Kelvin-Helmholtz Instability and Kinetic Internal Kink Modes in Tokamaks

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Abstract. The m=1 (poloidal mode number) and n=1 (toroidal mode number) kinetic internal kink (KIK) mode in the presence of a density gradient is studied with the cylindrical version of the gyro-reduced MHD code, which is one of the extended MHD codes being able to treat the physics beyond resistive MHD. Electron inertia and electron finite temperature effects are included. The unstable KIK mode is observed in the parameter range in which the linear theory predicts complete stabilization_due to the electron diamagnetic effect. The electrostatic potential profile in the linear stage of the KIK instability has the sheared poloidal flow with the m=1 mode structure. The vortexes are generated due to the Kelvin-Helmholtz (K-H) instability. The KIK is stabilized when the vortexes are formed, but it is destabilized again as the vortexes diminish due to the charge neutralizing electron motion along the magnetic field. These phenomena are observed in the early nonlinear stage of the KIK instability in which the width of the m=1 magnetic island is sufficiently small compared with the radial extent of the vortexes. The strong coupling between the vortexes and the KIK instability can be one of the candidates explaining the sudden onset of the sawtooth crash.

1. Introduction

There are a lot of mysteries in the sawtooth crash phenomena (or internal disruptions) in tokamaks. It is clear that the sawtooth crash is closely related to the nonlinear development of the $m=1$ (poloidal mode number) and $n=1$ (toroidal mode number) internal kink mode. The fast sawtooth crashes in the present-day high temperature tokamaks are now explained as the collisionless magnetic reconnection in which kinetic effects, which are outside the scope of the ideal MHD or resistive MHD theories, are essential. We can observe similar fast magnetic reconnection phenomena in the magnetosphere and the sun. We will call the internal kink mode in which kinetic effects are dominant as the kinetic internal kink (KIK) mode. Although extended MHD theory as well as the gyrokinetic theory have the possibility to elucidate the mysteries, there is no complete theory to explain the sudden onset of the instability and the slight change of the magnetic field structure accompanied by the temperature flattening in the core plasma inside the $q=1$ (q is the safety factor) magnetic surface. This paper focuses on the effects of vortexes generated by the Kelvin-Helmholtz (K-H) instability, which is a secondary instability of the nonlinear development of the unstable KIK mode with a density gradient. These vortexes can play important roles in the temporal evolution of the KIK mode because vortexes have the potential to flatten the density, temperature, and current profiles.

For the non-uniform density profile, the linear mode structure of KIK has the sheared poloidal flow with $m = 1$. Although it is difficult to excite the K-H instability in the system with magnetic shear because of the charge neutralizing electron motion along the magnetic field, the growth of the kinetic internal kink mode destabilizes the K-H mode. We have found, by the numerical study, that there is a strong coupling between the K-H and internal kink modes even in the early nonlinear phase in which the width of the $m = 1$ magnetic island is sufficiently small compared with the radial extent of the vortexes which are much smaller than the radius of the $q = 1$ magnetic surface. In the opposite extreme, in which vortexes are generated inside the magnetic island, Biskamp and Sato [1] showed numerically that the magnetic reconnection is stopped at a finite width of the magnetic island because of the K-H instability (partial reconnection).

2. Simulation Model

We used the gyro-reduced MHD code [2-5], which is one of the extended MHD codes that is able to treat some of the kinetic effects crucial to the MHD phenomena. We adopt a lowest order toroidal model in which a cylinder with minor radius a and height $L_z = 2\pi R$ (R is the major radius) surrounded by a perfectly conducting wall is assumed. The longitudinal constant magnetic field B_0 (toroidal magnetic field) is in the axial direction. The basic equations are a three-field model with the electrostatic potential ϕ, the longitudinal component of the vector potential A_z, and the electron density n_e:

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi = -\boldsymbol{b}\times\nabla\phi\cdot\nabla(\nabla_\perp^2\phi) - \boldsymbol{b}^*\cdot\nabla(\nabla_\perp^2 A_z) + D\nabla_\perp^2(\nabla_\perp^2\phi)\,, \tag{1}$$

$$\frac{\partial}{\partial t}A_z = -\boldsymbol{b}^*\cdot\nabla\phi + {d_e}^2\frac{d}{dt}(\nabla_\perp^2 A_z) + {\rho_s}^2\,\boldsymbol{b}^*\cdot\nabla n_e - \mu\nabla_\perp^2(\nabla_\perp^2 A_z)\,, \tag{2}$$

$$\frac{\partial}{\partial t}n_e = -\boldsymbol{b}\times\nabla\phi\cdot\nabla n_e - \boldsymbol{b}^*\cdot\nabla(\nabla_\perp^2 A_z) + D\nabla_\perp^2 n_e\,, \tag{3}$$

