Short Wavelength Temperature Gradient Driven Modes in Tokamak Plasmas

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Abstract. A new temperature gradient driven instability in the short wavelength region $k_\perp^2\rho_i^2 > 1$ is investigated. The mode is driven by the ion temperature gradient; it exists with adiabatic electrons but may be further enhanced by the non-adiabatic electron effects. In the slab plasma approximation, both local dispersion equation and non-local (differential equation) analysis indicate instability in the short wavelength region. In the toroidal case the mode is somewhat similar to the "ubiquitous mode" but does not require trapped electrons.

1. Introduction

Small scale instabilities driven by the ion and electron temperature gradients instabilities are believed to be responsible for particle and energy transport in a tokamak. Both types of modes, the ion temperature gradient (ITG) and electron temperature gradient (ETG), have been extensively studied in the last years. In this work we report on a new regime of the temperature gradient driven instability which occurs for large values of the Larmor radius parameter, $k_\perp^2\rho_\alpha^2 > 1$, $\alpha = (e,i)$ [1]. Both, electron and ion, modes exist in the respective regions, but the ion mode is of prime interest; the electron mode is suppressed by the finite Debye length effects.

2. Local shearless mode

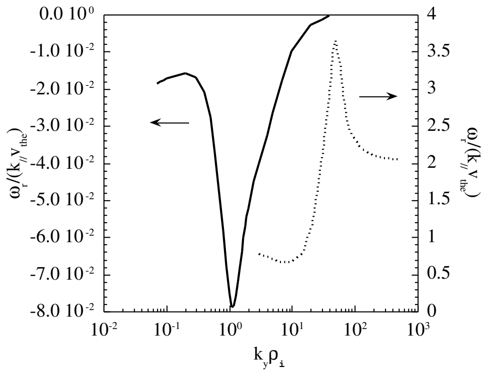
To illustrate the existence of new modes, first we consider a shearless slab case. Within the local theory the parallel velocity and density perturbations for each species are given by standard expressions

$$\frac{\widetilde{v}_{\alpha\|}}{v_{th\alpha}} = -\frac{e_\alpha\widehat{\phi}}{T_\alpha}s_\alpha D_\alpha, \qquad \frac{n_\alpha}{n_0} = -\frac{e_\alpha\phi}{T_\alpha}l_\alpha - \frac{e_\alpha\widehat{\phi}}{T_\alpha}D_\alpha, \tag{1}$$

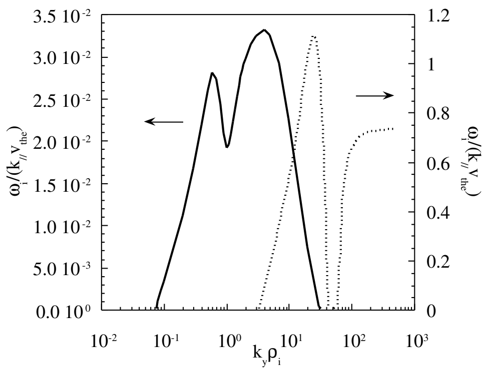
$$\begin{aligned} D_\alpha &= \left(1-\frac{\omega_{n\alpha}}{\omega}\right)(1+s_\alpha Z(s_\alpha))\,\Gamma_0(b_\alpha) + \frac{\omega_{T\alpha}}{\omega}s_\alpha\left(\frac{1}{2}Z(s_\alpha) - s_\alpha - s_\alpha^2 Z(s_\alpha)\right)\Gamma_0(b_\alpha) \\ &\quad + \frac{\omega_{T\alpha}}{\omega}(1+s_\alpha Z(s_\alpha))\,(\Gamma_0(b_\alpha)-\Gamma_1(b_\alpha))\,b_\alpha, \\ l_\alpha &= 1-\left(1-\frac{\omega_{n\alpha}}{\omega}\right)\Gamma_0(b_\alpha) - \frac{\omega_{T\alpha}}{\omega}(\Gamma_0(b_\alpha)-\Gamma_1(b_\alpha))\,b_\alpha. \end{aligned}$$

Various plasma parameters are: $\omega_{n\alpha} = -k_y cT_\alpha/e_\alpha B_0 L_n$, $\omega_{T\alpha} = -k_y cT_\alpha/e_\alpha B_0 L_{T\alpha}$, $L_n^{-1} = -n_0^{-1}\partial n_0/\partial x$, $L_{T\alpha} = -T_\alpha^{-1}\partial T_\alpha/\partial x$, $s_\alpha = \omega/k_\| v_{th\alpha}$, $b_\alpha = k_\perp^2\rho_\alpha^2/2$, $v_{tha}^2 = 2T_\alpha/m_\alpha$, $\rho_\alpha = v_{tha}m_\alpha c/(e_\alpha B_0)$; $\Gamma_{0,1}(b) = I_{0,1}\exp(-b)$, $\widehat{\phi} = \phi - \omega/(k_\| c)A$ is an auxiliary potential, ϕ is the electrostatic potential, A is the magnetic vector potential, and $Z(s)$ is the standard plasma dispersion function.

