A Tokamak with Nearly Uniform Coil Stress Based on Virial Theorem

H. Tsutsui, K. Nakayama, T. Ito, H. Ajikawa, S. Nomura, S. Tsuji-Iio, R. Shimada

Research Laboratory for Nuclear Reactors, Tokyo Institute of Technology, Tokyo, Japan

e-mail contact of main author: htsutsui@nr.titech.ac.jp

Abstract. A novel tokamak concept with a new type of toroidal field (TF) coils and a central solenoid (CS) whose stress is much reduced to a theoretical limit determined by the virial theorem has been devised, and a new small tokamak with the concept was constructed. According to the virial theorem, the best TF coil to produce the strongest magnetic field under the weakest stress requires equal averaged principal stresses in all directions. Applying this condition to a helical coil, its pitch number is determined as a function of the aspect ratio. The helical winding with the condition is modulated in such a way that poloidal field exists only outside of torus, which reduces the torsional force on the helical coil and makes plasma breakdown possible. Moreover, a helical coil with this modulation and a low aspect ratio is similar to CS and TF coil systems in conventional tokamaks, because its helical winding is nearly vertical in the outer side of torus. In the case of an aspect ratio $A = 2$, our optimal coil theoretically reduces the working stress in the coil to about one third smaller than those of conventional TF coils.

1 Introduction

Strong magnetic field is required to make a compact magnetically confined fusion reactor. In order to make a compact tokamak, we had recently developed a tokamak with force-balanced coils (FBCs) [1]-[6] which are multi-pole helical hybrid coils combining toroidal field (TF) coils and a central solenoid (CS) coil. The combination reduces the net electromagnetic force in the direction of major radius [1, 2] by canceling the centering force due to the TF coil current and the hoop force due to the CS coil current. This excellent feature of FBC and its applicability to a tokamak device were demonstrated by the first FBC tokamak "Todoroki-I" [3]-[5]. An operation scenario [6] and a capability of FBC tokamak reactors [7] were also investigated, while three problems came up as follows: 1) Working stress in coils has not yet been investigated while the reduction in the net electromagnetic force was demonstrated. 2) Poloidal fields to control plasma might break the force-balanced condition. 3) Expected values of some parameters were not achieved owing to poor plasma control due to error fields generated by FBC. In this work, we extend the concepts of FBC using the virial theorem[8, 9] in the next section. In Section 3, a small tokamak based on the virial theorem is designed, and its initial experiments are shown in Section 4. In Section 5, we summarize our results.

2 Virial Theorem

First of all, we have extended the FBC concept using the virial theorem [10, 11],

$$< \sum_i \tilde{\sigma}_i >= 1, \quad \tilde{\sigma} \equiv \frac{V_\Omega}{U_\mathrm{M}}\sigma, \ < \sigma >\equiv \frac{\int \sigma \mathrm{d}V}{V_\Omega}, \tag{1}$$

where $\sigma_i (i = 1, 2, 3)$ are the principal stresses in coils and their supporting structure Ω, U_M is magnetic energy, and V_Ω is the volume of Ω. This theorem shows that the magnetic field strength is restricted by working stress in Ω, and high-field coils should accordingly have the same averaged principal stresses in all directions, whereas conventional FBC reduces stress in the toroidal direction only [10].

Using a shell model in which helical coils and their supporting structures are assumed to be a uniform and axisymmetric toroidal shell, we have obtained the poloidal rotation number N of helical coils which satisfy the uniform stress condition [10], and named the coil as a virial-limit coil (VLC). VLC with a circular cross section of aspect ratio $A = 2$ reduces the maximum stress to 60% compared with that of TF coils [10], and VLC with a non-circular cross section also reduces working stress. Figure 1 shows poloidal distributions of principal stresses in coils with $A = 2$. Here we use the stress $\hat{\sigma}$,

$$\hat{\sigma} \equiv \frac{V_\Omega}{U_{\mathrm{TF}}}\sigma, \tag{2}$$

normalized by toroidal magnetic energy U_{TF} instead of the total magnetic energy U_{M} in order to compare the toroidal magnetic field strength B_ϕ. The maximum working stress in a coil with $N = 2$, elongation $\kappa = 1.5$ is reduced to one third of that in TF coils. In other words, the amount of supporting structure for CS and TF coils is reduced to less than about one third.

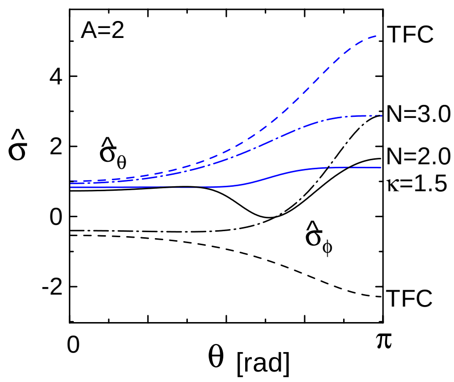

FIG. 1. Poloidal distribution of principal stress $\hat{\sigma}$ normalized by toroidal magnetic field energy. Dashed, dot-dashed and solid lines are in the cases of conventional TF coils with circular cross section and VLCs with poloidal rotation number $N = 3$ with circular cross section and $N = 2$ with $\kappa = 1.5$, respectively.

