Integrated Plasma and Coil Optimization for Compact Stellarators

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Abstract. The integration of the COILOPT model, based on explicit representations for modular coils and coil geometry constraints, into the stellarator optimization package STELLOPT, provides a unique and important computational tool for the design of compact stellarators. This self-consistent analysis ensures that physics and engineering criteria are simultaneously satisfied. The analysis to date has been based a local minimization method, a parallel version of the Levenberg-Marquardt algorithm, and is successful because it is implemented after the separate plasma and coil optimizations have identified a good starting point. The merged optimization technique has led to highly optimized designs for the quasi-axisymmetric National Compact Stellarator Experiment and the Quasi-poloidal Stellarator.

1. Introduction

Compact stellarators are toroidal confinement devices with low aspect ratio $2 < A < 5$ ($A = \langle R\rangle/\langle a\rangle$), a small number of toroidal field periods ($2 \le N_p \le 4$), and with bootstrap current producing a small fraction of the magnetic rotational transform. They have been developed to combine the advantages of stellarators (in particular, steady-state operation and the avoidance of disruptions) and tokamaks (e.g., good particle and energy confinement at high beta) in an efficient and cost effective plasma configuration. Specific examples are the National Compact Stellarator Experiment (NCSX), a proof-of-principle compact stellarator with $A = 4.4$, $N_p = 3$, and the $A = 2.7$, two-field period Quasi-poloidal Stellarator (QPS), a concept exploration device.

The stellarator optimization code STELLOPT [1] has been used in recent NCSX and QPS design studies to determine the shape of the outer magnetic flux surface, together with internal plasma pressure and current profiles, that approximate a prescribed rotational transform, limit particle drift trajectories, and lead to attractive plasma stability and confinement properties at significant beta. Some coil-related figures of merit (e.g., measures of coil complexity, curvature, and current density) are included in the optimization, using the NESCOIL [2] model, in order to guide the plasma configuration toward a region in parameter space that can be accessed in the subsequent coil design. In a separate step in the design process, the coil optimization code COILOPT [3] solves for the coil geometry and current distribution that will best approximate the physics solution in a free-boundary MHD equilibrium calculation. Both are computationally intensive optimization problems, and even after several iterations of the two-step process, it is often difficult to produce a coil design that meets practical engineering design standards.

In this paper we describe the results of a new method that includes both the plasma and coil models in a combined optimization to ensure that physics and engineering goals are simultaneously satisfied. The self-consistent analysis is successful largely because it is implemented only after the separate optimization studies have identified a good candidate for the design point, thus providing an initial “guess” for the local minimization that includes both plasma and coil models.

2. Intergated Optimization of Plasma and Coils

The compact stellarator coil system, consisting of modular coils to provide the helical field, together with toroidal field (TF) coils and vertical field (VF) coils for configuration flexibility, is designed to minimize the normal component of the magnetic field (which may include a component due to a net plasma current [4]) relative to a reference plasma surface. The coils are subject to engineering constraints such as minimum coil separation and minimum radius of curvature. The reference plasma shape is the result of a separate optimization targeting plasma stability and confinement at reference values of beta, consistent with a fixed-boundary MHD equilibrium obtained using VMEC [5]. A solution to the coil design problem is a feasible point with respect to engineering constraints that will accurately reconstruct the reference plasma boundary shape in a free-boundary VMEC MHD equilibrium calculation where, given the same plasma current and pressure profiles, the plasma shape is self-consistent with the external field due to the coils.

In the STELLOPT code, the stellarator design optimization problem is formulated as a least-squares problem to minimize $\chi^2 = \Sigma\ \chi_i(\mathbf{x})^2$, where the components, χ_i, are (generally nonlinear) functions of the system parameters **x**. The independent variables **x** include coefficients describing the MHD plasma equilibrium pressure and current profiles, as well as 1) Fourier coefficients of the plasma shape, in the case of a fixed-boundary optimization, or 2) coil currents, if the optimization is to be executed in free-boundary mode. The functions χ_i are stellarator physics figures-of-merit and engineering constraints which are evaluated numerically using a set of complex plasma physics and engineering models that depend on the solution of a 3D plasma MHD equilibrium. STELLOPT uses the Levenberg-Marquardt (LM) method to solve the nonlinear least-squares problem. Subroutines interface each physics and engineering model with the optimization code, and several models (e.g., NEO [6], COBRA [7], NESCOIL) are evaluated as executables through system calls from these subroutines.