The dispersion equation is obtained from (1) by using the Poisson and Ampere law equations. It is solved as a function of the $k_y\rho_i$ for fixed plasma parameters: $\beta = 2\times 10^{-4}$, $\rho_i/L_n = 2\sqrt{2}\times 10^{-2}$, $\rho_i/L_{Ti} = \sqrt{2}\times 10^{-1}$, $\rho_i/L_{Te} = \sqrt{2}\times 10^{-1}$, $k_x\rho_i = \sqrt{2}\times 10^{-1}$, $k_{\|}\rho_i = 2\sqrt{2}\times 10^{-3}$, $\tau = 1$, and λ_D(Debye lenghth)$= 0$. Two new, electron and ion, unstable branches exist in the regions $k_y\rho_i \geq 1$ and $k_y\rho_e \geq 1$ as shown in Fig. 1. Numerical solution shows that similarly to the standard ETG mode [2], the electron short wavelength mode is strongly stabilized in a high temperature plasma for $\lambda_D/\rho_e \geq 1$; it is completely suppressed for $\lambda_D/\rho_e \geq 3$.



(a)



(b)

Figure 1: Real (a) and imaginary (b) part of the eigen-frequency. Solid line – standard ITG and short wavelength ion mode (left panel); dotted line – standard ETG and short wavelength electron mode (right panel).

3. Ion short wavelength mode in the fluid regime, $\omega > k_{\|}v_i$

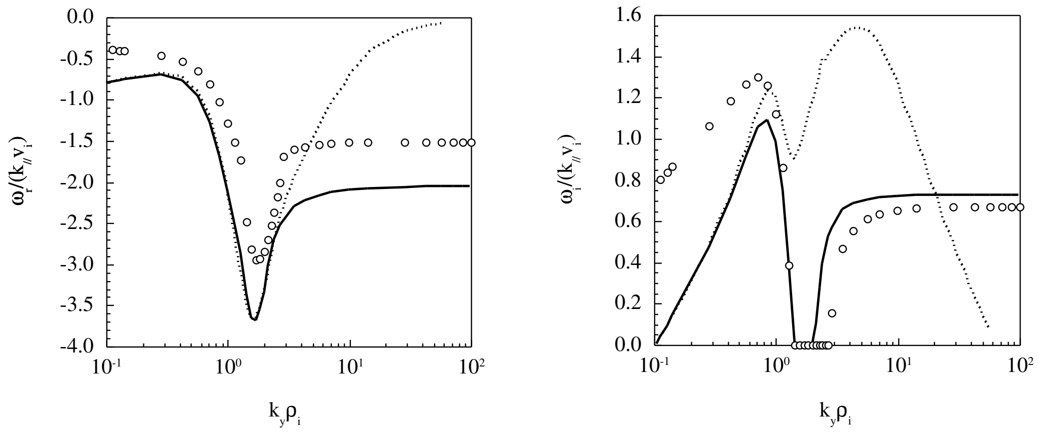
To investigate the nature of new modes we consider the slab plasma case assuming $\omega_D = 0$ and adiabatic electrons. Then, for $\omega/k_{\|}v_i > 1$ but keeping the full finite Larmor radius effects for ions we obtain the following local dispersion equation

$$2 + \frac{\omega_{ni}}{\omega}\Gamma_0(b_i)\left(1+\frac{1}{2s_i^2}\right) - \frac{\omega_{Ti}}{\omega}\left(\Gamma_0(b_i)-\Gamma_1(b_i)\right)b_i\left(1+\frac{1}{2s_i^2}\right) + \frac{\omega_{Ti}}{\omega}\Gamma_0(b_i)\frac{1}{2s_i^2} = 0. \quad (2)$$

