Dissipative Drift Ballooning Instabilities in Tokamak Plasmas

R.J. Hastie 1), J.J. Ramos 1) and F. Porcelli 2)

1) Plasma Science and Fusion Center, Massachusetts Institute of Technology, U.S.A.

2) Instituto Nazionale Fisica della Materia, Politecnico di Torino, Italy

e-mail contact of main author: jim.hastie@ukaea.org.uk

Abstract. The linear stability of high-toroidal-number drift-ballooning modes in tokamaks is investigated with a model that includes resistive and viscous dissipation, and assumes the mode frequency to be comparable to both the sound and diamagnetic frequencies. The coupled effect of ion drift waves and electron drift-acoustic waves is shown to be important, resulting in destabilization over an intermediate range of toroidal mode numbers. The plasma parameters where the assumed orderings hold would be applicable to the edge conditions in present day tokamaks, so these instabilities might be related to the observed quasi-coherent edge-localized fluctuations.

Experimental observations in the Alcator C-Mod tokamak indicate a well defined transition that separates ELM-free H-mode behavior, in which the edge plasma is fluctuation free, from Enhanced-D_α (EDA) behavior, in which a localized instability (the quasi-coherent mode) is present in the edge pedestal [1]. It is possible that these observations could be explained by a linear stability threshold. Even though ideal-MHD ballooning modes might be near their stability threshold, electron and ion diamagnetic corrections are likely to be important, as are interactions with acoustic waves. Dissipative effects (especially resistive effects) are also likely to be important. In this paper we investigate the linear stability of drift ballooning modes using a simple model for the plasma equilibrium (the $s-\alpha$ model [2]), but including a variety of non-ideal physical effects. We derive a simple eigenmode equation for drift ballooning modes by considering an optimal ordering in which $\omega \sim \omega_s \sim \omega_{*j} \sim \mu k_\perp^2 \sim \eta_\| k_\perp^2$, where ω is the mode frequency, $\omega_s \equiv C_s/Rq$ is the frequency for sound propagation over a connection length with $C_s^2 = (T_i + T_e)/m_i$ and R the major toroidal radius, $\omega_{*j} = (\mathbf{k} \times \mathbf{b}) \cdot \nabla p_j/(NeB)$ is the diamagnetic frequency for species j, $\mu = 0.3\nu_{ii}\rho_i^2$ is the classical perpendicular viscosity with $\rho_i = (T_i/m_i\omega_{ci}^2)^{1/2}$ the ion Larmor radius, and $\eta_\parallel$ is the longitudinal Spitzer resistivity. The $s-\alpha$ equilibrium model with $s = rq'/q$ and $\alpha = -2Rp'q^2/B^2 \sim O(1)$, is employed and a two length scale averaging formalism is developed, using $\omega/\omega_A \ll 1$ as the expansion parameter, with $\omega_A = V_A/Rq,\ V_A^2 = B^2/(Nm_i)$. The resulting eigenmode equation contains the following physical effects: (1) ideal-MHD instability drive (through the boundary condition), (2) resistive diffusion leading to resistive ballooning modes, (3) sound wave propagation, (4) two-fluid diamagnetic effects which modify sound waves into electron drift-acoustic modes and introduce finite ion Larmor radius effects, and (5) perpendicular ion viscosity. The two different classes of resistive ballooning modes, viz those driven locally (in radius) by the geodesic curvature (the Carreras-Diamond modes [3]) and those driven by the ideal-MHD energy (characterized by the stability index Δ_B' [4,5]) are both described by the eigenmode equation.

A simple set of reduced two-fluid equations has been given in Refs.[6,7]. We further simplify these equations by assuming that the equilibrium ion and electron temperatures are equal and constant, and ions and electrons are isothermal. We also specialize them to the $s-\alpha$ equilibrium model. Finally we adopt our "drift ordering", that assumes all the frequencies, $\omega,\ \omega_s,\ \omega_{*j},\ \mu k_\perp^2,\ \eta_\parallel k_\perp^2$ to be comparable and small compared to the Alfven frequency ω_A. It follows that the resistivity, the viscosity and the inertia only become important at large values of the extended ballooning variable θ. A two length scale analysis is performed by introducing the stretched variables $Z = \epsilon_\eta s\theta$ where $\epsilon_\eta = [\eta_\parallel n^2 q^2/(r^2\omega_A)]^{1/3}$, and $X^2 = i\epsilon_\eta\omega_A Z^2/(\omega-\omega_{*e})$ in the secular terms of the ballooning equations. The result is an averaged ballooning equation of the form:

