Ferromagnetic and Resistive Wall Effects on Beta Limit in a Tokamak

G. Kurita, T. Tuda, S. Ishida, S. Takeji, A. Sakasai, M. Matsukawa, T. Ozeki, M. Kikuchi, M. Azumi

Naka Fusion Research Establishment, Japan Atomic Energy Research Institute, Mukoyama, Naka-machi, Naka-gun, Ibaraki-ken, Japan 311-0193

e-mail contact of main author: kurita@naka.jaeri.go.jp

Abstract. Ferromagnetic and resistive wall effect on beta limit in a tokamak is investigated. It is shown that the beta limit is reduced to 90% of that without ferromagnetic effect for high aspect ratio tokamak, if the ferromagnetic wall of relative permeability of 2 is used. The effect of toroidal plasma flow is also investigated, and the flow velocity of $0.03v_{ta}$, v_{ta} is toroidal Alfvén velocity, is sufficient for the resistive wall to have stability effect of ideal wall. Both the resistive wall and ideal kink modes are destabilized by the ferromagnetic wall effects.

1. Introduction

In order to improve economic and environmental suitability of tokamak fusion reactors, both the accomplishment of high beta plasmas and the practical use of low activation materials to reduce the amount of radioactive waste are crucially important [1]. Although low radio-activation ferritic steel is considered as a most promising candidate for structural material in DEMO reactors, the influence of a ferromagnetic property in the ferritic steel on MHD stability and beta limits has been poorly investigated so far [2]. The effect of ferritic steel on MHD stability can be regarded as an additional factor to deteriorate the stability in a close relationship with stability for resistive wall mode (RWM) [3]. This paper finds substantial influences of residual magnetism in passively stabilizing wall on ideal MHD stability, i.e., "ferromagnetic wall mode", even though the ferromagnetism is sufficiently saturated at a high toroidal field (typically, μ/μ_0~2) and shows evaluations of deterioration of the beta limit due to the ferromagnetic property for the first time: where μ and μ_0 denote the permeability of ferromagnetic wall and vacuum, respectively. The toroidal flow effect on ferromagnetic and resistive wall mode is also investigated.

2. Basic analysis of ferromagnetic wall effect on kink mode

The roles of the ferromagnetic wall on the MHD stability is the twofold; the attraction of the perturbed magnetic field and the enhancement of the local skin time. The first one effectively moves the wall far from the plasma, even further than the infinity, and then widens the unstable regime of safety factor. The second reduces the growth rate and is

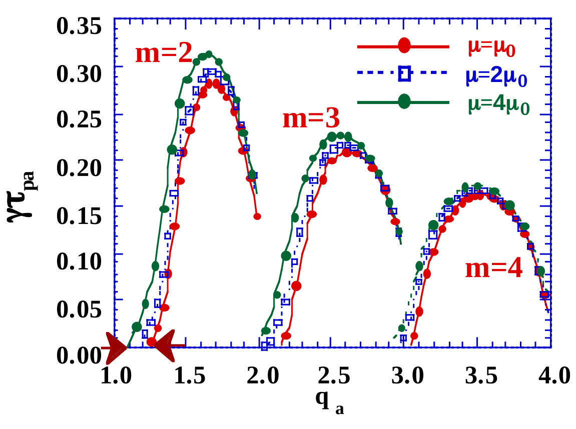

Fig.1 Growth rate of n=1 free boundary kink mode versus safety factor of uniform current cylindrical tokamak with permeability effect. The permeability increases the growth rate of kink mode, especially for low q_a region, and the stability window is reduced, amount of which is shown by distance between two arrows.

stabilizing. However, the analysis (see Appendix) shows that the enhancement of the local skin time is canceled out by the magnetic field compression and that the effective skin time on the MHD stability is expressed by the vacuum permeability; $\tau_s = \mu_0 r_w d/(2m\eta_w)$ where η_w is the wall resistivity, r_W and d are the plasma and wall minor radii and the wall thickness, m is the poloidal mode number. These features were confirmed by the numerical simulation of the free boundary kink mode in a cylindrical plasma with ferromagnetic and resistive wall (see FIG.1) and it was shown that, even for the almost saturated state of the permeability, the ferromagnetic wall has the considerable effect on the MHD stability. The increment of the unstable q regime for the uniform current is in good agreement with the analytic evaluation of $\delta\, nq_a=(a/r_w)^{2m}(\mu/\mu_0-1)md/(2r_w)$.

