship is found between the local and the global analysis for the 3D or helical ballooning mode. It is demonstrated that the magnetic shear has crucial role on the helical ballooning mode in a low shear stellarator.

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