# On the Physics of Runaway Particles in JET and MAST

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**Abstract.** This paper explores the physics of runaway particles observed in MAST and JET. During internal reconnection events in MAST, it is observed that the ion distribution function, as measured by a neutral-particle analyser, develops a high-energy tail, which subsequently decays on the time scale of collisional slowing down. These observations are explained in terms of runaway ion acceleration in the electric field induced by the reconnection – a phenomenon predicted theoretically by Furth and Rutherford in 1972 but not commonly noted in tokamaks. In JET, long-lived post-disruption currents carried by runaway electrons have been observed to decay on a time scale of 1-2 s. A relativistic kinetic theory is developed to explain this decay as a consequence of the combined action of Coulomb collisions and synchrotron radiation emission. It is also pointed out that substantial electron-positron pair production should occur in such discharges, which have also been made more recently on JT-60U. In fact, tokamaks may be the largest positron repositories made by man.

## 1. Runaway ions in MAST

The only fast ions that are ordinarily present in a tokamak plasma are those that are deliberately introduced for the purpose of heating: neutral-beam ions, RF-heated ions, or fusion products. However, in Ohmic MAST discharges it is routinely observed that some thermal ions are spontaneously accelerated to high energy (up to $\sim 10 - 20\ T_i$) when the plasma undergoes internal magnetic reconnection. MAST is equipped with a neutral-particle analyser detecting suprathermal particles moving in the direction of the plasma current. A Maxwellian ion distribution function is normally observed in Ohmic plasmas, but at each internal reconnection event (IRE), a high-energy tail extending to about 10 keV is formed, which then decays on the collision time scale, see Fig 1. The observed ion acceleration in MAST is reminiscent of observations of ion heating and tail formation in reversed field pinches, and could shed light on this long-standing puzzle.

IREs occur frequently in MAST and result in a temporary increase in total plasma current, a spike of negative loop voltage at the plasma edge, and a broadening (in fact, hollowing) of the current profile. The precise dynamics of IREs is not known, but its main features can be understood crudely by assuming that the plasma lowers its magnetic energy while conserving its magnetic helicity, as in Taylor relaxation of a reversed-field pinch [1].

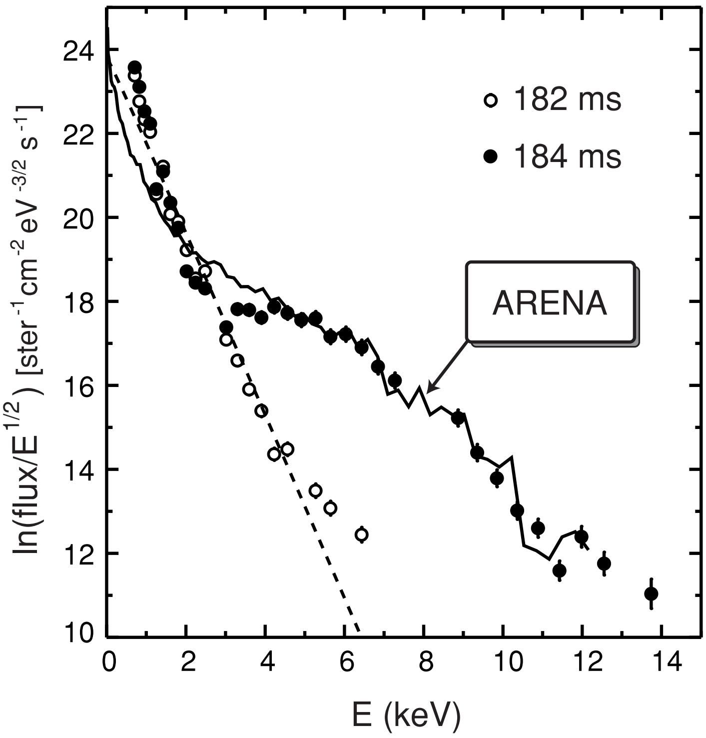


Figure 1: *Ion distribution function just before and after a reconnection event, as measured by the neutral-particle analyser and simulated with the ARENA code.*