3. Critical beta analysis with and without ferromagnetism

In the present paper, the stability for ferromagnetic and resistive wall modes are analyzed using the linear MHD code, AEOLUS-FT, based on the original resistive MHD equations developed at JAERI. The linearized resistive MHD equations with plasma flow and permeability effect are shown blow,

$$\rho_0 \frac{\partial \vec{v}}{\partial t} = -\rho_0\left(\vec{v}_0 \cdot \vec{\nabla}\right)\vec{v} - \vec{\nabla} p + \left(\vec{j}_0 \times \vec{b}\right) + \left(\vec{\nabla} \times (\vec{b}/\hat{\mu}) \times \vec{b}_0\right)$$

$$\frac{\partial \vec{b}}{\partial t} = \vec{\nabla} \times \left(\vec{v}_0 \times \vec{b} + \vec{v} \times \vec{b}_0 - \eta \vec{\nabla} \times (\vec{b}/\hat{\mu})\right)$$

$$\frac{\partial p}{\partial t} = -\left(\vec{v}_0 \cdot \vec{\nabla}\right)p - \left(\vec{v} \cdot \vec{\nabla}\right)p_0 - \Gamma p_0 \vec{\nabla} \cdot \vec{v}$$

Here, subscript 0 denotes the equilibrium quantity, $\hat{\mu}$ ($=\mu/\mu_0$) is relative permeability and Γ is specific heat ratio. Applicability and accuracy of this code for fixed boundary problem were confirmed by a benchmark test with the FAR code developed at ORNL[4]. For free boundary problem, the "pseudo-vacuum" model [5, 6] is used instead of "real vacuum" where the vacuum is replaced by highly resistive plasma, in the AEOLUS-FT code. In the following numerical calculations, the time is normalized to the poloidal Alfvén transit time $\tau_{pa}=\sqrt{\rho_0}R/B_t$, where R is major radius and B_t is toroidal magnetic field, and the ferromagnetic and resistive wall is assumed to surround the plasma uniformly and the distance between the wall and the plasma is also uniform.

3.1 Resistive wall mode without ferromagnetism

In order to look at MHD stability of the resistive wall mode without ferromagnetism in the wall, we investigate the plasma surface safety factor dependence on the growth rate for the plasma with a uniform current profile and parabolic pressure profile, a circular plasma cross section and a high aspect ratio without ferromagnetism. The AEOLUS-FT code analysis shows that, by changing the resistivity of the wall from $\eta_w=1$ (representing "pseudo-vacuum") to $\eta_w=10^{-4}$ and 10^{-6}, the growth rates of n=1 modes are reduced from the growth rate for free-boundary kink mode to that of resistive wall mode, where the obtained growth rate is consistently shown to be of the order of the inverse time constant of resistive wall. From these calculations, the dependence of the growth rate on nq_a, for low beta and high aspect ratio plasma, is found to be almost the same as cylindrical analysis [7], indicating the validity of the AEOLUS-FT code calculation. Here used are a circular plasma cross-section and a high aspect ratio, $q_a=2.5$ and the minor radius of resistive wall of 1.14a and a resistivity of the wall fixed at $\eta_w=10^{-4}$. The poloidal mode numbers are taken into account from m=1 to m=10. The number of non-uniform grid points in the minor radius direction is typically 2000.

3.2 Dependence of critical beta on permeability

Under the above conditions with parabolic current and pressure profiles, the dependence of the n=1 mode growth rate on the poloidal beta for the plasma is obtained from the AEOLUS-FT code analysis, where the thickness of the wall is fixed at d=0.07a and the permeability in the ferromagnetic and resistive wall is changed from $\mu/\mu_0=1$ to 8. As shown in FIG.2, the growth rates clearly increase and the critical poloidal beta values are substantially reduced down to 90% at $\mu/\mu_0=2$, 78% at $\mu/\mu_0=4$ in comparison with the critical beta value at $\mu/\mu_0=1$. Figure 3 shows the comparison of the mode structures of m=3/n=1 with and without ferromagnetism corresponding to the cases shown in FIG.2. This figure clearly represents a feature of magnetic field attraction due to ferromagnetism in the wall in comparison with the resistive wall without ferromagnetism.

