Two-Fluid And Nonlinear Effects Of Tearing And Pressure-Driven Resistive Modes In Reversed Field Pinches

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Abstract. We report results of five investigations covering two-fluid dynamos, toroidal nonlinear MHD computation, nonlinear computation of Oscillating Field Current Drive (OFCD), the effect of shear flow on tearing instability, and the effect of pressure on resistive instability. The key findings are (1) two-fluid dynamo arising from the Hall term is much larger than the standard MHD dynamo present in a single-fluid treatment, (2) geometric coupling from toroidicity precludes the occurrence of nested helical flux surfaces, except for nonreversed plasmas, (3) OFCD, a form of AC helicity injection, can sustain the RFP plasma current, although magnetic fluctuations are enhanced, (4) edge shear flow can destabilize the edge resonant m = 0 modes, which occur as spikes in experiment, and (5) pressure driven modes are resistive at low beta, only becoming ideal at extremely high beta.

1. Introduction

Large-scale tearing instabilities have long been considered to underlie transport and dynamo processes in the reversed field pinch (RFP). The vast majority of theoretical and computational RFP work has focused on pressureless, single-fluid MHD in cylindrical plasmas driven solely by a toroidal electric field. During the last two years significant progress has been achieved in analytical and numerical treatments of new approaches which are of practical importance for the RFP. Specifically, we report here results of five investigations covering two-fluid dynamos, toroidal nonlinear MHD computation, nonlinear computation of Oscillating Field Current Drive (OFCD), the effect of shear flow on tearing instability, and the effect of pressure on resistive instability. Each result is described below.

2. Two-Fluid Dynamo

Due to relatively low magnetic field and high plasma temperature, two fluid effects are important for the dynamics of tearing instabilities in the RFP. Although linear two-fluid MHD eigenfunctions are long known [1-3], quasilinear two-fluid theory of Hall and alpha dynamos has not yet been developed. These effects are important for high temperature RFPs where the nonlinear dynamo action flattens the equilibrium current profile toward the Taylor state of the minimum energy. The tearing mode dynamo effect driven by the $\mathbf{v} \times \mathbf{B}$ term in Ohm's law (sometimes known as alpha dynamo) has been investigated through the single fluid MHD theory [4], computation [5] and experiment [6]. Here we focus our attention on the case when the ion gyroradius is significantly larger than the electron skin depth so that electrons are decoupled from ions inside the linear tearing layer. This speeds up the instability and changes the spatial profiles of eigenfunctions in comparison the with single fluid MHD case. Similarly to quasilinear treatment of the fluid Reynolds stress [7], a two-fluid MHD theory is used to calculate quasilinearly **two-fluid dynamo** effects from tearing instability in a force-free (constant pressure) slab equilibrium relevant to the RFP. Fluctuations contribute to the parallel component of the mean field generalized Ohm's law as $\mathcal{E}_{\|} = <\mathbf{E}>_{\|} - \eta <\mathbf{j}>_{\|} = -1/c <\mathbf{v}^{(1)} \times \mathbf{B}^{(1)}>_{\|} + (1/en^{(0)}c) <\mathbf{j}^{(1)} \times \mathbf{B}^{(1)}>_{\|}$ where <…> denotes flux surface average, the parallel components are defined with respect to $\mathbf{B}^{(0)}$, and the superscripts $^{(0),\ (1)}$ indicate mean quantities and linear fluctuations, respectively. Two-fluid effects influence dynamos in two ways: by altering the MHD $< \mathbf{v}^{(1)} \times \mathbf{B}^{(1)} >_{\|}$ dynamo and by introducing a Hall $< \mathbf{j}^{(1)} \times \mathbf{B}^{(1)} >_{\|}$ dynamo that arises from perpendicular current density fluctuations consistent with the magnetic perturbations $\mathbf{B}_{||}^{(1)}$. The two-fluid tearing mode eigenfunctions are evaluated using the generalized Ohm's law in a compressible plasma. Instability dynamics relate to the kinetic Alfven wave in the range of parameters where this mode is decoupled from the compressional Alfven and slow magneto-acoustic modes. The growth rate is enhanced over the MHD growth rate, scaling as $g \propto d^{1/3} r_s^{2/3}$ at large values of the stability factor D¢ , $d^{2/3} r_s^{1/3}$ D¢ >>1 and as g µ d r_S D¢ in the opposite limiting case, where d is a combination of collisionless and collisional electron skin depth, $d^2 = c^2 / w_{pe}^2 + hc^2 /4pg$, and $r_s = c_S /w_{ci}$ is the ion-sound gyroradius, $c_S^2 = (g_e T_e + g_i T_i) / m_i$, $g_{e,i} = 5/3$.