As a simple illustrative model, we take the plasma to be cylidrical and the current profile before and after the IRE to be of the form

$$j_z(r) = j_0 \left(1 - \rho^2\right) \left(1 - \lambda\rho^2\right) \ ,$$

where $\rho = r/a$ is the normalized minor radius. Thus, the current density is constrained to be zero at the plasma edge, and $\lambda$ determines the peakedness of the profile. It is assumed that $\lambda$ and $j_0$ change as a result of the IRE in such a way as to keep the magnetic helicity constant,

$$K = \int_0^1 \mathbf{A} \cdot \mathbf{B} \rho d\rho \propto (4 - \lambda) j_0,$$

where $\mathbf{B} = \nabla \times \mathbf{A}$, and we have taken the toroidal field to be constant. The relaxed, minimum energy state has $\lambda = -27/7$, so that the final amplitude $j_{0f}$ is related to the initial value $j_{0i}$ by $55 j_{0f} = 7(4 - \lambda) j_{0i}$, and the final current profile is hollow, as observed in experiments. It follows that the relative change of the total plasma current is

$$\frac{\Delta I}{I} = \frac{7\lambda + 27}{55(3 - \lambda)},$$

and the axial electric field $E_z$ required to bring about this change can be calculated from Ampere's and Faraday's laws,

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E_z}{\partial r} \right) = \mu_0 \frac{\partial j_z}{\partial t},$$

giving

$$\int E_z dt \propto 1 - (36/7)\rho^2 + (45/7)\rho^4 - (16/7)\rho^6 \ .$$

Although this is clearly an approximate model of what might happen in an IRE, we note that it does successfully predict its three main characteristics: an increase in total plasma current, a hollowing of the current profile, and a negative voltage spike at the plasma edge. Indeed, the radial profiles of the relaxed current and the induced electric field are remarkably similar to those inferred from magnetic reconstruction of the equilibrium.

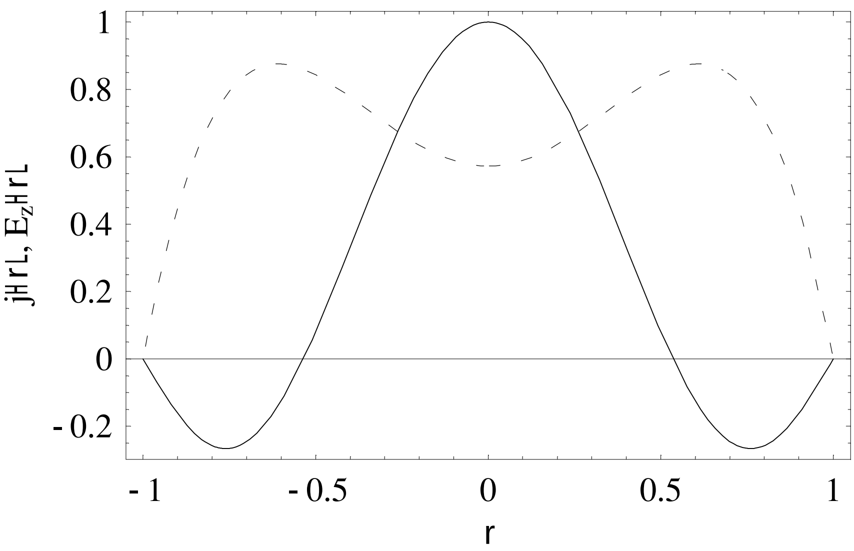


Figure 2: *Radial profile of parallel electric field (solid) and relaxed current profile (dashed) predicted by a simple helicity-conserving model of IREs.*