The COILOPT code is based on a parametric representation of stellarator coils confined to a coil-winding surface (CWS). The winding law for modular coils on this surface is given either by Fourier series, or a cubic spline representation $u(s) = \Sigma_j u_j B_j(s)$, $v(s) = \Sigma_j v_j B_j(s)$. In the spline representation, the basis functions $B_j(s) = B_j(s;t_j,\ldots,t_{j+4})$ are normalized cubic B-splines defined on the interval [0,1] with a prescribed set of N+4 non-decreasing knots $t_j \in [0,1]$. The N pairs of coefficients (u_j,v_j) are referred to as “control points”, and are constrained to satisfy the appropriate periodic end conditions. The winding law coefficients, CWS coefficients, and the coil currents, are all possible independent variables in the coil optimization problem. The spline representation allows the possibility of local changes in coil geometry, since a cubic B-spline $B_j(s)$ is non-zero only over the interval $t_j \leq s \leq t_{j+4}$. COILOPT also uses the LM method to solve the nonlinear optimization, where components of the objective function include residuals in the normal component of the magnetic field on the targeted plasma surface, together with engineering constraints on coil geometry.

In the integrated plasma/coil optimization the elements of the vector **x** consisting of Fourier coefficients of the plasma boundary in the STELLOPT calculation are replaced by the independent variables of the coil optimization problem. In addition, COILOPT is executed from STELLOPT in a mode where only the functions χ_i (**x**) for coil geometry constraints are evaluated. Here VMEC is executed in the free-boundary mode. A consistent solution is achieved by targeting the physics parameters of the reference plasma, and the geometric properties necessary for engineering coil design, while allowing the plasma boundary shape to vary from the original reference.

