Minimum Energy State of Plasmas with an Internal Transport Barrier

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Abstract. The condition for the minimum energy state of tokamak plasmas is examined under the constraint of the total angular momentum conservation. This problem can be treated as a variation problem and the solution gives a specific condition between density and rotation profiles. This condition has been tested against actual DIII-D and JT-60U experimental data with careful evaluation of second order variation values. It is concluded that the steady state tokamak plasmas with an internal transport barrier (ITB) are in the minimum energy state, but the transition states are not. These plasmas with ITB are likely self-organized, which explains stiff profiles of ITB plasmas seen in many tokamak devices.

1. Introduction

It was reported at the Sorrento IAEA Energy Conference that the profiles of the tokamak plasmas with an internal transport barrier (ITB) are explained by the constraint due to the total angular momentum conservation [1]. However, the physical mechanism presented at that time was not very solid. Since then, studies on second order variations and further analysis of DIII-D and JT-60U experimental data have revealed that steady state ITB plasmas correspond to the minimum energy state under the constraint of the total angular momentum conservation [2]. In this paper we present the theoretical derivation of the condition for the minimum energy state and the results of its tests against the actual tokamak plasmas with ITB.

2. Basic Theory

In a toroidally symmetric system such as a tokamak, the total toroidal angular momentum P_ϕ is conserved. Taking an average over the flux surfaces, one may write

$$P_\phi = 4\pi^2 R_0{}^2 \int_0^a nmu_\phi r dr\,, \qquad (1)$$

where R_0 is the major radius of the axis, n is the plasma density, m is the sum of the ion and electron masses, and u_ϕ is the toroidal component of rotation velocity. The total number of particles N is conserved:

$$N = 4\pi^2 R_0 \int_0^a nrdr\,. \qquad (2)$$

The total energy U, the sum of the total kinetic energy K and the total potential energy W, may also be conserved, i.e.,

$$U = K + W = const\,. \qquad (3)$$

The total kinetic energy averaged over flux surfaces may be written as

$$K = 4\pi^2 R_0 \int_0^a nmu^2 rdr\,, \qquad (4)$$

where $u^2 = u_\phi{}^2 + u_\theta{}^2$, and u_θ is the poloidal component of the rotation velocity. The total potential energy averaged over flux surfaces may be written as

$$W = 4\pi^2 R_0 \int_0^a \{\frac{3}{2} nk(T_i + T_e) + \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}\} r dr . \quad (5)$$

Since the total energy is conserved, the maximum total kinetic energy corresponds to the minimum total potential energy. Accordingly, the condition for the minimum (potential) energy state is obtained by solving the variation problem of finding the maximum kinetic energy state by use of the Lagrangian method

$$\delta(K - \lambda_1 N - \lambda_2 P_\phi) = 0, \quad (6)$$

where λ_1 and λ_2 are constant. Equation (6) is explicitly written as

$$(u^2 - \lambda_1 - \lambda_2 u_\phi)\frac{\partial n}{\partial u_\phi} + (2u_\phi - \lambda_2)n = 0 . \quad (7)$$

The solution for Eq. (7) gives a specific condition between the density and the toroidal rotation:

$$\frac{n}{n_0} = \frac{1 + \alpha + \beta}{1 + \alpha \frac{u_\phi}{u_0} + \beta\left(\frac{u}{u_0}\right)^2}, \quad (8)$$

where n_0 and u_0 are the density and the toroidal rotation velocity at $r = 0$, and α and β are adjustable constants. This solution generally means either the maximum or the minimum. The second order variation $\delta^2 K$ must be evaluated to distinguish the two:

$$\delta^2 K = 4\pi R_0{}^2 m \int_0^a (\frac{d^2 n}{du_\phi{}^2} u^2 + 4\frac{dn}{du_\phi} u_\phi + 2n)(\delta u_\phi)^2 r dr . \quad (9)$$

This can numerically be evaluated for Eq. (8) as

$$\delta^2 K = 4\pi R_0{}^2 a n_0 m \int_0^1 F x dx (\delta u_\phi)^2 , \quad (10)$$

where

$$F = \frac{2(1 + \alpha + \beta)(-\alpha\beta w^3 - 3\beta w^2 + \alpha w + 1)}{(1 + \alpha w + \beta w^2)^3} . \quad (11)$$

Here, $x = r/a$ and $w = u_\phi / u_0$. The minimum energy state corresponds to a negative $\delta^2 K$.