The loop voltage induced by an IRE is believed to exceed 200 V in the plasma core and is observed to accelerate runaway electrons, generating hard ( $>$ 80 keV) X-rays. Its effect on the plasma ions is more subtle and was investigated theoretically by Furth and Rutherford [2]. Applying an electric field to a pure hydrogen plasma in a straight magnetic field does not lead to any ion acceleration since the electric force on the ions is exactly balanced by friction against electrons. The "effective electric field" representing the total force on the ions vanishes,

$$E_* \equiv E_\| + R_{ie\|}/n_i e = 0,$$

where $R_{ie\|}$ is the ion-electron friction force and $n_i$ the ion density. However, in practice this balance is disturbed by two effects: toroidicity and the presence of impurity ions. Toroidicity leads to trapping of some electrons, removing some of the electron-ion friction, and impurity ions exert friction on the electrons, which reduces their friction on bulk ions correspondingly. The effective electric field acting on the bulk ions can be calculated by using tools from neoclassical theory, and is given by [3]

$$\frac{\langle E_* B\rangle}{\left\langle E_\| B\right\rangle} \simeq \frac{\alpha}{1+\alpha} + \frac{3.96 + 2.59x + \alpha(4.21 + 3.24x) + \alpha^2(1+x)}{2.59(0.65+x)(1.44+x) + \alpha(3.24+\alpha)(1+x)^2}\,\frac{x}{1+\alpha}, \qquad (1)$$

where $\alpha = Z_{\text{eff}} - 1$, and $x = f_c^{-1} - 1$ with

$$f_c = \frac{3\langle B^2\rangle}{4}\int_0^{B_{\max}^{-1}} \frac{\lambda d\lambda}{\langle\sqrt{1-\lambda B}\rangle}$$

the effective fraction of circulating particles. In practice, this implies the effective electric field is comparable to $E_{\|}$; typically $E_* \gtrsim 0.5\ E_{\|}$. As pointed out by Furth and Rutherford, runaway acceleration of suprathermal ions is expected to occur if $E_* > (3m_e/2\pi m_i)^{1/3} E_D$, where $E_D = n_e e^3 \ln\Lambda/4\pi\epsilon_0^2 T_e$ is the Dreicer field. This condition is satisfied by a wide margin during IREs in MAST.

Using the effective electric field given by (1), with $E_{\|}$ inferred from equilibrium reconstruction of an IRE, we have calculated the fast-ion distribution function using the three-dimensional Monte Carlo code Arena, originally developed for the study of runaway electrons [4]. This code solves the orbit average of the drift kinetic equation in toroidal geometry, and has been applied to calculate the distribution function of fast ions in MAST following an IRE. The background plasma density was represented by the density profile $n_e = n_0(1 - 0.9\rho^2)^{1/2}$ and temperature profile $T_i = T_0(1 - 0.9\rho^2)$, with $n_0 = 6 \cdot 10^{19}$ m$^{-3}$, $T_0 = 400$ eV, and effective ion charge $Z_{\text{eff}} = 2$, all in accordance with experimental observations. The simulation was thus performed without using any "free parameters". Figure 1 shows the resulting distribution function of fast ions moving in a cone around the forward direction, as observed by the NPA. As can be seen, the agreement is excellent. The conclusion that runaway ion acceleration is taking place is further corroborated by supplementary experiments, where the NPA was turned 90 degrees so as to detect particles moving perpendicularly to the magnetic field. No tail formation was then observed, as expected from a beam of runaway ions travelling in the direction of the plasma current.

## 2. Decay of runaway electron currents in JET

Relativistic runaway electron beams are frequently generated in tokamak disruptions, and can severely damage the vacuum vessel on impact. In JET disruptions, up to half the plasma current can be converted to runaway electrons with energies in the range 10-20 MeV provided the runaway current column remains stable [5]. Most frequently, these electrons are quickly dumped onto the first wall, which leads to intense hard X-ray generation. In other cases, a substantial runaway current remains for several seconds, showing a smooth decay with a time scale of one or two seconds. An example is shown in Fig 3. This decay cannot be explained by collisional drag alone since an accelerating electric field is induced during the decay which almost balances the drag, thus leading to a very slow net damping. Instead, it appears that the emission of synchrotron radiation plays a key role in damping the runaway electron current. To account for the JET observations, we have developed a kinetic theory to describe the combined effects of Coulomb collisions and synchrotron radiation emission on beams of runaway electrons by including the Abraham-Lorentz radiation reaction force in the relativistic drift kinetic equation [6].