3.3 Dependence of critical beta on thickness of ferritic wall

Effects of the thickness of the ferromagnetic and resistive wall on the critical beta can appear as a competition between stabilizing and destabilizing effects caused by skin time and ferromagnetism, respectively. When the wall thickness is increased with the inner wall radius fixed from d=0.07a to 0.11a and 0.14a, the critical beta increases as 1.49, 1.67 and 1.70, respectively, due to increasing the skin time if the permeability effect is not taken into account ($\mu/\mu_0=1$). However, with the permeability effect, the critical beta saturates or even decreases with the thickness of the wall above a threshold value of the thickness since the destabilizing effect due to ferromagnetism becomes larger. Indeed, for the case of $\mu/\mu_0=2$, the critical beta value is increased from 1.34 at d=0.07a to 1.41 at d=0.11a, but is decreased to 1.39 at d=0.14a.

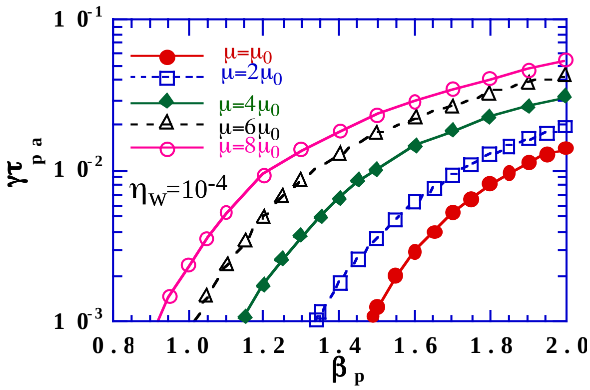

Fig.2 Growth rate of n=1 free-boundary kink mode of parabolic current high aspect ratio tokamak versus poloidal beta.

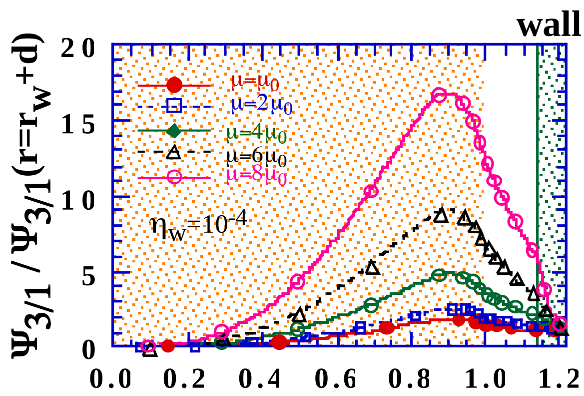

Fig.3 Comparison of eigen-function of $\Psi_{3/1}$ in Fig.2 calculation at βp=2.

4. Effect of toroidal plasma flow

The effect of toroidal or poloidal plasma flow has been considered to play an important role in stabilizing resistive wall mode [8, 9] with the effect of viscous damping [3, 10]. We investigate the effect of toroidal plasma flow on resistive wall and ideal kink modes using AEOLUS-FT code, which solves the complex eigen-value problem. We use the same analytical equilibrium as that in section 3 (large aspect ratio with parabolic profiles for both plasma current and pressure) with poloidal beta of 1.8. The equilibrium is sufficiently unstable for b/a=1.43. In these calculations, we use the rigid plasma rotation and change the position of resistive wall for different flow velocity. The calculated growth rates versus the position of resistive wall, r_w/a, are shown in FIG.4. For no rotation case, the growth rate shows the monotonous decreasing function of resistive wall position. For high rotation case, on the other hand, they appears steep decreasing function at large values of r_w/a as ideal wall branch, and appears again at small value region as resistive wall branch. The growth rates of ideal wall branch tends to that of ideal wall case for sufficient large flow speed, $v_{\Phi 0}=0.03v_{ta}$ (v_{ta} is toroidal Alfvén velocity). The growth rates in the intermit region are expected to be stabilized, if we incorporate the effect of viscous damping for equilibria of small aspect ratio tokamak [3,10]. Figure 5 shows the growth rate for $v_{\Phi 0}=0.06v_{ta}$ flow velocity case with the ferromagnetic effect. Growth rates are more increased for larger values of relative permeability for all calculation region.

