Integral Equation Based Stability Analysis of Short Wavelength Drift Modes in Tokamaks*

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Abstract. Linear stability of electron skin-size drift modes in collisionless tokamak discharges has been investigated in terms of electromagnetic, kinetic integral equations in which neither ions nor electrons are assumed to be adiabatic. A slab-like ion temperature gradient mode persists in such a short wavelength regime. However, toroidicity has a strong stabilizing influence on this mode. In the electron branch, the toroidicity induced skin-size drift mode previously predicted in terms of local kinetic analysis has been recovered. The mode is driven by positive magnetic shear and strongly stabilized for negative shear. The corresponding mixing length anomalous thermal diffusivity exhibits favourable isotope dependence.

1. Introduction

In recent reflectometer studies of density fluctuations in JET (Joint European Torus), it has been observed that after formation of an internal transport barrier (ITB), low frequency, long wavelength modes are suppressed while high frequency, short wavelength modes are not [1]. The significant reduction in the ion thermal diffusivity concurrently observed is attributed to the suppression of long wavelength modes by the velocity shear. The electron thermal diffusivity still remains large and it has been speculated that the electron thermal loss is caused by short wavelength modes.

In this paper, results of tokamak stability analysis of short wavelength $[(k_\perp \rho_i)^2 \gtrsim 1,\ (k_\perp \rho_e)^2 \ll 1]$ drift type modes based on a fully kinetic, electromagnetic integral equation code will be reported. The familiar kinetic ion density perturbation

$$n_i \simeq -\frac{e\phi}{T_i} n_0 + \frac{\omega + \omega_{*i}}{\omega + \omega_{Di}} e^{-\lambda} I_0(\lambda) \frac{e\phi}{T_i} n_0, \tag{1}$$

does not approach adiabatic form for ω falling in the regime $\omega_{Di} < |\omega| < \omega_{*i}$ even if $\lambda = (k_\perp \rho_i)^2 \gg 1$. The electron response

$$n_e = \frac{e\phi}{T_e} n_0 - \int \frac{\omega - \omega_{*e}(v^2)}{\omega - \omega_{De}(\mathbf{v}) - k_\parallel v_\parallel} \left(\phi - \frac{v_\parallel}{c} A_\parallel\right) d\mathbf{v} \frac{e}{T_e} n_0, \tag{2}$$

is not adiabatic either because the condition ω, $\omega_{De} \ll k_\parallel v_{Te}$ tends to be violated as both the mode frequency ω and ω_{De} increase. A new type of instability may emerge in which both ions and electrons participate in nonadiabatic manner and the electron/ion mass ratio is expected to manifest itself in the growth rate. In order to handle electron and ion kinetic resonances and the parallel gradient operator $k_\parallel = -i\frac{1}{qR}\frac{\partial}{\partial\theta}$ in a satisfactory manner, a high resolution, fully kinetic, electromagnetic integral equation code has been developed [2] following the method developed by Rewoldt and co-workers [3]. Short wavelength drift modes propagating in both the electron and ion diamagnetic drifts have been identified. The ion mode has recently been reported [4] and in this paper, the electron mode will mainly be described.

2. Formulation

We consider a high temperature, low β tokamak discharge with eccentric circular magnetic surfaces. The frequency regime of interest is $\omega_{bi} \ll \omega \lesssim \omega_{be}$, where $\omega_{bi(e)}$ is the trapped ion (electron) bounce frequency. The magnetosonic perturbation ($\mathbf{A}_\perp$) is ignored in light of the low β assumption and we employ the two-potential (ϕ and $A_\parallel$) approximation to describe electromagnetic modes. The basic field equations are the charge neutrality condition and parallel Ampere's law,

$$n_i(\phi, A_\parallel) = n_e(\phi,\, A_\parallel),\;\; \nabla^2 A_\parallel = -\frac{4\pi}{c} J_\parallel(\phi,\, A_\parallel), \tag{3}$$

where the density perturbations are given in terms of the perturbed velocity distribution functions f_i and f_e as $n_i = \int f_i d\mathbf{v}$, $\quad n_e = \int f_e d\mathbf{v}$, and the parallel current by $J_\parallel = e\int v_\parallel (f_i - f_e) d\mathbf{v}$. The distribution functions f_i and f_e are given by

$$f_i = -\frac{e\phi}{T_i} f_{Mi} + g_i(v,\theta) J_0(\Lambda_i),\;\; f_e = \frac{e\phi}{T_e} f_{Me} + g_e(v,\theta) J_0(\Lambda_e), \tag{4}$$