Since actual helical coils do not satisfy the assumptions in the shell model, we have numerically investigated the distribution of stress in one-dimensional cable with flexural rigidity like a cable-in-conduit (CIC) cable. Here we use a uniaxial model in which equilibrium of forces acting in every infinitesimal small region of the cable is satisfied.

$$\begin{aligned} \frac{\mathrm{d}T}{\mathrm{d}s} + \frac{F_u}{R_\mathrm{c}} &= 0, && (3)\\ \frac{\mathrm{d}F_u}{\mathrm{d}s} + \frac{T}{R_\mathrm{c}} + f_u &= 0, && (4)\\ \frac{\mathrm{d}F_v}{\mathrm{d}s} + f_v &= 0, && (5) \end{aligned}$$

where T, F and f are tension, sharing force and electromagnetic force, respectively. R_c is the radius of curvature, and coordinates s, u, v are determined by directions of the coil orbit, its curvature and the other orthogonal direction. The poloidal distributions of tension and bending moment in coils with circular cross section are depicted in Fig. 2, where VLC ($N = 3$) also realizes nearly uniform distribution of stress in the CIC configuration. Next, we calculate the von Mises stress, which is an index for breaking strength of materials based on a distortional energy theory, by means of a finite element method. The results are depicted in Fig. 3, where orbits of normal helical coils are θ(poloidal angle) $= N\phi$(toroidal angle), while those of VLC and FBC are modified not to create poloidal field in the torus. Figure 3 also shows that VLC has the minimum stress and achieves a nearly uniform distribution of stress.

Since the analysis in this work is basically non-dimensional and independent of size and magnetic field strength, the VLC concept and its advantage are applicable to fusion reactors.

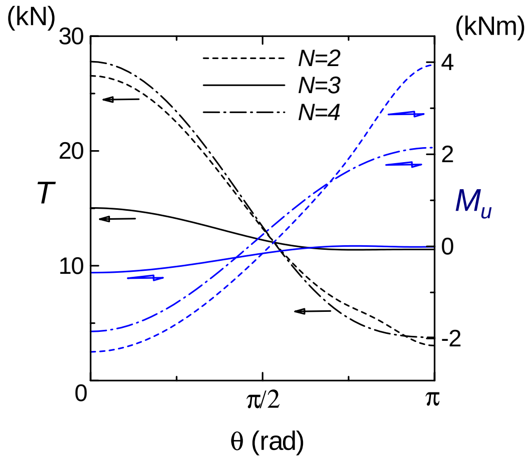

FIG. 2. Poloidal distributions of tension T and bending moment M_u in coils with circular cross section, $R = 0.30$ m, $a = 0.14$ m, $B_\phi = 1.55$ T at $r = R$. A solid line of the winding pitch $N = 3$ corresponds to VLC, while a dash-doted line of $N = 4$ corresponds to conventional FBC.

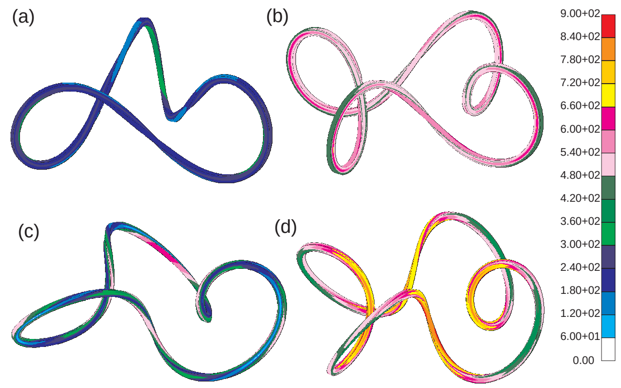

FIG. 3. Distributions of the von Mises stress in VLC ($N = 3$) (a), FBC ($N = 4$) (b), and normal helical coils with $N = 3$ (c), $N = 4$ (d).

TABLE I Size and operational parameters.

Parameters	Todoroki-I	Todoroki-II
Major radius	0.297 m	0.300 m
Minor radius of FBC/VLC	0.115 m	0.140 m
Winding pitch of FBC/VLC	5	3
Pole number of FBC/VLC	8	8
Plasma minor radius	0.055 m	0.070 m
Maximum toroidal field at axis	0.90 T	1.55 T
Maximum plasma current	10 kA	40 kA
Stray field level B_z/B_ϕ	1×10^{-2}	1×10^{-3}

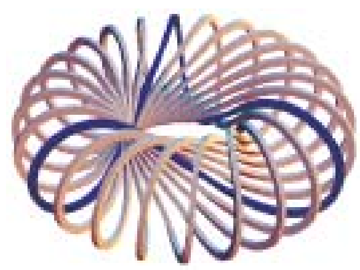

FIG. 4. VLC winding of Todoroki-II.

