Analytical Study of RWM Feedback Stabilisation with Application to ITER

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Abstract. An analytical model for studying the feedback control of Resistive Wall Modes (RWMs) in a tokamak with single or double conducting wall is presented. The model is based on a cylindrical approximation. It is shown that the outer conducting shell, in the case of ITER-like double wall vacuum vessel, does not significantly reduces the RWM instability growth rate but deteriorates the feedback stabilisation. It is also shown that six side saddle coils with the nominal voltage 40 V per turn is capable of stabilizing the RWM for the expected range of normalized beta.

1. Introduction

An analytical model for studying the feedback stabilisation of RWM in a tokamak with single or double conducting wall is presented. The model is based on a cylindrical approximation - a single mode with poloidal number m and "toroidal" number n is considered. The model comprises a cylindrical plasma with radius a_p, two thin cylindrical conducting shells with radii a_1, a_2, thickness d_1, d_2, electrical conductivity σ_1, σ_2, and the ideal feedback coils producing the same harmonic (m, n). Thus, all currents and magnetic fields are proportional to $\exp[i(m\vartheta - n\varsigma)]$, where ϑ and ς are the poloidal and "toroidal" angles, so that r, ϑ and $z = R\varsigma$ are the cylindrical coordinates (R is equivalent to tokamak major radius).

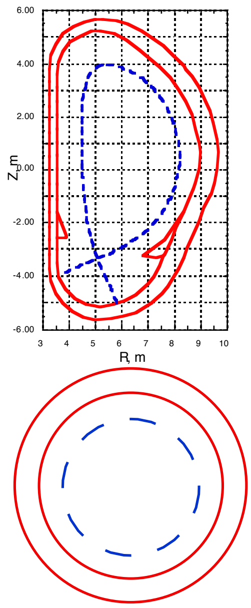

FIG. 1. ITER plasma, vacuum vessel and their simplified cylindrical models.

Numerical estimates have been made for ITER, which has elongated plasma and double wall vacuum vessel. The upper part of Fig. 1 shows an ITER plasma of 9 MA steady-state scenario and two shells of the vacuum vessel. The cylindrical circular model of ITER, used in the analytical study of RWM stabilisation, is shown in the lower part of Fig. 1. The model has a plasma radius $a_p = 3.5$ m and shell radii $a_1 = 1.35\ a_p$, $a_2 = 1.7\ a_p$. Each shell of the vacuum vessel has a thickness of 60 mm and resistivity 0.825 μΩ•m.

2. Equation for RWM

The equation modeling the feedback control of the (m, n) mode in the double-wall tokamak, derived in [1], can be written as:

$$\frac{d^2 B_r}{d\tau^2} - (\gamma - \lambda)\frac{dB_r}{d\tau} - \gamma\lambda B_r = \frac{\gamma\lambda}{\gamma_0} B_f . \tag{1}$$

Here B_r is the radial component of the total magnetic field of the mode on the 1st shell, $\tau = t/T_m$ is the dimensionless time normalized by resistive time constant of the 1st shell $T_m = \mu_0\sigma_1 a_1 d_1/(2m)$, B_f is the radial component of the magnetic field of the (*m, n*) harmonic produced by the feedback coils on the 1st shell, and $\gamma_0 = \Gamma_0 T_m$ with Γ_0 being the growth rate of RWM in the presence of only the 1st shell without the feedback stabilisation. Parameters γ and λ are, correspondingly, the normalized growth rate and decay rate of the two branches of RWM. They depend on the wall parameters and on γ_0: $\gamma - \lambda = \gamma_0 - \alpha\xi$, $\gamma\lambda = \gamma_0\xi(\alpha - 1)$, where $\xi = \left[(a_2/a_1)^{2m} - 1\right]^{1}$ and $\alpha = 1 + (\sigma_1/\sigma_2)(d_1/d_2)(a_2/a_1)^{2m-1}$.

The model for ITER is characterized by $\xi = 0.66$, $\alpha = 3.0$ for the $m = 2$ mode and by $\xi = 0.33$, $\alpha = 4.2$ for the $m = 3$ mode.