where $g_{i,e}$ are the nonadiabatic parts that satisfy

$$\left(i\frac{v_\parallel(\theta)}{qR}\frac{\partial}{\partial\theta} + \omega + \widehat{\omega}_{Di}\right) g_i = (\omega + \widehat{\omega}_{*i}) J_0(\Lambda_i)\left(\phi - \frac{v_\parallel}{c}A_\parallel\right)\frac{e}{T_i} f_{Mi}, \tag{5}$$

$$\left(i\frac{v_\parallel(\theta)}{qR}\frac{\partial}{\partial\theta} + \omega - \widehat{\omega}_{De}\right) g_e = -(\omega - \widehat{\omega}_{*e}) J_0(\Lambda_e)\left(\phi - \frac{v_\parallel}{c}A_\parallel\right)\frac{e}{T_e} f_{Me}. \tag{6}$$

Here, θ is the extended poloidal angle, ϕ is the scalar potential, $A_\parallel$ is the parallel vector potential, J_0 is the Bessel function with argument $\Lambda_{i,e} = k_\perp v_\perp/\omega_{ci,e}$, $\widehat{\omega}_{Di,e}$ are the magnetic drift frequencies defined by

$$\widehat{\omega}_{Di,e} = \frac{cm_{i,e}}{eBR}\left(\frac{1}{2}v_\perp^2 + v_\parallel^2\right)[\cos\theta + (s\theta - \alpha\sin\theta)\sin\theta], \tag{7}$$

with

$$s = \frac{r}{q}\frac{dq}{dr},\;\; \alpha = -q^2 R\frac{d\beta}{dr},\;\; k_\perp^2 = \left(\frac{nq}{r}\right)^2[1 + (s\theta - \alpha\sin\theta)^2], \tag{8}$$

$\widehat{\omega}_{*i,e}$ are the diamagnetic drift frequencies,

$$\widehat{\omega}_{*i,e} = \omega_{*i,e}\left[1 + \eta_{i,e}\left(\frac{m_{i,e}v^2}{2T_{i,e}} - \frac{3}{2}\right)\right],\;\; \omega_{*i,e} = \frac{cT_{i,e}}{eBL_n}k_\theta,\;\; k_\theta = \frac{nq}{r}, \tag{9}$$

$L_n^{-1} = -d(\ln n_0)/dr$ is the inverse density gradient scale length, and $\eta_{i,e} = d\ln T_{i,e}/d\ln n_0$ are the temperature gradient parameters.

For circulating particles, g_j $(j = i, e)$ can be integrated as

$$v_\parallel > 0, \qquad g_j^+ = -i\frac{e_j f_{Mj}}{T_j}\int_{-\infty}^{\theta} d\theta' \frac{qR}{|v_\parallel|} e^{i\beta_j}(\omega - \widehat{\omega}_{*j}) J_0(\Lambda_j')\left(\phi(\theta') - \frac{|v_\parallel|}{c}A_\parallel(\theta')\right), \tag{10}$$

$$v_\parallel < 0, \qquad g_j^- = -i\frac{e_j f_{Mj}}{T_j}\int_{\theta}^{\infty} d\theta' \frac{qR}{|v_\parallel|} e^{-i\beta_j}(\omega - \widehat{\omega}_{*j}) J_0(\Lambda_j')\left(\phi(\theta') + \frac{|v_\parallel|}{c}A_\parallel(\theta')\right), \tag{11}$$

where

$$\beta_j(\theta,\theta') = \int_{\theta'}^{\theta} \frac{qR}{|v_\parallel|}[\omega - \widehat{\omega}_{Dj}(\theta'')]d\theta''.$$

Substitution of perturbed distribution functions into charge neutrality and parallel Ampere's law yields

$$\sum_j \left(-\frac{e_j}{T_j}\phi + \int \left[g_j^+(\theta) + g_j^-(\theta)\right] J_0(\Lambda_j) d\mathbf{v} \right) = 0, \tag{12}$$

$$\nabla_\perp^2 A_\parallel(\theta) = -\frac{4\pi}{c}\sum_j e_j \int v_\parallel \left[g_j^+(\theta) - g_j^-(\theta)\right] J_0(\Lambda_j) d\mathbf{v}, \tag{13}$$

where $\int d\mathbf{v} = 2\pi \int_0^\infty v_\perp dv_\perp \int_0^\infty dv_\parallel$. This system of inhomogeneous integral equations, subject to the charge neutrality condition $k \ll k_{De,i}$ (Debye wavenumber), has been solved by employing the method of Fredholm in which the integral equations are viewed as a system of linear algebraic equations.