In the limit of small $b_i < 1$, equation (2) reduces to the standard local dispersion equation for the long wavelength ion temperature gradient mode [3] that has the maximal growth rate for $k_\perp\rho_i < 1$. New modes occur due to a specific plasma response for $k_y\rho_i > 1$ where it is usually assumed that the ion density response is Boltzmann due to decaying asymptotics of $\Gamma_{0,1}(b_i) \sim 1/\sqrt{b_i}$ for large b_i. In fact, this is not necessarily true for modes whose frequency increases with $k_\perp\rho_i$ slower than linear function of $k_\perp\rho_i$. In the latter case the combination $(1-\omega_*/\omega)\Gamma_0(b_i)$ which enters the density expression remains finite, and the density response is not Boltzmann. This leads to a short wavelength branch which has not been investigated earlier. In the limit of large $k_\perp\rho_i > 1$, by expanding the Bessel function in (2) one obtains the following dispersion equation for the ion short wavelength mode

$$\frac{\omega_{ni}}{\omega}\frac{1}{2s_i^2}\frac{1}{\sqrt{\pi}k_\perp\rho_i}\left(1+\frac{\eta_i}{2}\right) + 2 + \frac{\omega_{ni}}{\omega}\frac{1}{\sqrt{\pi}k_\perp\rho_i}\left(1-\frac{\eta_i}{2}\right) = 0. \quad (3)$$

In the leading order we have from (3) $\omega^3 = v_i^3 k_{\|}^2\left(1+\eta_i/2\right)/\left(8\sqrt{\pi}L_n\right)$. A simplified dispersion equation (2) qualitatively reproduces the behavior of the eigen-frequency in the whole range of values of the parameter $k_\perp\rho_i$ (shown in Fig. 2 by circles).



(a) (b)

Figure 2: Real part (a) and growth rate (b) of the short wavelength ion mode. Solid line – local kinetic result from (1) assuming adiabatic electrons, circles – fluid approximation given by (2). Dotted line - effect of non-adiabatic electrons for $\eta_e = 5$. Other parameters: $k_{\|} L_n = 0.1$, $\alpha_* \equiv k_{\|} v_{ti}/\omega_s = 0.1$, where $\omega_s = v_i/L_n$, $\eta_i = 5$, $k_\perp^2 = k_y^2 + k_x^2$, $k_x \rho_i = \sqrt{2} \times 10^{-1}$.

4. **Ion short wavelength mode in the local toroidal limit**

From the standard gyrokinetic equation one can obtain the following equation in the ballooning space

$$2\phi(\theta) = \int F_m J_0(k_\perp v_\perp/\omega_c) d^3 v \frac{\omega - \widehat{\omega}_*}{\omega - \widehat{\omega}_D + i v_{\|}/qR\partial/\partial\theta} \left(J_0(k_\perp v_\perp/\omega_c)\phi(\theta)\right), \tag{4}$$

where $\widehat{\omega}_* = \omega_{ni} + \omega_{Ti}\left(v^2/v_{ti}^2 - 3/2\right)$, $\widehat{\omega}_D = \omega_D(\cos\theta + \widehat{s}\theta\sin\theta)\left(v_\perp^2/2v_i^2 + v_{\|}^2/v_i^2\right)$, θ is the standard ballooning space variable, $\omega_D = 2\varepsilon_n \omega_{ni}$ is the toroidal drift frequency, $\varepsilon_n = L_n/R$ is the toroidicity parameter. The perpendicular wave vector is $k_\perp^2 = k_y^2 \rho_i^2 \left(1 + \widehat{s}^2\theta^2\right)$. The adiabatic electrons are assumed in (4). In the local limit $i v_{\|}/qR\partial/\partial\theta \to -k_{\|} v_{\|}$. A simple insight into the toroidal short wavelength modes can be obtained from a local analysis. In the fluid limit, $\omega > \omega_D$ and neglecting the ion parallel motion one can obtain from (4) the following local dispersion equation

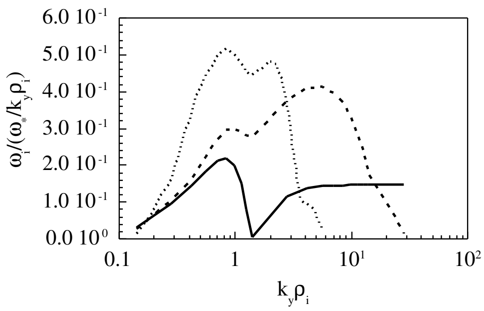


Figure 3: Ion mode growth rate from the equation (4) in the local limit; solid line – $s_D = 0$, dashed line – $s_D = 1$, dotted line – $s_D = 6$

$$\omega^2\left(2 - \Gamma_0\right) + \omega\left(\omega_{ni}\Gamma_0 + \omega_{Ti}G_1 - \omega_D G_2\right) + \omega_D\omega_{Ti}G_3 + \omega_D\omega_{ni}G_2 = 0, \tag{5}$$