We examine the dynamo terms for two cases: the experimental case for which $r_s >> d$ and, for comparison, the single-fluid MHD case for which $r_s << d$. For $r_s >> d$ the Hall dynamo effect is larger than the single-fluid MHD dynamo by a factor $(r_S /d)^2$ (for large d D¢ >>1), which is typically more than an order of magnitude in RFP plasmas. The Hall dynamo is localized to within a short distance $d^{4/3}/r_s^{1/3}$ from the resonant surface. In two-fluid theory, the <**v** × **B**> term is also enhanced by a smaller factor $(r_S /d)^{2/3}$ and is broadened to a spatial scale of order r_s. At smaller D¢ (d D¢ <<1), the ratio of Hall dynamo to the single-fluid MHD dynamo decreases with D¢ proportional to (D¢r_S)^2 for (d /r_S)$^{1/3}$<< d D¢ <<1 and then increases as r_s /D¢ d^2 for d D¢ << (d /r_S)$^{1/3}$ that corresponds to the MST RFP case (D¢~ 5) and provides an order of magnitude enhancement factor for Hall dynamo. The enhancement factor for two fluid <**v** × **B**> dynamo monotonically decreases with D¢ for all d D¢ <<1 passing through unity at d D¢~(d /r_S)$^{1/3}$. These results motivate experimental studies of two-fluid effects in the RFP.

2. Toroidal Geometry Effects

It has been established that solutions to the time-independent resistive MHD equations in a periodic cylinder include helical Ohmic equilibria with average axial field reversal [8]. Further, Cappello and Escande have found a transition between these steady single-helicity laminar states and unsteady multi-helicity states at Hartmann number H~2500 for aspect ratio R/a=4 and pinch parameter Θ=1.9 [9]. In toroidal geometry, pure single-helicity solutions do not exist, due to the geometric coupling among different poloidal Fourier components (*m*-numbers) for any toroidal Fourier component (*n*). Nonetheless, laminar states with a significant volume of nested helical flux surfaces may exist. Here we report on a numerical study with the NIMROD code [10] that investigates laminar pinch states in toroidal geometry.

To explore conditions where toroidal geometry effects may be important, we have run a number of pinch simulations in both toroidal and periodic cylindrical geometry at S=2000 while varying viscosity (Pm=1, 10, and 100), aspect ratio (1.1≤R/a≤5), and pinch parameter (1.4≤Θ≤2). At Pm=1 (H=2000) and Θ≥1.6, we find that the usual nonlinear multi-helicity coupling among resonant fluctuations dominates geometric coupling, and there is little to distinguish the toroidal results from cylindrical results. At Pm=10 in cylindrical geometry, single-helicity states with nested helical flux surfaces result, but the same conditions in toroidal geometry produce magnetic stochasticity over most of the domain (see Fig. 1a). The mean magnetic field in these cases exhibits reversal, so *m*=0 fluctuations are resonant. An *m*=0 island chain is evident, and the *n*-number of the chain matches that of the dominant, helical *m*=1 perturbation, so we infer that the excitation is geometric, rather than nonlinear. If Pm is increased to 100 or Θ is decreased to 1.4, the resulting mean field loses reversal, hence *m*=0 is not resonant. With *m*=0 islands precluded, nested helical flux surfaces form (Fig. 1b).

The results are then qualitatively similar to the corresponding cylindrical results. The importance of the *m*=0 resonance for the formation of nested helical flux surfaces in toroidal geometry has been confirmed over aspect ratios 1.25≤R/a≤5.

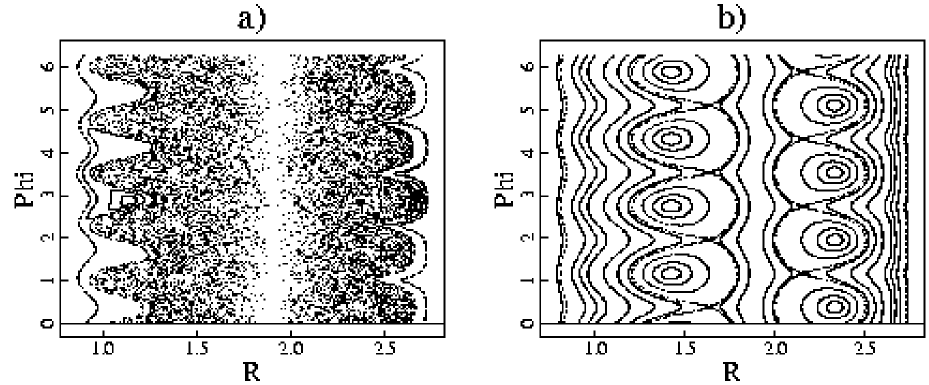

FIG.1. Poincaré surfaces of section for magnetic field resulting from NIMROD toroidal pinch simulations with R/a=1.75, Pm=10, and a) Q=1.8 and b) Q=1.4.