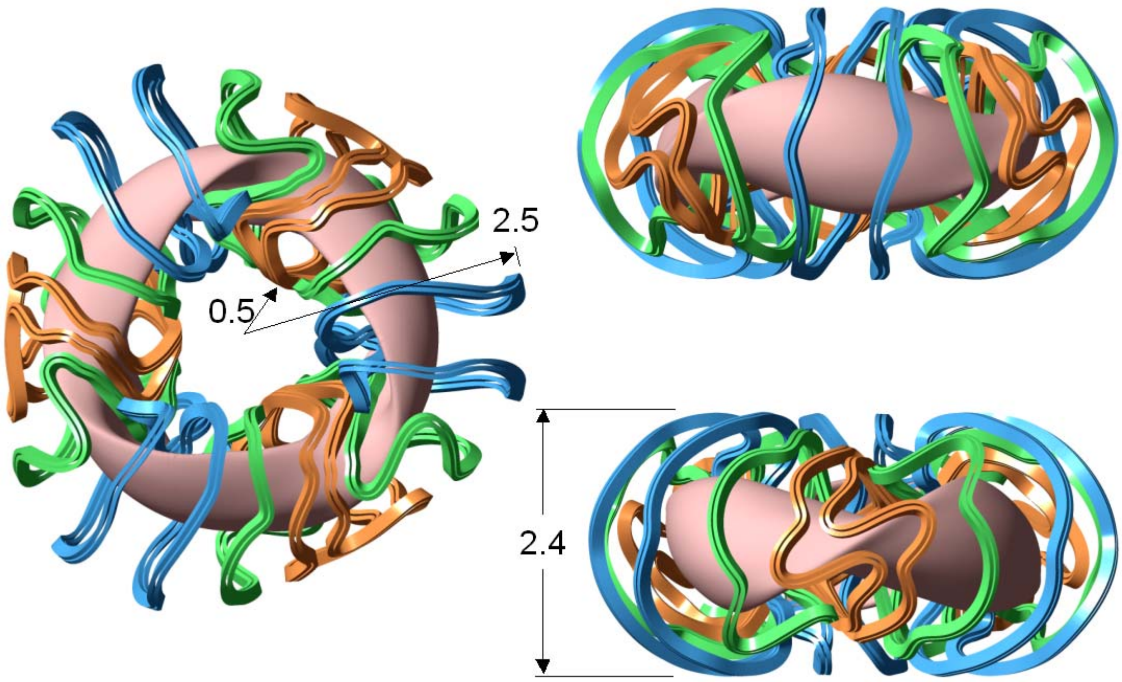

FIG. 1. The NCSX modular coils consist of three distinct coil types.

3. Configuration Optimization of the NCSX

The goal of the NCSX is to explore the physics of three-dimensional plasma shaping at high beta with rotational transform due to both internal and external currents, and to study the effect of quasi-axisymmetry on plasma confinement. The NCSX is a proof-of-principle device with major radius $\langle R \rangle = 1.4$m, magnetic field $\langle B \rangle = 1.7$T, and $\langle \beta \rangle = 4\%$. The NCSX magnetic coil design consists of 18 modular coils (3 coil types, Fig. 1) and 18 toroidal field (TF) coils, together with six pairs of poloidal field (PF) coils, and trim coils. In addition to the usual coil geometry constraints, the NCSX coil design targets include constraints on the winding surface imposed by an internal vacuum vessel, and access requirements for neutral beam injection. Coil optimization studies have considered: (1) the number of modular coils per field period, (2) the position of the coils relative to the plasma symmetry planes, and (3) the role of TF and PF coils in the modular coil optimization and plasma flexibility analysis. Representative configurations from the sequence of numerical optimizations leading to design M45 for NCSX are summarized in Table 1. The LI383 plasma configuration was the result of an extensive set of fixed-boundary STELLOPT calculations, and provided an initial target for COILOPT investigations. Experience showed that to obtain levels of confinement and stability comparable to the LI383 target required an approximation of the normal magnetic field at the plasma boundary ∂S in the coil optimization with an average error $\langle \delta B \rangle = (1/A)\int_{\partial S} |\mathbf{B \bullet n}| / |\mathbf{B}|\, dA \leq 0.6\%$. Coil designs with 21 (M12) and 18 (M25) modular coils were found with COILOPT that adequately reconstructed the plasma properties of LI383 in free-boundary STELLOPT studies, including, e.g., marginally stable beta (for $\langle \beta \rangle = 4\%$), effective ripple factor ($\varepsilon_{eff}^{\ 3/2}$), and neutral beam energy loss fraction (f_{NB}), but did not fully meet engineering feasibility requirements. It was not until the coil engineering targets in COILOPT were combined with the plasma optimization in STELLOPT that a family of consistent solutions M45 (Fig. 1) were found. The LI383 plasma configuration and the M25 coils provided a starting point for the M45 combined optimization. The final step in the NCSX physics design was to make corrections in the modular coil geometry to reduce the width of targeted magnetic islands. Hudson. et. al, describe an algorithm [8] using the PIES [9] code to make small adjustments in the modular coils, also subject to COILOPT constraints on coil geometry, in order to recover good magnetic surface quality.

TABLE I: DESIGN OPTIMIZATION OF THE NCSX.

Configuration	LI383	M12	M25	M45
A	4.36	4.36	4.34	4.37
$<\beta>$ (%)	4.19	4.09	4.05	4.08
$\varepsilon_{eff}^{3/2}$ ($\times 10^4$), center	0.24	0.32	0.41	0.96
f_{NB} (%)	14.5	14.2	13.5	15.5
Δ_{min}, coil-coil (cm)		12.8	11.9	16.0
Δ_{min}, coil-plasma (cm)		17.5	19.9	18.7
Δ_{min}, NBI access (cm)			30.9	37.4
Min. bend radius (cm)		7.7	9.4	10.5