where $G_1 = b\left(\Gamma_1(b) - \Gamma_0(b)\right)$, $G_2 = G_1/2 + \Gamma_0(b)$, and $G_3 = \Gamma_0(b) + b\left[3\Gamma_1(b)/2 - 2\Gamma_0(b)\right] + bG_1 + \Gamma_0(b)$, where $b = k_y^2\rho_i^2(1+\widehat{s}^2\theta^2)/2$. In the short wavelength limit $k_y\rho_i > 1$ equation (5) predicts the interchange type mode with a growth rate $\gamma \simeq \left(3(1+\eta_i/2)\omega_D v_{ti}/(16\sqrt{\pi}L_n)\right)^{1/2}$. It can be readily observed that the toroidal short wavelength ITG mode is closely related to the "ubiquitous" modes [4]: the dispersion equation (5) is analogous to the dispersion equation in Refs. 4 (except the contribution of trapped electrons which is omitted in (5)).

The applicability of the fluid equation (5) is limited by the condition $\omega > \omega_D$ which requires either small values of the magnetic drift frequency ω_D or large temperature gradient parameter η_i. In a general case with a finite value of the parallel wave vector, the instability tends to be stabilized around $k_\perp \rho_i \simeq 2 \div 3$ for larger values of the toroidicity parameter $s_D \equiv \omega_D / k_\| v_i$. [5]. This is shown in Fig. 3 which is obtained by solving equation (5) in the local limit without expansion in ω_D and $k_\| v_\|$.

5. Nonlocal differential equation analysis

To investigate the effect of the magnetic shear we employ nonlocal differential equation in the fluid limit $\omega > k_\| v_i$ and $\omega > \omega_D$

$$\left(1-\frac{\omega_{ni}}{\omega}\right)\Gamma_0(b)\phi(\theta) - \frac{\omega_{Ti}}{\omega}G_1\phi(\theta) + \left[\left(1-\frac{\omega_{ni}}{\omega}\right)G_2 - \frac{\omega_{Ti}}{\omega}G_3\right]\frac{\omega_D}{\omega}\left(\cos\theta + s\theta\sin\theta\right)\phi(\theta)$$
$$-\frac{v_{ti}^2}{q^2R^2\omega^2}\left(1-\frac{\omega_{ni}}{\omega}\right)\left(C_1\frac{\partial^2\phi(\theta)}{\partial\theta^2} + C_2\frac{\partial\phi(\theta)}{\partial\theta} + C_3\ \phi(\theta)\right)$$
$$+\frac{v_{ti}^2}{q^2R^2\omega^2}\frac{\omega_{Ti}}{\omega}\left(D_1\frac{\partial^2\phi(\theta)}{\partial\theta^2} + D_2\frac{\partial\phi(\theta)}{\partial\theta} + D_3\ \phi(\theta)\right) = 2\phi(\theta). \tag{6}$$

Here various coefficients are defined as follows: $C_1 = \Gamma_0(b)/2$, $D_1 = \Gamma_0/2 + G_1/2$, $C_2 = k_\perp^{-1}\partial k_\perp/\partial\theta G_1$, $D_2 = -\,k_\perp^{-1}\partial k_\perp/\partial\theta\ G_4$, $G_4 = 2\left[bG_1 + b\Gamma_0 - b\Gamma_1/2\right]$, $C_3 = -\left(k_\perp^{-1}\partial k_\perp/\partial\theta\right)^2 G_5 + k_\perp^{-1}\partial^2 k_\perp/\partial\theta^2 G_1/2$,$D_3 = -\left(k_\perp^{-1}\partial k_\perp/\partial\theta\right)^2 G_7 -\ k_\perp^{-1}\partial^2 k_\perp/\partial\theta^2 G_6$, $G_5 = (2bG_1 + b\Gamma_0 + b\Gamma_1)/2$, $G_6 = b(G_1 + \Gamma_0 - \Gamma_1/2)$, $G_7 = b(-2bG_1 + \Gamma_0 + \Gamma_1/2 + 2b\Gamma_1 - 3b\Gamma_0)$. Equation

