# Sausage Instabilities in Electron Current Channels and the problem of fast ignition

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**Abstract.** The stability of a current channel to sausage like perturbations at fast electron time scales (where Electron Magnetohydrodynamic model is applicable) is studied. The effect of relativistic electron flow speed in the current channel is taken into account for the linear instability analysis. This instability gets driven due to the shear in flow velocity and is similar to the fluid Kelvin Helmholtz (KH) mode, with subtle differences due to the magnetized character of the electron fluid. For relativistic electron flow velocity the hydrodynamic character of the instability attains predominance, due to effective increase in the electron skin depth by the relativistic variation of the electron mass. This problem has direct relevance to the fast ignition concept of laser fusion where electrons heated on the surface of the target by the laser, move inward towards the core, and the cold plasma electrons provide a return shielding current. This leads to the formation of a sheared current channel whose instability and subsequent nonlinear development can hinder the propagation of fast electrons towards the core influencing the location of the hot spot for ignition. Nonlinear studies to get an insight into these phenomena are made by a 2-d EMHD fluid simulation, which shows growth of the instability and the subsequent saturation as well as relaxation of the equilibrium shear profile.

## 1 Introduction

In the fast ignition concept of laser fusion, an intense picosecond laser pulse is incident on an overdense precompressed pellet with the objective of initiating a hot spot in its core. The laser pulse energy is absorbed by nonlinear mechanisms near the pellet surface and gets converted into inward propagating fast electron currents with current densities of order $10^{12} Amps/sq.cm.$ The plasma responds to these currents by generating shielding cold plasma return currents which interact with the incoming currents through collective effects. Three dimensional PIC simulations [?] have shown that intense Weibel, tearing and coalescence instabilities take place which organize the current into a few current filaments. In each of these filaments the central core region constitutes a current due to the fast electrons propagating towards the pellet core, while the outer cylindrical shell region carries the return shielding current due to the cold electrons. The stability of these current channels is thus a topic of great interest to fast ignition concept of laser fusion. We show that such a sheared current profile is susceptible to 2-d sausage like instabilities. We employ electron magnetohydrodynamic (EMHD) [2, 3] model to uncover the fastest growing modes. We present here a simplified treatment in which the incoming fast and outgoing cold electron fluids are treated identically. A two fluid description for hot and cold electrons will be presented in a detailed paper elsewhere. The linear stability calculation is carried out analytically and numerically for step and tangent hyperbolic velocity profiles respectively, and is presented in the next section. Section III contains the nonlinear studies on the saturation and the reaction of the interacting unstable modes back on equilibrium velocity profiles. This relaxation flattens the profile implying a reduction in the current moving in either direction. An estimate of the stopping length of inward moving fast electrons is made on its basis. The relevance of these studies to the problem of fast ignitor concept of laser fusion is also outlined.

## 2 Linear Studies

We study the dynamics of the sheared current channel using Electron Magnetohydrodynamics (EMHD) model, which describes fast electron time scale phenomena at which the ion motion can be ignored [2, 3]. We use slab representation for the current channel in which the radial direction of current shear is taken as $\hat{x}$, the poloidal direction as $\hat{y}$ and the current flow direction as $\hat{z}$. The sausage like perturbations have variations along $x$ and $z$ directions only and are independent of the $y$ coordinate. For this case it can be shown that only the $\hat{y}$ component of magnetic field is relevant; the other components can be taken as zero, in equilibrium as well as for perturbations. Linearizing the EMHD evolution equation about the equilibrium electron flow velocity $v_0(x)$, and fourier analyzing in time and $z$ we obtain

$$\bar{\omega}\left\{\gamma_0^3\frac{d^2B_y}{dx^2}+3\gamma_0^2\frac{d\gamma_0}{dx}\frac{dB_y}{dx}-(1+\gamma_0k_z^2)B_y\right\}=k_z\left\{v_0-\frac{d^2}{dx^2}(\gamma_0v_0)\right\}B_y \tag{1}$$

Here $\bar{\omega}=\omega-k_zv_0$ is the Doppler shifted frequency. We have chosen to normalize lengths by the electron skin depth $d_e=c/\omega_{pe}$, magnetic field by some typical value $B_{00}$ and time by the inverse of the corresponding electron gyrofrequency $\omega_{ce}=eB_{00}/mc$. It can be shown by integration that the following two functions $f_1=\gamma_0^3\bar{\omega}dB_y/dx+k_zd(\gamma_0v_0)/dxB_y$ and $f_2=B_y/\bar{\omega}$ ought to be continuous at the point of discontinuity in the equilibrium velocity profile. For analytical tractibility, a step profile in $x$ for $v_0$ is chosen with region I as $-\infty<x<0$ where $v_0=-V_0$ and region II as $0<x<\infty$ where $v_0=V_0$ . The solutions in two regions are then $B_{yI}=A\exp(\alpha_1x)$ and $B_{yII}=C\exp(-\alpha_2x)$ respectively. Here $\alpha_1^2=(1/\gamma_0^3)\{1+\gamma_0k_z^2-(k_zV_0)/(\omega+k_zV_0)\}$; $\alpha_2^2=(1/\gamma_0^3)\{1+\gamma_0k_z^2+(k_zV_0)/(\omega-k_zV_0)\}$. Using the matching conditions for $f_1$ and $f_2$ we obtain the following dispersion relation