3 Design of a VLC tokamak

In order to prove the advantage of the VLC concept, we have designed and made a small VLC tokamak “Todoroki-II” whose parameters are listed in Table I, where the achieved values of Todoroki-I are also presented. The virial-limit winding of Todoroki-II is illustrated in Fig. 4, where a VLC is shown by darker hatch. Because the power supply of Todoroki-I and the same coil conductors are used, the magnetic field strength, plasma current and shape of cross section are restricted. Its winding-frame boards are also made of the same GFRP as Todoroki-I.

In the previous section, the nearly uniform distribution of stress in VLC was achieved without poloidal field (PF) coils and plasma. Since VLC is only a hybrid coil of CS and TF coils, PF coils are required in a tokamak system though the virial-limit condition might be broken by them. In order to evaluate the influence of them, distributions of electromagnetic forces were calculated in Fig. 5 where PF coil currents and the plasma current are determined to satisfy an equilibrium condition. This figure shows that their influence was small enough to conserve the validity of VLC.

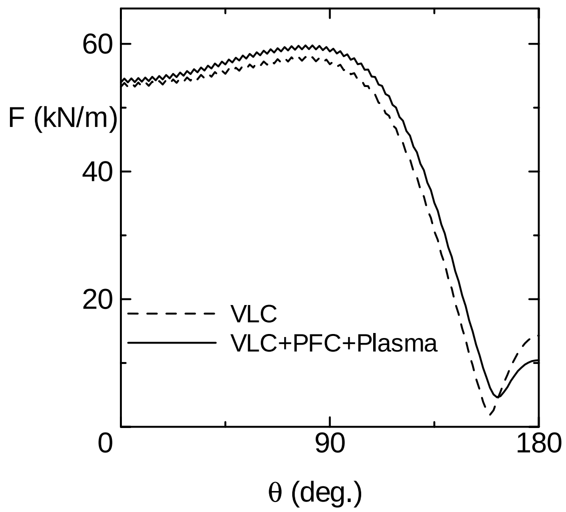

FIG. 5. Poloidal distributions of electromagnetic force acting on VLC with circular cross section, $R = 0.30$ m, $a = 0.14$ m, $B_\phi = 1.55$ T at $r = R$. Forces due to only VLC and both VLC, PF coil and plasma are represented by dashed and solid lines, respectively.

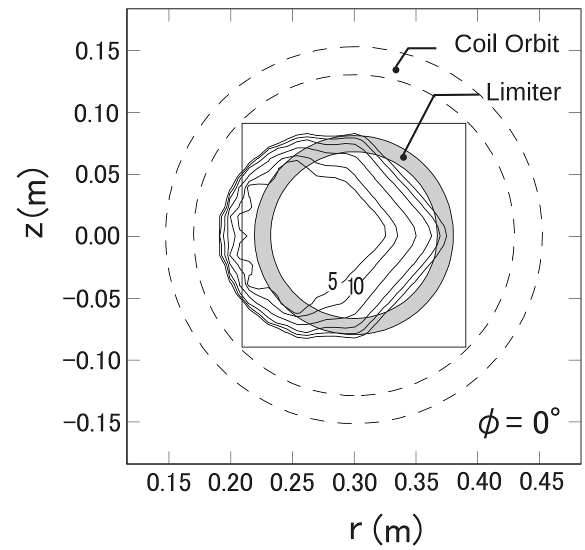

FIG. 6. Contour plot the poloidal field strength created by the helical coil with $R = 0.30$ m, $a = 0.14$ m, $B_\phi = 1.55$ T at $r = R$. The difference of the contour level is 5 G.

The other important problem of FBC tokamak was stray magnetic fields due to the helical winding. Since strong vertical field was generated in the previous FBC tokamak Todoroki-I, an auxiliary coil set to cancel the vertical field and an electrode for pre-ionization were required for plasma production. To minimize the stray fields, we have modulated the helical winding in such a way that the poloidal field is generated only out of the torus [4]. In Todoroki-II, the level of the stray field B_z/B_ϕ is less than 10^{-3} which is 1/10 of that in Todoroki-I, and is small enough to cause the breakdown. This modulation also reduces the torsional force acting on the coils, and makes the winding directions to be nearly vertical and more horizontal in the outer and the inner sides of torus, respectively, as shown in Fig. 4.