$$s^2\frac{d}{dX}\left(\frac{X^2}{1+X^2}\frac{dU_0}{dX}\right) - \left[X^2Q + X^4T\right]U_0 = 0, \qquad (1)$$

with

$$Q(X) \;=\; i\hat{\omega}(\hat{\omega}-\hat{\omega}_{*i})(\hat{\omega}-\hat{\omega}_{*e})\left[1+2q^2P(X)\right] - \frac{\alpha^2}{2}\left[1-P(X)\right], \qquad (2)$$

$$P(X) \;=\; \frac{\omega_s^2}{\omega_s^2 - [\omega + 4\hat{\mu}(\omega-\omega_{*e})X^2](\omega-\omega_{*e})}, \qquad (3)$$

and

$$T(X) \;=\; i\hat{\mu}(\hat{\omega}-\hat{\omega}_{*e})^2(\hat{\omega}-\hat{\omega}_{*i})[1+8q^2P(X)]. \qquad (4)$$

In these expressions $\hat{\mu}$ is a normalized perpendicular viscosity and the notation $\hat{\omega}_j$ denotes the dimensionless frequency $\hat{\omega}_j = \omega_j/(\epsilon_\eta \omega_A)$. Equations (1-4) describe the coupling of visco-resistive ballooning modes to drift-acoustic waves. The novel feature of the foregoing two-scale analysis, lies in the ordering $\omega/\omega_s \sim 1$. In much previous analyses either the ordering $\omega/\omega_s \ll 1$ was assumed with the result that the geodesic curvature (Carreras-Diamond) drive was lost, or the limit $\omega/\omega_s \gg 1$ was taken so that sound wave propagation was neglected. Our equations provide a bridge between these two extremes, not only for studying visco-resistive ballooning modes, but also for investigating finite Larmor radius effects on ideal modes. The relevant dispersion relations are obtained by asymptotically matching the solution of Eqs.(1-4) to the solution in the ideal-MHD region:

$$U_0(Z \to 0) = 1 + \frac{\epsilon_\eta \Delta'_B}{Z}, \tag{5}$$

where the quantity $-1/\Delta'_B$ is a measure of the ideal-MHD energy, δW, available to drive the $n \to \infty$ ballooning mode.

In the absence of resistive and viscous dissipation we obtain the following dispersion relation:

$$\omega(\omega - \omega_{*i}) \left[(1 + 2q^2)\omega_s^2 - \omega(\omega - \omega_{*e})\right] + \left(\frac{s\omega_A}{\Delta'_B}\right)^2 \left[\omega_s^2 - \omega(\omega - \omega_{*e})\right] = 0 \tag{6}$$

Ignoring the coupling to drift-acoustic waves (i.e in the $\omega_s \to \infty$ limit) we recover the standard finite Larmor radius dispersion relation which implies that all sufficiently short wavelength modes with $n > n_{critical}$ are stabilized. Taking into account the coupling to drift-acoustic waves modifies the standard result in such a way that, assuming ideal instability, both a high-n and a low-n range of modes are stabilized but an intermediate range of toroidal mode numbers is always unstable. Specifically, the finite Larmor radius stabilization of ideal modes is lost when the ion drift wave is resonant with one of the electron drift acoustic wave branches: $\omega_{*i}(\omega_{*i} - \omega_{*e}) = (1 + 2q^2)\omega_s^2$. At this point the dispersion relation reduces to:

$$\omega = \omega_{*i} \pm \frac{2iqs\omega_A}{\Delta'_B[3(1 + 2q^2)]^{1/2}} + O\Big(\frac{\omega_A^2}{\Delta'^2_B \omega_{*i}}\Big), \tag{7}$$

so that the growth rate of the ideal instability is actually enhanced by a factor $(2/3^{1/2})q$ above the value it would have in the absence of diamagnetic effects.

With finite resistivity but still zero viscosity, we obtain the following dispersion for drift-resistive ballooning modes:

$$\epsilon_\eta \Delta'_B \;=\; -\left(\frac{\omega-\omega_{*e}}{i\epsilon_\eta\omega_A}\right)^{1/2} \frac{Q_0^{1/4}}{8s^{1/2}} \frac{\Gamma[(Q_0^{1/2}/s-1)/4]}{\Gamma[(Q_0^{1/2}/s+5)/4]}, \tag{8}$$

where $Q_0 = Q(\hat{\mu}=0)$. These modes can be driven unstable by a positive Δ'_B or, for even negligible $\epsilon_\eta \Delta'_B$ but finite pressure gradient, by the α term in Q_0. For negligible $\epsilon_\eta\Delta'_B$, their dispersion relation is given by $Q_0 = 0$ or

$$(\omega-\omega_{*i})[(1+2q^2)\omega_s^2 - \omega(\omega-\omega_{*e}] \;-\; \frac{i}{2}\alpha^2\omega_A^3\epsilon_\eta^3 \;=\; 0. \tag{9}$$