$$\omega^2=-\{k_z^2V_0^2(1+4\gamma_0k_z^2)\}/\{3+4\gamma_0k_z^2\} \tag{2}$$

The nonrelativistic limit can be obtained by putting $\gamma_0=1$. For $\gamma_0k_z^2>>1$ we recover the fluid like Kelvin Helmholtz (KH) growth rate $\gamma=k_zV_0$. In the opposite limit we obtain a reduced growth rate of $\gamma=k_zV_0/\sqrt{3}$. This difference is due to the magnetized character of the electron fluid. As $\gamma_0$ increases, it becomes easier for longer scales to satisfy the conditions for hydrodynamic KH like excitations. In Fig.1 we plot the growth rate for a tangent hyperbolic equilibrium velocity profile $v(x)=V_0tanh(x/\epsilon)$ as obtained from the numerical eigenmode evaluation for $\gamma_0=1.67$ ( relativistic case) and $\gamma_0=1.0$ (non relativistic case). This figure shows a limiting value of $k_z\epsilon$ beyond which the growth rate vanishes. This limit is less for the relativistic case.

## 3 Nonlinear Studies and Discussion

As the amplitude of unstable modes acquires appreciable level, nonlinear terms in the evolution equation start playing a role and permit interaction amidst a variety of excitation

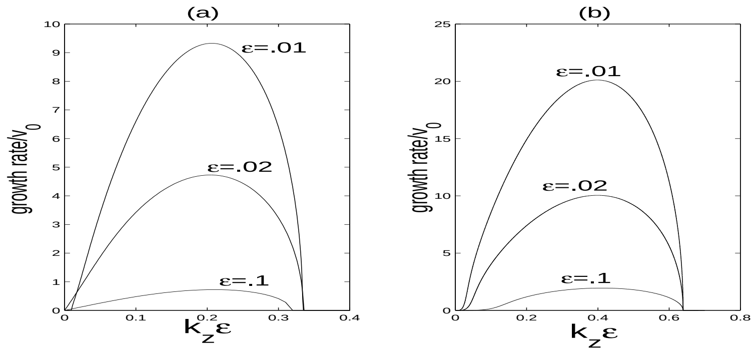


*FIG.1. Plot of growth rate $\gamma$ vs. $k_z\epsilon$ for (a) $\gamma_0 = 1.67$ and (b) $\gamma_0 = 1.0$*

scales. The nonlinear studies are carried out with the help of 2-d fluid simulation. These studies are presently done only for the non relativistic case. The modification of the equilibrium velocity profile due to nonlinear effects can be tracked by plotting the $z$ independent electron flow velocity defined by $\bar{V}(x)\hat{z} = 1/2L_z \int_{-L_z}^{L_z} \vec{v}_{ez} dz$ as a function of $x$ with time shown in Fig.2 (left side plot). At $t = 0$ $\bar{V} = V_{eq} = V_0 \tanh(x/\epsilon)$ and has