3. MHD Dynamics of Oscillating Field Current Drive (OFCD)

Oscillating Field Current Drive (OFCD), also called F-Θ pumping, was first proposed in [11] as a technique to drive steady-state current in the RFP. In OFCD helicity is injected by oscillating the toroidal and poloidal surface voltages 90^o out of phase. We investigate the full nonlinear dynamics of OFCD using 3-D resistive MHD computation in a cylinder, (the DEBS code) at Lundquist numbers up to $S = 5 \times 10^5$, and aspect ratio R/a = 1.6. The penetration of driven axisymmetric oscillating fields, the response of the helical tearing instabilities, and the driven current are examined. We also evaluate the 1D response of the plasma to compare the actual 3D situation with 1D case in which the tearing modes are absent. In the absence of tearing fluctuations, oscillating fields generate a steady-state edge current driven by the dynamo effect ($\langle \mathbf{v}_{00} \times \mathbf{B}_{00} \rangle$) from the axisymmetric velocity and magnetic field oscillations.

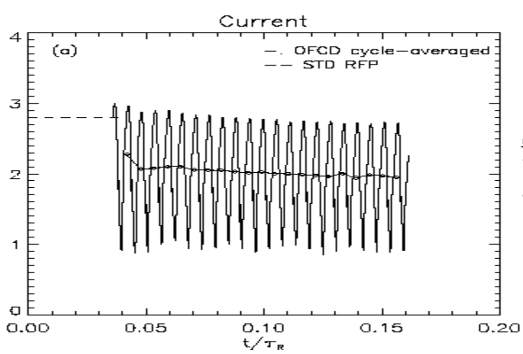

FIG.2 Total axial current vs. time. The ac fields $E_z = e_z$ sin(w t), $E_q = e_q$sin(w t + p/2) are applied at $t=0.035t_R$ (S = 5 ´ 10^5)

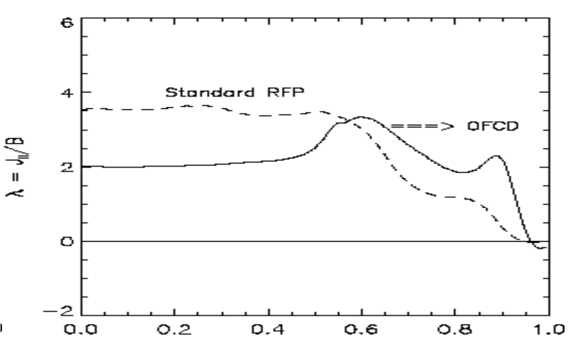

FIG.3 Time averaged current density profiles for standard RFP and OFCD plasma

The 3D computation shows that the tearing fluctuations transport the edge current towards the center (by the tearing mode dynamo), resulting in a steady-state current over the entire cross-section. Fig.2 illustrates the total plasma current sustained by OFCD in the absence of a dc electric field . Although OFCD is able to sustain the plasma current, the current oscillations are large (at $S=10^5$ the current oscillates by 100%). The oscillating amplitudes required to drive a given amount of total current decrease at high S. We find that the current oscillations decrease to about 50% at $S = 5 \times 10^5$.

The radial profile of time averaged current in an OFCD plasma is shown in Fig.3. The mechanism of current penetration into the core is investigated by analyzing the dynamo terms. The dynamo from the axisymmetric oscillations drives a time averaged edge current (Fig.4 (a)) while the core current is sustained by the dynamo from the helical tearing fluctuations (Fig.4 (b)). As the reversal parameter F deepens through a cycle, edge resonant modes (modes resonant outside the reversal surface) are excited, resulting in large fluctuation amplitudes. It is expected that this global (nearly ideal) edge mode may be suppressed in high S plasmas where the reversal is weak. OFCD is presently being tested in MST experiments.