3. Results of Stability Analysis

Figure 1 (a) shows the mode frequency $(\omega_r + i\gamma)/\omega_{*e}$ as a function of the ion finite Larmor radius parameter $b_s = (k_\theta \rho_s)^2$ ($\rho_s = \sqrt{T_e/M}/\Omega_i$) when $L_n/R = 0.2$, $s = 1$, $q = 1.5$, $\eta_e = \eta_i = 2.5$ and $\beta_i = \beta_e = 0.1\%$ (low β case). The instability sets in at $b_s \simeq 2$ or $(k_\theta c/\omega_{pe})^2 \simeq 2.2$ for the assumed parameters. This is consistent with the local kinetic analysis reported earlier [5]. The instability requires a finite (electron) temperature

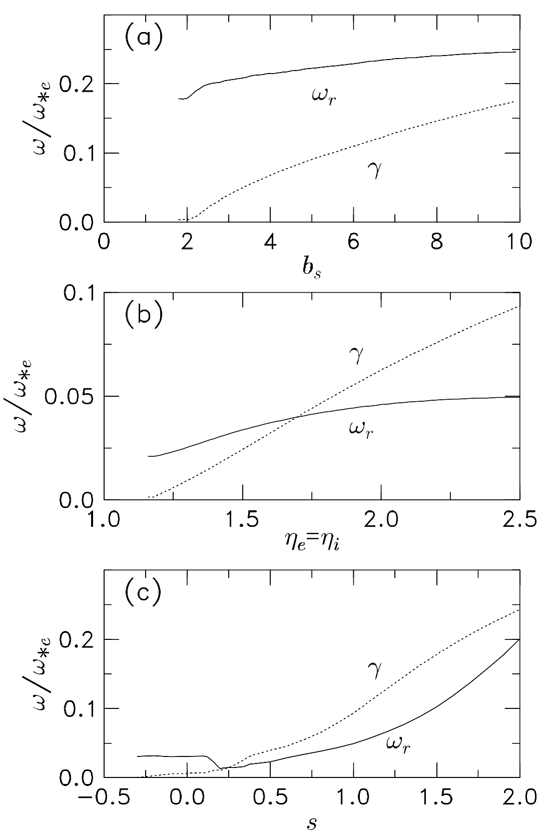

Fig. 1

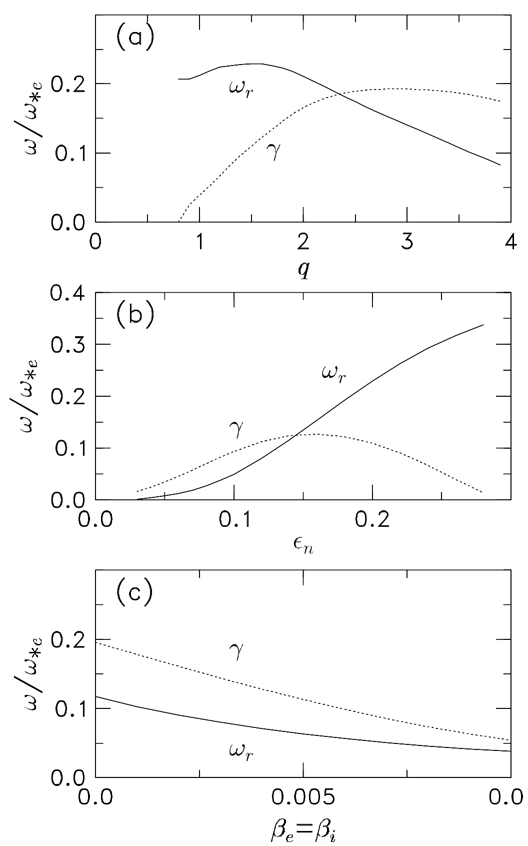

Fig. 2

gradient as shown in Fig. 1 (b) where $\eta_e = \eta_i$ is assumed. (The instability does not require a finite ion temperature gradient η_i. η_i alone is actually weakly stabilizing.) Dependence of ω on the magnetic shear parameter s is depicted in Fig. 1 (c) for the mode $b_s = 6$ when $\eta_e = \eta_i = 2.5$. (Other parameters are unchanged from those in Fig. 1 (b).) It is evident that the instability is driven by positive magnetic shear, but it is strongly stabilized by negative shear. Dependence of the mode frequency and growth rate on the safety factor q is shown In Fig. 2 (a). The instability exists only in toroidal geometry. This is demonstrated in Fig. 2 (b) by varying the magnitude of the magnetic drift frequency $\omega_{Dj} = 2\varepsilon_n \omega_{*j}$ where $\varepsilon_n = L_n/R$. For small ε_n (*i.e.*, in slab geometry), the instability disappears. Too strong toroidicity (large ε_n) also deactivates the instability because the ion dynamics tends to be adiabatic if $\omega_{Di} \simeq \omega_{*i} > \omega$ and $b_s \gg 1$. Stabilization at large ε_n seen in Fig. 2 (b) is thus an expected result. The mode is distinct from the conventional η_e mode which is operative at even shorter wavelength $(k_\perp \rho_e)^2 \lesssim 1$ where ions are adiabatic. Dependence of the mode frequency and growth rate on the plasma β factor is shown in Fig. 2 (c). The predominantly electrostatic mode shown in Figs. 1 and 2 is subject to finite β stabilization. For β factor relevant to high performance tokamaks $\beta \simeq 1$ %, the mode becomes highly retarded and the mode frequency and growth rate become comparable to the ion acoustic