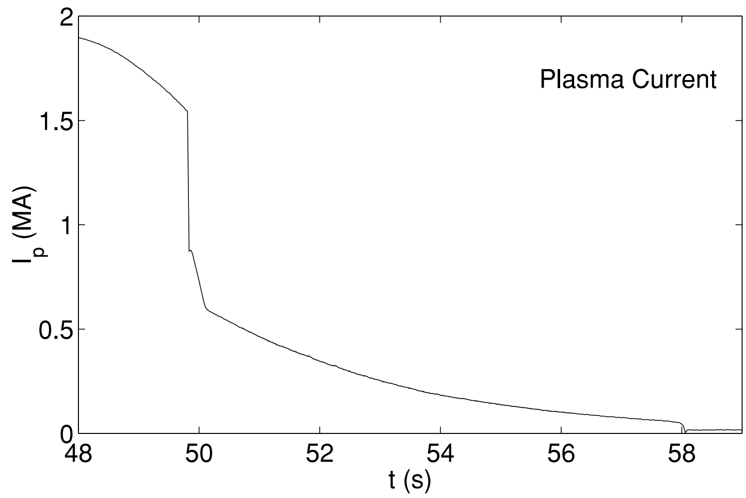


Figure 3: *Post-disruption runaway current in JET (pre-divertor) discharge 14248, where the runaway column remained stable for 8 seconds.*

On a gyro-average, this force causes the normalised relativistic momentum, $p = \gamma v/c$, and pitch-angle variable $\xi = v_{\|}/v$ of a beam electron to change according to

$$\left.\frac{dp}{dt}\right|_{\rm rad} = -\frac{\gamma}{\tau_r p}\left(p_\perp^2 + \frac{\rho_0^2}{R^2}p_\|^4\right), \tag{2}$$

$$\left.\frac{d\xi}{dt}\right|_{\rm rad} = \frac{1}{\tau_r \gamma p^2}\left(p_\perp^2 + \frac{\rho_0^2}{R^2}p_\|^4\right), \tag{3}$$

where $\gamma = (1-v^2/c^2)^{-1/2}$, $\tau_r = 6\pi\epsilon_0(m_e c)^3/e^4B^2$, and $\rho_0 = m_e c/eB$. The terms involving $R$, which denotes the radius of curvature of the magnetic field, are unimportant in JET. The relativistic drift kinetic equation thus becomes

$$\frac{\partial f}{\partial t} - \frac{eE_\|}{m_e c}\left(\xi\frac{\partial f}{\partial p} + \frac{1-\xi^2}{p}\frac{\partial f}{\partial \xi}\right) - cp\frac{\nabla_\| B}{B}\frac{1-\xi^2}{2\gamma}\frac{\partial f}{\partial \xi} - \frac{\gamma p_\perp^2}{\tau_r p}\left(\frac{\partial f}{\partial p} - \frac{1}{\gamma^2 p}\frac{\partial f}{\partial \xi}\right) = C(f), \tag{4}$$

where $E_\|$ is the electric field induced by the decay of the runaway current, the term involving $\nabla_\| B$ represents the mirror force, and the terms containing $\tau_r$ accounts for radiation reaction. The collision operator for relativistic electrons in the cool, post-disruption plasma is [7]

$$C(f) = \frac{1}{\tau}\left[\frac{1}{p^2}\frac{\partial}{\partial p}\left(\gamma^2 f\right) + \frac{\gamma(1+Z_{\rm eff})}{2p^3}\frac{\partial}{\partial \xi}\left(1-\xi^2\right)\frac{\partial f}{\partial \xi}\right], \tag{5}$$

where $\tau = 4\pi\epsilon_0^2 m_e^2 c^3/n_e e^4 \ln\Lambda$ is the collision time.