4. Effect Of Toroidal Flows On Tearing Mode Stability

Linearized MHD equations in cylindrical geometry are solved numerically to determine the stability factor Δ' for toroidal flows with shear localized away from the rational surface in the external region [12]. Both *m*=1 and *m*=0 modes are destabilized by edge-localized flows. For flows whose shear is consistent with the edge shear layer of enhanced confinement in MST, the growth rate of *m*=1 modes is increased by a small factor, while *m*=0 modes change from damped to growing. The destabilization of *m*=0 modes by edge localized flow shear may thus account for the *m*=0 bursts observed in the experiments. The steady state rotation profiles with shear in the region of modes with *n*>6, but outside the *n*=6 surface is shown to destabilize the *n*=6 mode and stabilize modes with n>6. This may account for an observed propensity toward the formation of quasihelicity *n*=6 states during pulsed poloidal current drive.

5. Resistive-Ideal Transition of Pressure-Driven Instabilities

In experimental RFP plasmas with improved confinement beta is increasing to the point that pressure-driven instabilities can begin to be significant. We examine the linear MHD stability of local and global resistive pressure-driven instabilities computationally in a cylinder [13]. A localized pressure-driven instability in a bad curvature region is excited if the stability parameter $D_S > 0.25$ [14]. The analytical calculation [15,16] shows that the growth rate depends on D_S (which is proportional to beta). It is exponentially small near the ideal limit ($D_S = 0.25$), becoming large for D_S values well above this limit. Here we employ initial value computation (DEBS code in the linear regime) to evaluate the growth rate and radial structure, for arbitrary wave number. To isolate the pressure driven modes, an equilibrium which is stable to resistive current driven modes is chosen (by the Δ' criterion).

The dependence of growth rate on D_S for the m=1, k=10.5 mode at $S=10^6$ is shown in Fig. 5. We find that the transition from resistive to ideal interchange modes occurs at high $D_S \sim 1.0$. For a rather wide range of beta, from zero to several times the Suydam limit, the high-k interchange mode is resistive. It is resistive in its radial structure i. e. the radial field is non-zero at the resonant surface, and its growth rate, which is small and scales as $S^{1/3}$ at low D_S, whereas at very high D_S, γ is roughly independent of S (ideal scaling).

We have examined the growth rate and radial structure of global modes, low-k pressure-driven modes, and find that they also display a transition from resistive to ideal instability as beta increases. We observe that the growth rate for the global modes is about equal to that of the localized interchange. Since the localized modes are more subject to stabilization by finite Larmor radius, the global modes will likely be more influential at high beta.

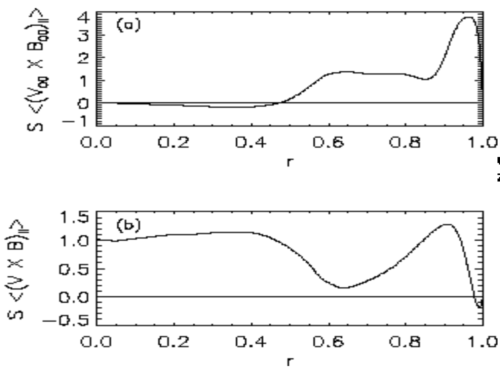

FIG.4 Cycle- averaged dynamo terms from: (a) symmetric oscillations, (b) helical tearing fluctuations

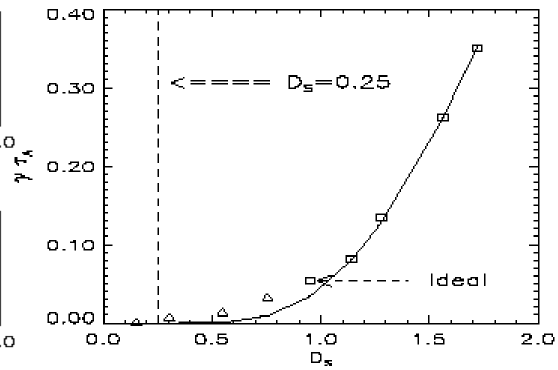

FIG.5. The growth rate vs. D_S. Computational results for resistive modes (D), pure ideal modes (□), analytical result for idea linterchange modes (solid line), the Suydam limit (dashed line)

6. Summary

The key results obtained for two-fluid, toroidal and nonlinear effects in RFP: (1) quasilinear theory predicts that the two-fluid dynamo arising from the Hall term is much larger than the standard MHD dynamo present in a single-fluid treatment, (2) geometric coupling from toroidicity precludes the occurrence of nested helical flux surfaces, except for nonreversed plasmas, (3) OFCD, a form of AC helicity injection, can sustain the RFP plasma current, although magnetic fluctuations are enhanced, (4) edge shear flow can destabilize the edge resonant m = 0 modes, which occur as spikes in experiment, and (5) pressure driven modes are resistive at low beta, only becoming ideal at beta values several times the Suydam limit.

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