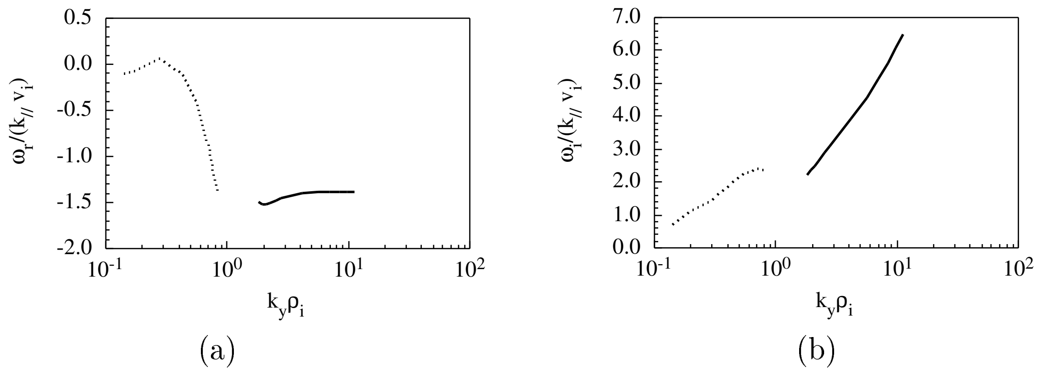


Figure 4: Normalized real (a) and imaginary (b) part of the eigen-frequency as obtained from shooting code solution of (6)

similar to (6) was obtained in Ref.6. In the limit of $k_y\rho_i < 1$ it has been investigated for the standard long wavelength modes [3]. We solve the equation (6) with a shooting code with full FLR effects for arbitrary values of $k_y\rho_i$. The eigen-frequency obtained from the shooting code solution is shown in Fig. 4 as a function of $k_y\rho_i$. for $\eta_i = 5$, $\hat{s} = 0.8$, $\alpha_* \equiv k_\| v_{ti}/\omega_s = 0.1$, $k_\| R = 1$, and $k_\| \equiv 1/(qR)$. A typical eigen-function in the short wavelength limit ($k_y\rho_i = 5\sqrt{2}$) is shown in Fig 5a. For comparison, in Fig. 5b we show a typical eigen-function in the long wavelength limit ($k_y\rho_i = 0.5\sqrt{2}$) [3].

6. Summary.

In summary, we have investigated new short wavelength temperature gradient driven modes that are active in the region $k_\perp^2\rho_\alpha^2 > 1$, $\alpha = (e, i)$. Both electron and ion modes may exist, but the electron short wavelength mode is strongly suppressed by the finite Debye length. The instability is investigated in the local approximation, with non-local differential eigenmode equation (shooting code solution) as well as with non-local integral equation [5,7]. Results of these approaches are in general agreement. The particle gyrokinetic simulations [8] are in progress. The ion mode is destabilized primarily by the ion dynamics but it is

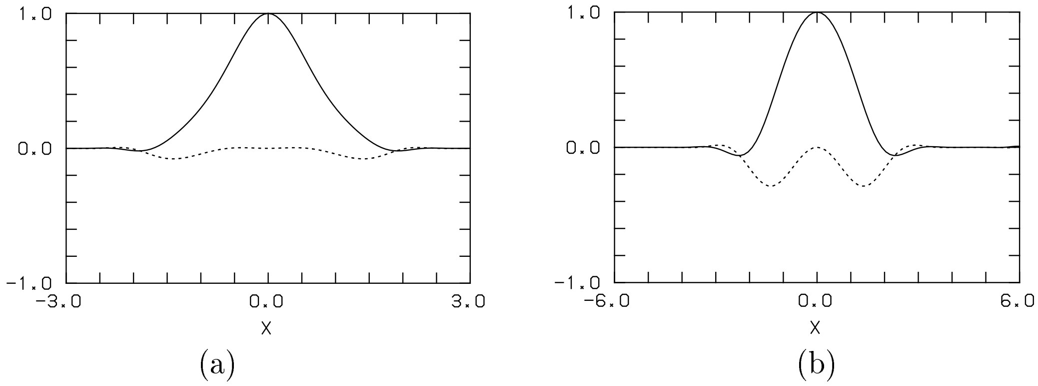


Figure 5: The eigen-function (in arbitrary units) as a function of the ballooning variable $\theta = x$ in the short wavelength limit – (a) , $k_y\rho_i = 5\sqrt{2}$, and the standard long wavelength limit – (b), $k_y\rho_i = 0.5\sqrt{2}$; other parameters are $\eta_i = 5$, $\hat{s} = 0.8$, $\alpha_* \equiv k_\parallel v_{ti}/\omega_s = 0.5$, $k_\parallel R = 1$. Solid line - real part and dashed line - imaginary part.

also affected by the electron temperature gradient. It is unstable in the region between the standard ITG and ETG modes, thus providing an opportunity for interaction of modes with disparate scale (such as standard ITG and ETG). One of the most interesting features of the ion short wavelength mode is its ability to affect the electron transport. Note that for a typical plasma pressure in a tokamak the characteristic scale length of the ion short wavelength mode approximately corresponds to the electron skin depth size.

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