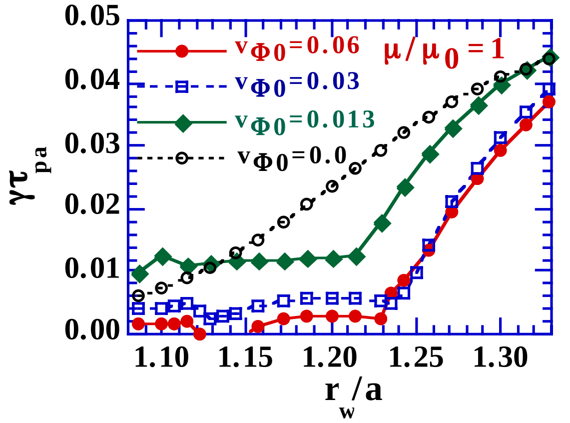

Fig.4 Growth rate versus wall position for 3 toroidal flow velocity values. Relative permeability value is 1.

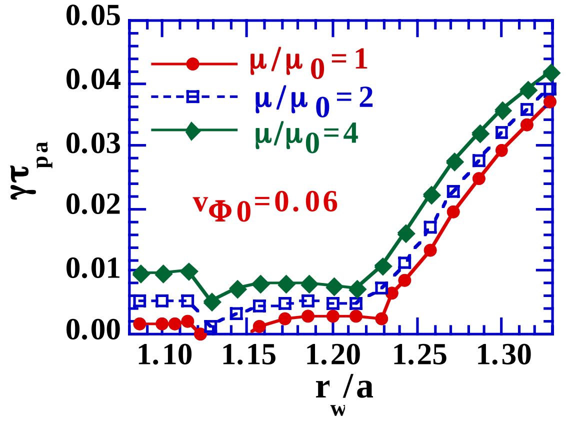

Fig.5 Growth rate versus wall position for 3 relative permeability values. Toroidal flow velocity is $0.06v_{ta}$.

5. Conclusion

In conclusion, the presence of ferromagnetic wall mode is identified as the critical beta is reduced to 90% of that without ferromagnetism with a wall thickness of 0.07a for $\mu/\mu_0=2$ at which the ferritic steel is sufficiently saturated. Even though the skin time in the wall is increased with the wall thickness, the ferromagnetism can suppress the improvement in the critical beta or decrease it if the wall thickness becomes larger than a threshold value. The effect of toroidal plasma flow is also investigated, and the flow velocity of $0.03v_{ta}$, v_{ta} is toroidal Alfvén velocity, is sufficient for the resistive wall to have stability effect of ideal wall. Both the resistive wall and ideal kink modes are destabilized by the ferromagnetic wall effects. These results would have an impact on reactor designs utilizing ferritic steel material with ferromagnetism. Finally, we note the above roles of ferromagnetic wall, that is, the attraction of the magnetic perturbation and the resultant reduction of the MHD stability, is basically independent on the plasma model, like the inclusion of the viscous damping term, and in this sense, the ferromagnetic wall mode is the generic one. However, the critical beta value is affected by details of the plasma model and the configuration, and the quantitative evaluation of it is now under way, including the effect on the feedback control.

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References

[1] Seki, Y., et al., Proc. 13th IAEA Conf. on Controlled Nuclear Fusion and Plasma Physics, Washington, 1990, IAEA-CN-53/G-1-2.
[2] Gimblett, C.G., Nucl. Fusion, 26, (1986) 617.
[3] Bondeson, A., and Ward, D. J., Phys. Rev. Lett., 72, (1994) 2709.
[4] Leboeuf, J-N. G., and Kurita, G., JAERI-Research, 98-010, 1998.
[5] Pogutse, O.P., and Yurchenko, E.I., Sov. J. Plasma Phys., 3, (1977) 283.
[6] Kurita, G., Azumi, M., et al., Nucl. Fusion, 26, (1986) 449.

[7] Tanaka, M., Tuda, T., and Takeda, T., Nucl. Fusion, 13, (1973) 119.
[8] Finn, J.M., Phys. Plasmas, 2, (1995) 198.
[9] Betti, R., Phys. Plasmas, 5, (1998) 3615.
[10] Chu, M.S., et al., Phys. Plasmas, 2, (1995) 2326.