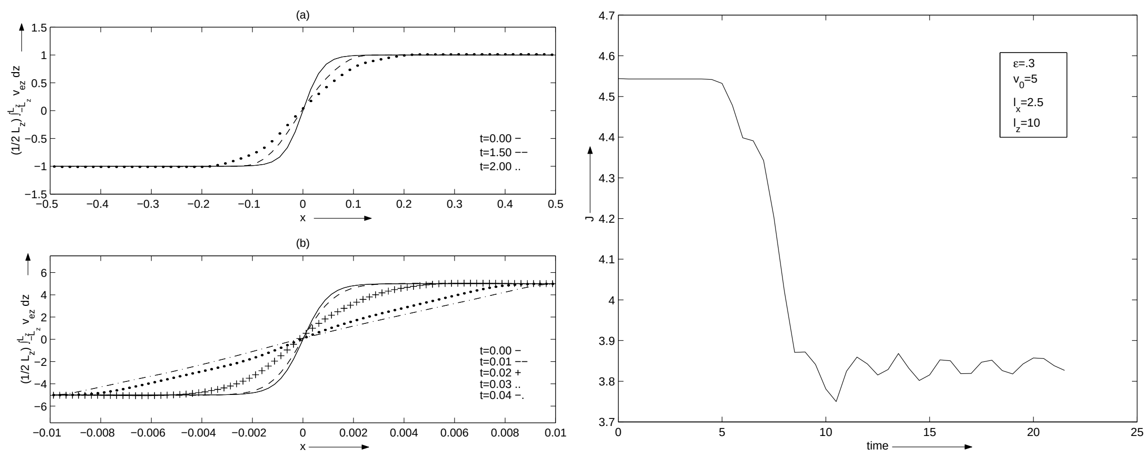


*FIG.2. The plot on the left side illustrates the modifications of the equilibrium (z independent) velocity flow profile with time. Parameters for subplot (a) $\epsilon = 0.0384$, $V_0 = 1$, $2L_x = 2L_z = 1.0$ and (b) $\epsilon = 0.001$, $V_0 = 5$, $2L_x = 0.02$ , $2L_z = 1.0$. The right hand side plot shows the reduction of $J = (\int_0^{L_x} \int_{-L_z}^{L_z} v_{ez} dz dx)/(2L_x L_z)$ with time. Here $\epsilon = .3$, $V_0 = 5$, $2L_x = 5.0$ and $2L_z = 20$.*

the specified tangent hyperbolic form in $x$. At later times the profile flattens and the instability is also seen to saturate. Basically, the flattening of the profile modifies the effective value of the shear scale length such that $\epsilon_{eff} > \epsilon$. It thus becomes increasingly difficult to satisfy the condition of $k_z\epsilon_{eff} < 0.693$ even for the longest mode $k_z = k_{zl}$ in the box, for instability. This halts the linear growth of the modes. A repetition of the simulation by allowing a self consistent evolution of the boundary points also yields similar adjustment of the shear structure. The fact that the overall magnitude of the velocity (e.g. the values at the edges) remains largely unaltered suggests that the evolution of $\bar{V}$ occurring via nonlinear scale interactions can be mocked up by an effective viscous or even higher order derivative dissipation.

The modification in the profile can have crucial implications for the problem of fast ignition in which one is interested in knowing the evolution of inward moving fast electrons. The right hand side plot of Fig.2 shows the current evolution due to inward moving electrons.

This is done by plotting $J = 1/L_x \int_0^{L_x} \bar{V} dx$, (the half space integration over $x$) with time. It is noted that $J$ is significantly reduced but does not go to zero. Thus, although there is collective stopping of the electron flow, it does not lead to complete stopping. We ascribe the incompleteness of the stopping to two dimensionality of our simulations. In 2-d, the tendency for the power is to accumulate at long scales (limited by simulation box sizes) leading to artificial coherence effects and lack of turbulence as the longest scale hits the box. We speculate that the initial development of the velocity profile in $2-D$ simulations thus gives us a reasonable indication of the overall collective stopping due to the development of EMHD turbulence; this can however best be pinned down only in actual three dimensional EMHD simulations where turbulence would have an opportunity to fully develop.
We now make an estimate of the expected stopping length due to collective effects. From Fig.2, the typical shear width is a fraction of electron skin depth (i.e. $\epsilon = 0.3$ in units of $c/\omega_{pe}$) we observe a reduction in normalized $J$ of $\sim \Delta J = 0.7$ in a time duration $\Delta t = 10$, i.e. a collective deceleration of fast electrons from mean velocity $J_i$ to $J_f$ (4.5 to 3.8) at a rate $a = \mid J_f - J_i \mid / \Delta t = 0.07$. For the parameters of the PIC simulation [1] (where plasma density is $4 \times 10^{21}/cm^3$ and the electron skin depth $c/\omega_{pe} \sim 0.1 \mu m$), this corresponds to kinetic energy reduction by 30% in a decorelation length of order $S = (J_f^2 - J_i^2)/2a = 83$ (i.e. 8.3 microns for simulation parameters). This is in reasonable agreement with the $3D$ PIC simulation result of collective stopping in [1, 4] and is also consistent with a model of collective stopping due to EMHD turbulence through a quasilinear friction term [4, 5] of order $\nu_{eff}/\omega_c \sim \delta B/B_0 \sim 10^{-2}$ which can be obtained as follows. We write the components of the electron momentum equations as $\partial \bar{V}/\partial t + \nu_{eff} \bar{V} = e\vec{E}/m + (e/mc) < \delta \vec{V} \times \delta \vec{B} >$ and $\partial \delta \vec{V}/\partial t + \nu_{eff} \delta \vec{V} = (e/mc) < \bar{V} \times \delta \vec{B} >$. Looking for the case with $\nu_{eff} >> \partial/\partial t$, we find $\nu_{eff} \sim (e/mc) \mid \delta B \mid$ as observed in the PIC simulations. If we *extrapolate* and *assume* that in actual fast ignition experiments such collective stopping effects continue to operate till complete stopping, we get a stopping length $\sim$ tens of $\mu m$, a result which is very reasonable and acceptable. It is therefore important to carry out large scale $3-D$ fluid EMHD and PIC simulations to further confirm and quantify these collective effects.

# References


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