where $\boldsymbol{b}$ is a unit vector in z, $\boldsymbol{b}^* = \boldsymbol{b} + \nabla A_z \times \boldsymbol{b}$, and $d/dt = \partial/\partial t + \boldsymbol{b}\times\nabla\phi\cdot\nabla$. The above equations are expressed in terms of normalized quantities:

$$z/L_z \to z\,,\quad r/a \to r\,,\quad tv_A/L_z \to t\,,\quad A_z L_z/(a^2 B_0) \to A_z\,,$$
$$\phi\, L_z/(v_A a^2 B_0) \to \phi\,,\quad n_e L_z/(n_{e0} d_i) \to n_e\,,$$

where v_A is the Alfven velocity, d_i is the collisionless ion skin depth, n_{e0} is the average electron density. While the left-hand-sides of the arrows are physical quantities, the right-hand-sides represent normalized values. Note that the normalized equations include only two parameters, namely $d_e/a \to d_e$ (normalized collisionless electron skin-depth) and $\rho_s/a \to \rho_s$ (normalized ion Larmor radius estimated by the electron temperature).

Equation (1) is the vortex equation. Equation (2) is the generalized Ohm s low along the magnetic field, which is equivalent to the equation of motion along the magnetic field for electrons. The inertia term appears in the second term on the right-hand-side of equation (2). The third term represents the electron pressure gradient (assuming a constant electron temperature) along the magnetic field. The fourth term is included to avoid the localization of the negative current layer beyond the resolution of the code. Equation (3) is the conservation equation for electrons. We also included the diffusion terms in the last terms of equations (1) and (3). We selected the following equilibrium safety factor and density profiles,

$$q(r) = q_0 \left[1 - 4(1-q_0) r^2 \right]^{-1} , \tag{4}$$

$$n(r)/n_{e0} = 1 - \varepsilon_n \tanh\left[(r - r_0)/l_n \right] . \tag{5}$$

The gyro-reduced MHD equations are solved using pseudo-spectral methods with high spatial and temporal resolution.

3. Simulation Results

Before presenting the simulation results, in order to elucidate the selected parameter set, we summarize the linear stability. Theory predicts that the KIK mode is completely stabilized for $\omega_{*e} > 2\gamma_0$, where ω_{*e} is the electron diamagnetic angular frequency and γ_0 is the growth rate for uniform density. The simulation results in our previous paper [4], however, confirmed that the residual instability appears for $\omega_{*e} > 2\gamma_0$ since the region with the density gradient is limited in the radial direction and the stabilization by the outgoing drift-wave like mode becomes incomplete. When the ion diamagnetic effect is included, we have obtained similar results [5].

A nonlinear simulation with the assumption of a single helicity is performed for $\omega_{*e}/(2\gamma_0) = 1.10$. A deuterium plasma is assumed. Other parameters are $d_e = 0.01$, $\rho_s = 0.03$, $q_0 = 0.85$, $r_0 = 0.5$, $\varepsilon_n = 0.5$, and $D = 0$. Note that the rational surface with $q = 1$ is at $r = 0.5$. A very small hyper resistivity of $\mu = 10^{-10}$ is added in the nonlinear phase. (A larger value of μ tends to suppress the vortex formation.) The linear growth rate for these parameters is 0.024, which is smaller than the value of 0.079 for uniform density; the electron diamagnetic effect is stabilizing but the stabilization is not complete. The time evolution of the electrostatic potential is shown in figure 1. The graphs on the left show snap shots of the profiles in a poloidal cross section for different times. The number in the lower left corner corresponds to the normalized time after the hyper resistivity is included. The graphs on the right are the squared potentials integrated over the poloidal cross-section for the modes ($m \geq 2$) normalized by the respective $m = 1$ component.