Appendix

In order to understand the basic features concerning the ferromagnetic wall effects on the MHD stability, we study MHD mode with poloidal number m in a cylindrical plasmas, under the assumption of long wavelength limit. We locate the ferromagnetic wall with resistivity η_w, permeability μ and width d, at $r=r_w$ and the perfect conductor wall at r=b, while the plasma boundary at r=a. The perturbed helical magnetic flux ψ in the vacuum regions ($a<r<r_w$ and $r_w+d<r<b$) and inside the ferromagnetic wall ($r_w<r<r_w+d$) are analytically solved and these are connected with each other by using the boundary conditions: $[\psi]=0$ and $[\psi'/\mu]=0$ at both sides of the ferromagnetic wall ($r=r_w$ and $r=r_w+d$) , where ψ' is the radial derivative of ψ and $[f]=f(x+0)-f(x-0)$. Then the resultant vacuum solution takes the following form;

$$\frac{\Delta_a^* + 1}{\Delta_a^* - 1} = \frac{\left(I'_{m+} + \hat{\sigma} I_{m+}\right)\left(K'_{m+} + \sigma I_{m+}\right) - \left(K'_{m+} + \hat{\sigma} K_{m+}\right)\left(I'_m + \sigma I_m\right)}{\left(I'_{m+} + \hat{\sigma} I_{m+}\right)\left(K'_{m+} - \sigma I_{m+}\right) - \left(K'_{m+} + \hat{\sigma} K_{m+}\right)\left(I'_m - \sigma I_m\right)} \left(\frac{a}{r_w}\right)^{2m}$$

where I_m and K_m are the 1st and 2nd modified Bessel function of m-th order with argument κr_w, and I_{m+} and K_{m+} are those with argument $\kappa(r_w+d)$,

$$\kappa^2 = \gamma\ \mu/\eta_w, \quad \sigma = \hat{\mu}\left(m/\kappa\ r_w\right), \quad \hat{\sigma} = \sigma r_w/\left(r_w + d\right), \quad \hat{\mu} = \mu/\mu_0, \quad \Delta_a^* = a\psi'/m\psi\big|_{r=a+c}.$$

γ is the growth rate of the mode. By connecting Δ^*_a with the solution of ψ in the plasma, the dispersion relation of the mode is obtained. In the case of the thin ferromagnetic wall, taking the first order of d/r_w, the above equation is expressed by the following simplified form;

$$\frac{\Delta_a^* + 1}{\Delta_a^* - 1} = \frac{\gamma\tau_w - \left(md/2r_w\right)\left(\hat{\mu} - \hat{\mu}^{-1}\right)}{1 + \gamma\tau_w + \left(md/2r_w\right)\left(\hat{\mu} + \hat{\mu}^{-1} - 2\right)} \left(\frac{a}{r_w}\right)^{2m}$$

where $\tau_w = \mu_0\, dr_w/(2m\eta_w)$ is the skin time of the ferromagnetic wall. Note that this skin time is independent on the permeability of the wall, which means the enhancement of the local skin time $\tau_w=\mu dr_w/(2m\eta_w)$ is compensated by the flux compression through the boundary condition. This equation also shows that the ferromagnetic wall makes the mode unstable even for the high conductivity. The growth rate of the MHD mode can be expressed as,

$$\gamma\tau_W = \Gamma_w + \Gamma_\mu$$

where

$$\Gamma_w = \frac{\alpha}{\left(a/r_w\right)^{2m} - \alpha}, \quad \Gamma_\mu = \left(\frac{md}{2r_w}\right)\frac{\alpha\left(\hat{\mu} + \hat{\mu}^{-1} - 2\right) + \left(\hat{\mu} - \hat{\mu}^{-1}\right)\left(a/r_w\right)^{2m}}{\left(a/r_w\right)^{2m} - \alpha}$$

and $\alpha=(\Delta^*_a +1)/(\Delta^*_a -1)$ is generally the function of growth rate determined by the plasma dynamics. For the range where the ideal mode is stable and the plasma inertia is neglected, the parameter α is independent on γ (for uniform current case, $\alpha= nq_a- m + 1$). Then the first term of the right hand side gives the resistive wall mode and the second term shows that the high permeability enhances the mode growth rate and widens the unstable region.