3. Examination against Experimental Data

The conditions described by Eq. (8) and Eq (10) for the minimum energy state have been examined against actual DIII-D and JT-60U plasmas with an internal transport barrier. Figure 1 shows the density profile of the DIII-D discharge #105893 at 3.975 s in a Quiescent Double Barrier (QDB) Mode [3]. The blue circle data points with error bars measured by Thomson scattering systems are plotted against the normalized radius x. Figure 2 shows the measured rotation velocity at 3.975 s. Then the density n is re-plotted against the rotation velocity u as shown in Fig. 3 and the best fit satisfying Eq. (8) is also shown. Note that u is nearly the same as u_ϕ because of small u_θ. The χ^2 value for the best fit is 0.014. The density corresponding to the best fit (red curve) is drawn in Figure 1. The fit seems excellent. The sign of the second order variation for the best fit is numerically evaluated by use of Eq. (10) and it is indeed negative since the calculated G value is –0.29, where G is defined as

$$G = \int_0^1 F x dx. \qquad (12)$$

The sensitivity of the G value sign against choices of the fitted parameters n_0, α, and β are carefully examined. For example, the n_0 value must be higher by almost 10 % for the G value slightly positive. This over-evaluated case is shown in Figure 4 and the fit is clearly poor as indicated by the higher χ^2 value of 0.043. Therefore, it is concluded that the DIII-D QDB is in the minimum energy state under the constraint of the total angular momentum conservation.

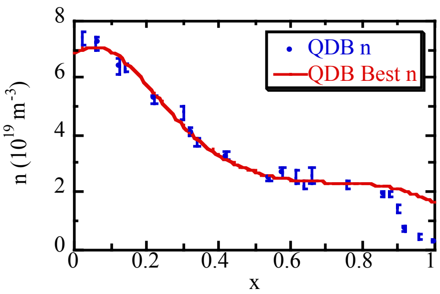

Fig. 1: QDB density profile and the best fit.

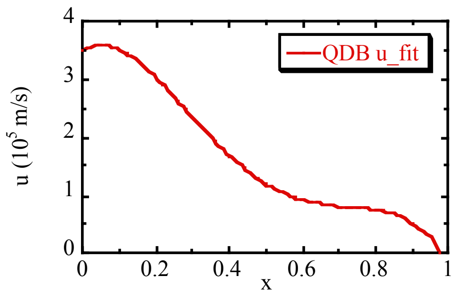

Fig. 2: QDB rotation profile.

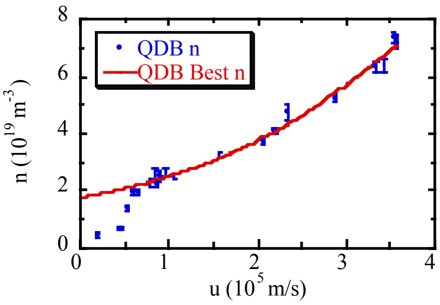

Fig. 3: QDB density vs. rotation with the best fit.

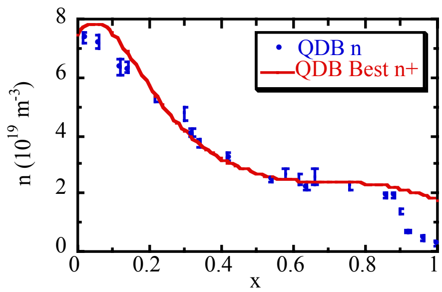

Fig.4: An example of a fit with a slightly positive second variation value.

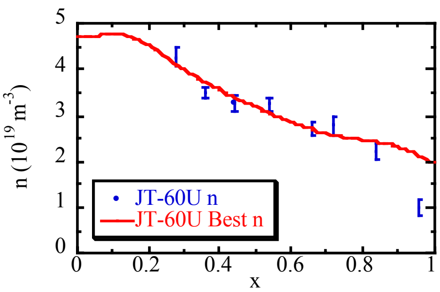

Fig. 5: JT-60U density profile and the best fit.

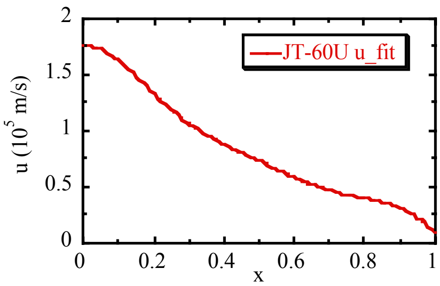

Fig. 6: JT-60U rotation profile.

Similar tests have been conducted against JT-60U discharges with ITB [4]. The JT-60U steady state density profiles with a non-box-type ITB satisfy the condition given by Eq. (8) very well and the second order variations calculated from Eq. (10) are confirmed to be negative. An example of (#E36715 at 6.4 s) is shown in Fig. 5. The blue circle points with error bars are the measured density and the red curve represents the best fit satisfying Eq. (8) (χ^2 = 0.0073 and

$G = -0.22$). In this case the central density is chosen so as to match the measured line density since the direct Thomson measurement near the center is missing. The measured line density must be higher by about 5% (about 20% higher in the central density) in order to give a slightly positive G value. Therefore, it is very likely that the JT-60U discharges with ITB are in the minimum energy state also. In these analysis ITB plasmas with extremely steep density gradients (box type ITB modes) are excluded since the measured impurity rotation velocity may not represent the main ion (deuteron) rotation velocity.