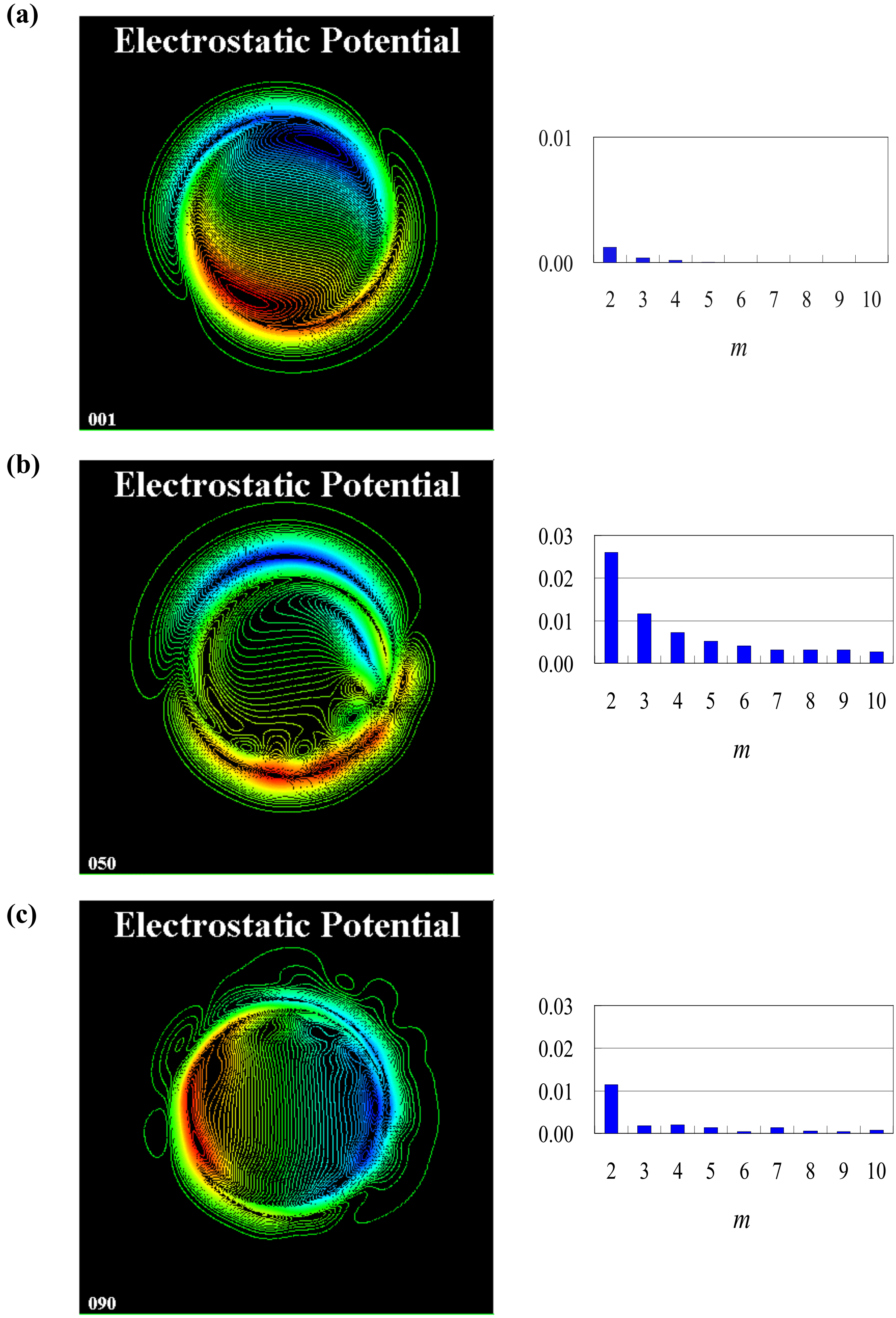

FIG. 1. Time evolution of electrostatic potential.

The mode pattern rotates in the electron diamagnetic (counter-clockwise) direction. Figure 1(a) shows the mode pattern very close to the linear stage. The $m = 1$ component is dominant and high m modes are negligible. We can see the strong $m = 1$ sheared poloidal flow near the rational surface. When the vortexes are generated due to the K-H instability as shown in figure 1(b), the internal kink mode is stabilized. The vortexes with $5 \le m \le 10$ are clearly seen. Damping due to the electron motion along the magnetic field is of great importance for the vortexes. In figure 1(c), the vortexes in the region far from the rational surface have disappeared and the internal kink mode is unstable again in this stage of the nonlinear development. The vortexes close to the rational surface are still surviving.

3. Conclusions and Discussion

The nonlinear development of the KIK mode in the early nonlinear stage of the instability is studied by using the cylindrical version of the gyro-reduced-MHD code, which includes an electron inertia as well as the finite temperature effects of electrons (pressure balance along the magnetic field and the electron diamagnetic effect). The simulation is performed in the parameter range in which theory predicts complete stability, although the KIK mode is still unstable in accordance with our previous study [4]. The sheared poloidal flow pattern with $m = 1$ intrinsic to the KIK mode with a density gradient, excites vortexes due to the K-H instability. Hence, the KIK mode is stabilized in this stage. Once the KIK is stabilized, the vortexes starts damping due to the charge neutralizing electron motion along the magnetic field. The KIK mode is destabilized again. The strong coupling between these two modes may explain the sudden onset of the KIK instability. One limit of this study is that the simulation is executed for large values of d_e and ρ_s which are one order of magnitude larger than those in the present-day high temperature large tokamaks. So we are planning to reduce these values to more realistic ones. The other limit is the assumption of a single helicity. If toroidal effects are included, there will be mode coupling between the vortexes with different helicities. In the toroidal model, the radial propagation of vortexes may also occur. A study in this direction is ongoing by adding kinetic terms to the FAR code. The results will be reported in a future article.

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