transit frequency $|\omega| \simeq c_s/qR$. In this case, it is expected that isotope effect manifests itself and growth rate should decreases with the mass of hydrogen isotopes. This is demonstrated in Fig. 3. In Fig. 3 (a), dispersion relations for hydrogen (H) and tritium (T) discharges are shown. ω is normalized by the electron transit frequency $\omega_{Te} = \sqrt{T_e/m_e}/qR$ and the wavenumber by the inverse skin depth, $k_\theta \to ck_\theta/\omega_{pe}$. It is clearly seen that the maximum growth rate rapidly decreases with the ion mass. Electron thermal diffusivity χ_e estimated from the mixing length theory, $\chi_e = \gamma^3/\left(\omega_r^2 + \gamma^2\right)/k_\perp^2$, is shown in Fig. 3 (b) in units of $\omega_{Te}(c/\omega_{pe})^2$. Favourable isotope effect in the thermal diffusivity is apparent.

In the local analysis [5], the role of magnetic shear could not be clarified. In the case of skin size ion mode [4], the shear is destabilizing also with the growth rate approximately proportional to $\sqrt{|s|}$. This is expected from the slab nature of the skin-size η_i instability in which magnetic shear enters through the parallel operator $k_\parallel$. In contrast, the electron mode is driven only by a positive shear and strongly suppressed by negative shear. This may be explained qualitatively by the shear dependence of the electron magnetic drift frequency. The norm of $\omega_{De} = 2\varepsilon_n \omega_{*e}[\cos\theta + (s\theta - \alpha \sin\theta)\sin\theta]$ can become negative for $s < 0$ and extended eigenfunctions [7].

4. Conclusions

In summary, stability of collisionless tokamaks against skin-size ($k_\perp \simeq \omega_{pe}/c$) drift modes has been investigated in terms of fully kinetic, electromagnetic integral equations. The electron drift mode previously predicted in terms of local kinetic dispersion relation has been confirmed. Neither electrons nor ions are adiabatic and the growth rate and resultant mixing length anomalous electron thermal diffusivity exhibit favourable isotope effect. Some features of the instability, e.g., stabilization in negative shear and favourable isotope effect) are consistent with the general confinement properties of tokamaks [6].

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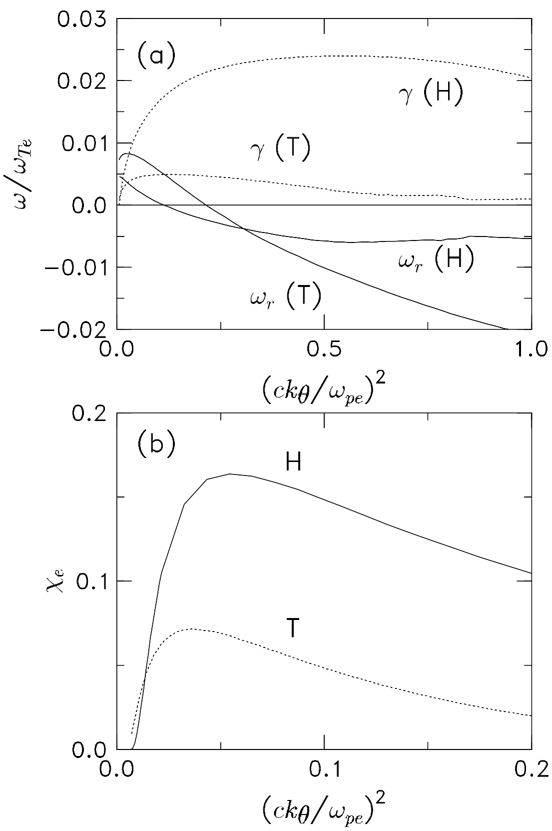

Fig. 3