Recently, control of the QDB density profiles by applying central electron cyclotron heating (ECH) has been reported [3]. A very intriguing observation is that the toroidal rotation profile significantly flattens as the density profile flattens after ECH while the ion temperature profile does not change much. It should also be noted that the ECH does not provide any angular momentum input either.

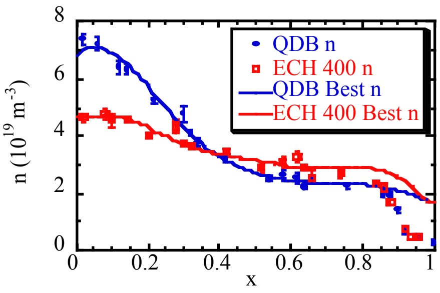

Fig. 7: QDB density profiles and the best fits just before (blue) and 400 ms after (red) ECH .

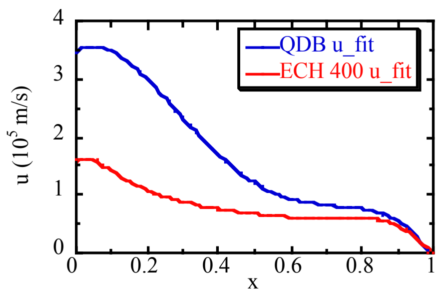

Fig. 8: QDB rotation profiles just before (blue) and 400 ms after (red) ECH.

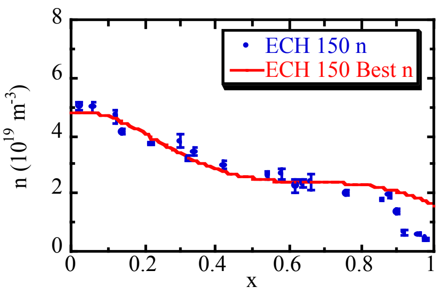

Fig. 9: QDB density profile at 150 ms after ECH.

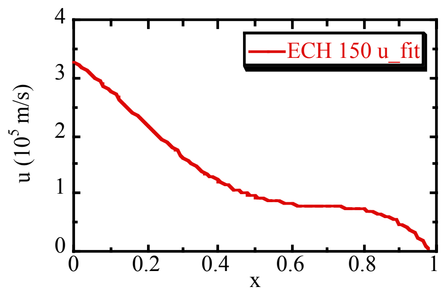

Fig. 10: QDB rotation profile at 150 ms after ECH.

An example is illustrated in Fig. 7 and Fig. 8. The blue circle data points in Fig. 7 represent the QDB density profile just before the ECH application (exactly the same as Fig. 1). The red squares are taken at 400 ms after the ECH application where the density profile settles down to a new steady state condition. Figure 7 shows the corresponding toroidal rotation profiles. The analysis of the data at 400 ms after the ECH application indicates that the condition Eq. (8) also holds between the density profile and the rotation profile as shown by the red best fit curve in Fig. 7. The second order variation is confirmed to be negative ($G = -0.37$). Figure 9 and Fig. 10 respectively shows the density and toroidal rotation profiles taken during the transition period at 150 ms after the ECH application. The density profile has nearly settled down to the new steady state profile, but the rotation profile has not. These data at 150 ms after ECH seemingly satisfy the condition Eq. (8) as indicated by the red curve in Fig. 9. However, the χ^2 value is somewhat higher compared to the other two cases (0.042 compared to 0.014 and 0.034) and the G value is only marginally negative (–0.06). This suggests that the QDB plasma once settled down in the minimum energy state before ECH shifts into another minimum energy state after ECH is applied, but the condition for the minimum energy state is not well satisfied during the transition.

4. Discussions

The result indicates that the density profile for the ITB plasma corresponds to a very particular rotation profile. In these tokamak plasmas toroidal rotation is supposed to be generated by neutral beam injection. Since the local momentum source term is determined from the injected neutral beam distribution and the density profile, there is no guarantee that the rotation profile matches to the desired particular profile. Therefore it is very likely that the ITB plasmas are self-organized or self-adjusted. This explains stiff profiles for ITB plasmas claimed in many tokamak devices. A possible physical mechanism for this process is as follows: An instability occurs which changes the density profile. Accordingly, the momentum and particle deposit profiles change. This loop continues until the plasma finds the particular minimum energy state condition between the density and toroidal rotation profiles. Therefore this seems consistent with some theoretical claims that the plasma profile corresponds to a marginally stable condition for a certain instability, such as a trapped particle mode [5].

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