4. Design Optimization of the QPS

A parallel effort uses the STELLOPT, COILOPT, and merged optimization codes to design the QPS, a concept exploration device to investigate the effects of three-dimensional shaping and quasi-poloidal symmetry ($\partial|\mathbf{B}|/\partial\theta \approx 0$, where θ is a poloidal angle) on neoclassical confinement at moderate beta. The QPS plasma has $N_p = 2$, aspect ratio A = 2.7, average major radius $\langle R\rangle = 0.9$m, magnetic field B = 1T, and target $<\beta> = 1.8\%$. The QPS coil design (Fig. 2) consists of 16 modular coils (4 coil types, where the winding packs of the pair of coils near the center of the long section in Fig. 2 follow independent paths), 12 TF coils, and 2 pairs of circular VF coils. In addition to the usual coil engineering constraints, modular coil optimization targets QPS specific constraints, such as space in the center of the device for the TF legs and solenoid coils.

Results are summarized in Table 2. Based on significantly improved confinement properties with respect to the original GB4 solution, as well as plasma flexibility considerations (e.g., vacuum field properties), configuration 0411 was chosen as an initial design point for an integrated plasma and coil optimization with STELLOPT to produce the QPS reference configuration 0718. Configuration 0411 was a result of the separate plasma and coil design optimizations, and provided an adequate reconstruction of the targeted fixed-boundary plasma shape and plasma properties. While coil geometry metrics were also acceptable, the coil design required a large number of control points (28) and CWS coefficients (41), resulting in a very complex winding surface shape. In this case the merged optimization was an attempt to obtain a solution with comparable plasma properties using a smaller number of free parameters in the coil winding law and CWS (leading to less complex coil manufacturing properties), while allowing the equilibrium plasma shape to change in a self-consistent manner. Solution 0718 achieved this goal with 15 control points and 23 CWS coefficients.

TABLE II: DESIGN OPTIMIZATION OF QPS

Configuration	GB4	0411	0718
A	2.62	2.71	2.67
<β> (%)	1.88	1.78	1.83
$\varepsilon_{eff}^{3/2}$ ($\times 10^3$), center	33.1	4.3	2.4
ι (axis)	0.27	0.29	0.28
ι (edge)	0.40	0.38	0.34
Δ_{min}, coil-coil (cm)	11.4	10.1	9.6
Δ_{min}, coil-plasma (cm)	13.4	14.6	13.0
Min. bend radius (cm)	7.2	10.1	9.3

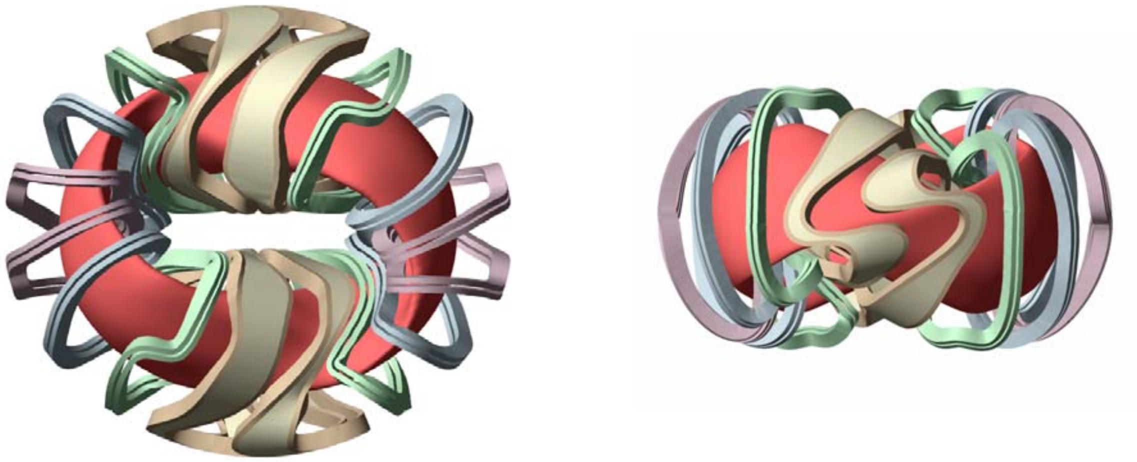

FIG. 1. QPS plasma boundary and modular coils for configuration 0718.

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