For vanishing α this predicts three waves: an ion drift wave and a pair of toroidally modified electron drift-acoustic waves. In the $\omega_s \to 0$ limit, the pressure gradient α drives instability of the low frequency branch of the electron drift-acoustic wave [8]. However, our Eq.(9) shows that, allowing for finite ω_s, their growth rate is strongly enhanced when the low frequency branch of the electron drift-acoustic wave is degenerate with the ion drift wave. This is the same mechanism responsible for the loss of finite Larmor radius stabilization of ideal modes.

Perpendicular viscous effects enter Eqs.(1-4) in two ways. First, the term T contains the viscous drag on the perpendicular velocity in agreement with Refs.[9,10]. Second, the factor P contains the drag on the parallel flow and introduces, as noted in [11], a strong toroidal enhancement of the viscosity in the limit $\omega_s \to \infty$. Our expression for $\hat{\mu}$ assumes its classical value and is therefore much less than unity. So we can expand $P(\hat{\mu}) = P_0 + \hat{\mu}P_1$ and approximate Eq.(1) by

$$s^2\frac{d}{dX}\left(\frac{X^2}{1+X^2}\frac{dU_0}{dX}\right) - \left[X^2 Q_0 + X^4 T_0\right] U_0 = 0 \tag{10}$$

with $Q_0 = Q(P_0)$ and $T_0 = T(P_0) + 2\hat{\mu}P_0^2[\alpha^2 + 4iq^2\hat{\omega}(\hat{\omega}-\hat{\omega}_{*i})(\hat{\omega}-\hat{\omega}_{*e})][(\omega-\omega_{*e})/\omega_s]^2$. Now we can obtain a variational expression for the dispersion relation using the trial function

$$U_0(X) = X^{1/2}K_{1/4}(\sigma X^2)\left\{1 - \left[\left(\frac{i\epsilon_\eta\omega_A}{\omega-\omega_{*e}}\right)^{1/2}\epsilon_\eta\Delta'_B - (\sigma/2)^{1/2}\frac{\Gamma(3/4)}{\Gamma(5/4)}\right]X\right\}, \tag{11}$$

where $K_{1/4}$ is the modified Bessel function and σ is a variational parameter. This choice of trial function incorporates the correct boundary condition matching the ideal-MHD solution as $X \to 0$, and reduces to the exact eigenfunction in the inviscid limit $T_0 = 0$. For negligible Δ'_B drive, the resulting variational dispersion relation is

$$Q_0 + 2.46(sT_0)^{2/3} = 0. \quad (12)$$

We have obtained numerical solutions of this variational dispersion relation. They show that viscosity does not supress the intermediate-n instability associated with the coupling of ion drift and electron drift-acoustic waves.

Acknowledgements

We thank Peter Catto for very helpful discussions and encouragement. This work was supported by the U.S. D.O.E. grant No. DE-FG02-91ER-54109 at M.I.T.

References

[1] D.A. Mossessian et al., Plasma Phys. and Controlled Fusion **44**, 423 (2002).

[2] J.W. Connor, R.J. Hastie and J.B. Taylor, Phys. Rev. Lett. **40**, 396 (1978).

[3] B. Carreras, P. Diamond and M. Murakami, Phys. Rev. Lett. **50**, 503 (1983).

[4] J.W. Connor et al., in Plasma Physics and Controlled Fusion Research, Proc. 9th IAEA Conf. (IAEA, Vienna, 1983), vol. III, p. 403.

[5] J.F. Drake and T.M. Antonsen, Phys. Fluids **28**, 544 (1983).

[6] J.W. Connor and R.J. Hastie, Plasma Phys. and Controlled Fusion **27**, 621 (1985).

[7] A. Zeiler, J.F. Drake and B. Rogers, Phys. of Plasmas **4**, 2134 (1997).

[8] P.H. Diamond, P.L. Similon, T.C. Hender and B.A. Carreras, Phys. Fluids **28**, 1116 (1985).

[9] G. Ara, B. Basu and B. Coppi, Phys. Fluids **22**, 672 (1979).

[10] B. Coppi and F. Porcelli, Phys. Rev. Lett. **57**, 2272 (1986).

[11] V. P. Lakhin and A. B. Mikhailovskii, Phys. Lett. A **191**, 162, (1994).