This winding is convenient to make room for ports. Because the winding directions were found to become more vertical and horizontal with increasing elongation and decreasing aspect ratio, the configuration of VLC with high elongation and low aspect ratios is similar to that of CS and TF coil systems in conventional tokamaks. Therefore, a VLC tokamak reactor can afford more room for blanket and other parts in conventional tokamak reactors with much reduced volume of coils and their supporting structure.

4 Operation of a VLC tokamak

In a usual tokamak, CS is used to initiate and ramp up the plasma current under nearly constant toroidal magnetic field, while a VLC tokamak cannot keep the toroidal field in the ramp-up phase since VLC is a hybrid coil of CS and TF coils. In the tokamak operation with the hybrid coil, the rise of the toroidal field synchronizes with the ramp up of plasma current. Because plasma initiation requires toroidal field, the VLC tokamak uses a two-step discharge [5], which was demonstrated in Todoroki-I. In a two-step discharge, toroidal field is created in the first ramp, and the loop voltage is utilized in the second ramp. The loop voltage of Todoroki-II was 50 V shown in Fig. 7, and is large enough to initiate plasma in a stray field level of $B_z/B_\phi \sim 10^{-3}$ shown in Fig. 6.

5 Summary and Conclusions

In order to prove the advantage of the VLC concept, we have designed a small VLC tokamak Todoroki-II, and its experiments started.

As has been shown in this work, the problems in Todoroki-I are resolved in Todoroki-II. Although the cross section of Todoroki-II is circle owing to restrictions of budget, high elongation and low aspect ratio make directions of VLC winding become more vertical and horizontal in the outer and the inner sides of torus, respectively. Since the configuration of VLC is similar to that of CS and TF coil systems of conventional tokamaks, a VLC tokamak reactor can afford more room for blanket and other parts in conventional tokamak reactors with much reduced volume of coils and their supporting structure.

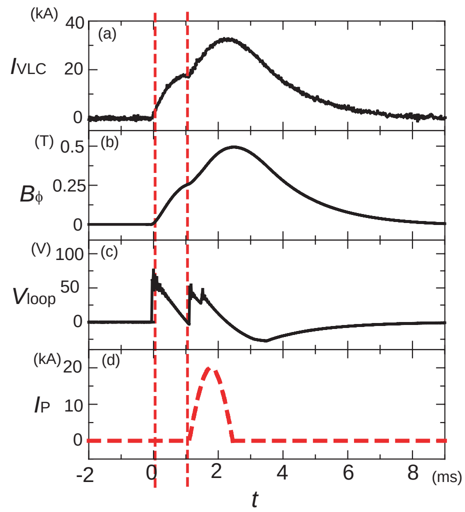

FIG. 7. Time evolutions of (a) VLC current, (b) toroidal field, (c) loop voltage, (d) expected plasma current in a two-step discharge of Todoroki-II without plasma.

Acknowledgment

This work was supported in part by New Energy and Industrial Technology Development Organization (NEDO), Japan.

References

[1] MIURA, Y., KONDOH, J., SHIMADA, R.: in Fusion Technology (Proc. 18th Eur. Symp. Karlsruhe, 1994), Vol. 2, Elsevier, Amsterdam (1995) 957-960.

[2] FUJII, K., KONDOH, J., *et. al.*: T.IEE Japan, **117-D**(8) (1997) 1041 (in Japanese).

[3] KONDOH, J., *et. al.*: Fusion Eng. Des., **42** (1998) 417-423.

[4] KONDOH, J., FUJITA, T., TSUTSUI, H., *et. al.*: Elec. Eng. Japan, **130**(3), (2000) 39-48.

[5] TSUJI-IIO, S., *et. al.*: Fusion Energy 1998 (2001 Edition) (Proc. 17th IAEA Fusion Energy Conf. Yokohama, 1998) IAEA, Vienna (2001) FTP-30.

[6] MURAKAMI. T., *et. al.*: Fusion Eng. Des., **51-52** (2000) 1059-1064.

[7] TSUJI-IIO, S., TSUTSUI, H., KONDOH, J., *et al.*: Fusion Energy 1996 (Proc. 16th IAEA Fusion Energy Conf., Montreal, 1996), Vol. 3, IAEA, Vienna (1997) 685-692.

[8] YOSHIKAWA, S.: Phys. Fluid, **7** (1963) 278.

[9] LONGMIRE, C.L.: *"Elementary Plasma Physics"*, (Interscience Publishers, John Wiley and Sons, New York, London, 1963), 68-73.

[10] TSUTSUI, H., NOMURA, S., SHIMADA, R.: J. Plasma and Fusion Research, **77**(3) (2001) 300-308 (in Japanese).

[11] TSUTSUI, H., NOMURA, S., TSUJI-IIO, S., SHIMADA, R.: IEEE Tran. Appl. Supercond. **12**(1) (2002) 644-647.