Unlike collisional friction, the radiation reaction force increases with increasing energy and therefore always becomes important at sufficiently high energies, where it changes the electron dynamics in a qualitative way. Just after the disruption, the runaways move practically parallel to the magnetic field and therefore emit very little synchrotron radiation. However, the velocity vector needs only be scattered slightly by

Coulomb collisions to acquire a Larmor rotation that can lead to substantial emission of synchrotron radiation. In this way, the combined effect of pitch-angle scattering and radiation reaction can damp beam currents more efficiently than does ordinary friction. This occurs in spite of the fact that for highly relativistic electrons, the friction term in the collision operator (5) is larger than the scattering term by a factor $p \gg 1$ because of the relativistic mass increase, which makes pitch-angle scattering a fast electron more difficult. It is important to note that since radiation from a relativistic particle is emitted in a cone centred around its velocity vector, the reaction force is mainly in the direction parallel to the magnetic field (if $|\xi| \simeq 1$), as follows from taking $p \gg 1$ in (3), although it is the perpendicular motion that causes the radiation. The efficiency

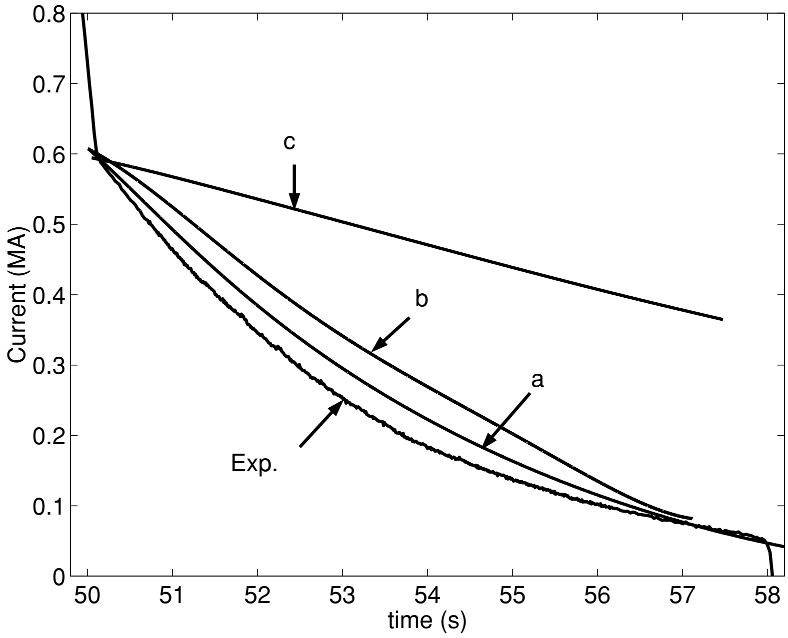


Figure 4: *Post-disruption plasma current in JET discharge No. 14248 and numerical results from the Monte Carlo code ARENA. Curve a shows a simulation where the density just after the disruption is six times higher than the pre-disruption value* ($6n_{e0}$) *and decreases exponentially. Curves b and c show simulations where the density is kept fixed at* $6n_{e0}$ *and* $n_{e0}$*, respectively.*

of this damping mechanism can be estimated in the following way. Because of pitch-angle scattering, an average electron initially moving along the magnetic field acquires a perpendicular momentum $p_\perp$ in the time

$$t_1 = \tau p_\perp^2/(1+Z).$$

According to Eq (2), emission of radiation then causes the particle to slow down on a time scale

$$t_2 = \tau_r p_0/p_\perp^2,$$

if the initial momentum was $p_0$. According to runaway avalanche theory $p_0 = 2 \ln \Lambda$ [8]. Equating $t_1$ and $t_2$ gives the time scale for the combined action of these two processes,

$$t_{\rm rad} = \sqrt{\frac{\tau \tau_r \ln \Lambda}{1+Z}}$$

which is significantly shorter than the slowing-down time

$$t_{\text{fric}} = 2\tau \ln \Lambda.$$

These scalings are verified by the solution of the kinetic equation (4), which has been solved both by analytical approximation and by full Monte Carlo simulation, using the ARENA code [6]. In both cases, the electric field induced by the current decay, entering in the second term of Eq (4), needs to be calculated self-consistently. This makes the problem nonlinear since this electric field both enters into the kinetic equation and is determined by its solution. The calculated current decay agrees with the measurements in JET (Fig 4) if it is assumed that the electron density increases temporarily following the disruption, which is thought to be the case for other other reasons [5].

## 3. Electron-positron pair production

Since the energy of runaway electrons is so large, one expects electron-positron pair production to take place in collisions between the runaways and background plasma particles in post-disruption plasmas. Indeed, the threshold energy for pair production is $3m_ec^2$ in a collision between a fast electron and a stationary ion, and $7m_ec^2$ in a collision with a thermal electron, i.e., far below the average runaway energy. The cross section for positron creation is [9]

$$\sigma_p^s \simeq \frac{28(Z_s\alpha r_e)^2}{27\pi} \ln^3 \gamma_e, \qquad (\gamma_e \gg 1) \tag{6}$$

where $\alpha = e^2/4\pi\epsilon_0\hbar c \simeq 1/137$ is the fine-structure constant, $Z_s$ the charge of the stationary particle (ion or electron, $s = i$ or $e$), $r_e = e^2/4\pi\epsilon_0 m_ec^2$ the classical electron radius, and $\gamma_e = E/m_ec^2$ the Lorentz factor for the fast electron. The number of runaway electrons in a post-disruption plasma with a current $I_r \sim 1$ MA is roughly $N_r = I_r/ec$, so the source strength of positrons can be estimated as

$$S_p = N_r \left(n_e\sigma_p^e + n_i\sigma_p^i\right) c \simeq 2I_r\sigma_p^e n_e/e \simeq 3 \cdot 10^{12}\ s^{-1}$$

for $n_e = 5 \cdot 10^{19}$ m$^{-3}$, and $\gamma_e \simeq 30$. The positron life time is determined by the annihilation cross section [9],

$$\sigma_a = \frac{\pi r_e^2}{1+\gamma}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right], \tag{7}$$

which is sensitive to the positron energy.

Most positrons are born with relativistic energy, and their subsequent fate depends on the magnitude of the parallel electric field. In JT-60U, this field can exceed the critical field for electron runaway long after the disruption [10]. In this case, the positrons will run away and form a beam in the direction opposite to that of the runaway electrons. The life time is then relatively long (several seconds), and the number of positrons can be estimated by multiplying the source strength by the duration of the post-disruption plasma,

$$N_p \sim S_p\tau_{\text{discharge}} \sim 10^{13}.$$

To our knowledge, this number exceeds that in other physics experiments, including particle accelerators and laser plasmas.

In JET, the post-disruption field is probably too low to cause runaway acceleration, and the positrons are instead expected to slow down under the action of radiation reaction and Coulomb collisions. The former is dominant above energies of a few MeV, see Eq (2), while collisions are more important at non-relativistic energies. The positron life time then becomes comparable to the slowing-down time since the positrons are quickly annihilated once they reach thermal energies. At such low energies, the cross section (7) is increased by the the focusing effect of Coulomb interaction, and positrons may also be lost by the formation of positronium [11], through charge-exchange with neutral deuterium atoms. The number of positrons in JET is approximately equal to

$$N_p \sim \frac{S_p}{n_e v_{Te} \sigma_a} \sim 10^{12}.$$

The positrons in a post-disruption plasma could, in principle, be detected either through their annihilation radiation or, if they run away, through their bremsstrahlung. In either case, the detection is complicated by the presence of background bremsstrahlung radiation from the runaway electrons.

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