3. RWM without Feedback Control

Without the feedback control, the 2nd shell with high resistivity ($\sigma_2 \to 0$) would not affect the RWM growth rate, $\gamma \to \gamma_0$ (the case of a single wall), whereas it would reduce the growth rate in the opposite case: $\gamma \to \gamma_0 - \xi$ when $\sigma_2 \to \infty$ [1].

The estimated effect of the 2nd shell of the ITER vacuum vessel on the RWM growth rate for $m = 2$ and $m = 3$ is shown in Fig. 2. The outer shell of the ITER vacuum vessel does not significantly reduce the RWM growth rate ($\gamma \approx \gamma_0$), but, as shown below, it may deteriorate RWM active stabilisation, screening the feedback-produced magnetic field.

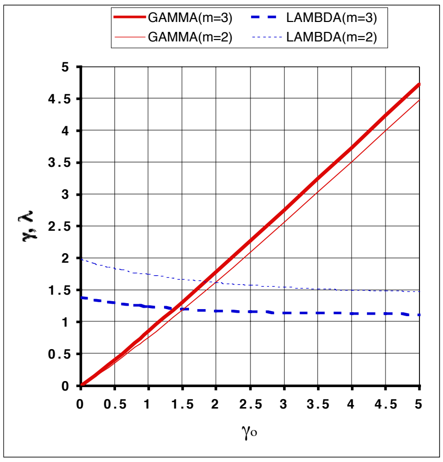

FIG. 2. RWM unstable (γ) and stable (λ) roots of equation (1) for RWM modes m = 2 and m = 3 in ITER cylindrical model as function of γ_0.

4. Ideal Feedback Coils

For active stabilization of the mode (*m, n*), the feedback coils must produce the field with the same (*m, n*). Static efficiency of this ideal feedback coils can be characterized by a parameter $b_{m,n}$ defined as $B_f = b_{m,n} I_f$, where I_f is the current in feedback coil producing mode (*m, n*). We study the feedback control of RWM assuming $T_f >> T_m$, where $T_f = L_f / R_f$ (L_f and R_f are the effective inductance and resistance of the feedback circuit), since in ITER the time constant of the 1st shell for $m = 1$ mode is $T_1 \approx 0.17$ s, while $T_f \approx 5$ s.

The feedback circuit equation can be written as

$$\frac{dB_f}{d\tau} + \frac{T_m}{T_f} B_f = \frac{b_{m,n} T_m}{L_f} V_f, \qquad (2)$$

where V_f is the voltage applied to the feedback circuit. The second term is small under ITER conditions.

5. Feedback Control with Radial Field Sensors

We describe the feedback control of RWM in terms of the radial component B_r of the total magnetic field on the 1st shell, consisting of several parts: B_p from the plasma, B_1 from the 1st shell, B_2 from the 2nd shell, and B_f from the feedback coil, $B_r = B_p + B_1 + B_2 + B_f$.
The following feedback algorithm is studied:

$$\frac{dB_f}{d\tau} = -k_1 B_r - k_2 \frac{dB_r}{d\tau} - k_3 \frac{d^2 B_r}{d\tau^2}. \tag{3}$$

Here the term proportional to k_2 is a conventional term used for RWM stabilization (see, for example, [1] and [2]). It is shown below that the term proportional to k_3 is needed in the case of double wall for stabilizing a highly unstable RWM. The term with k_1 determines the desired level of B_r (zero, in this case). This feedback algorithm is equivalent to the voltage control through equation (2).

Using (3) with constant gains k_i, one can get from (1) the following characteristic equation for RWM in the **double wall** tokamak when radial field sensors are used in the feedback system:

$$s^3 + c_2 s^2 + c_1 s + c_0 = 0, \tag{4}$$

$$c_2 = \xi(\alpha - 1)k_3 - \gamma_0 + \alpha\xi, \quad c_1 = \xi(\alpha - 1)(k_2 - \gamma_0), \quad c_0 = k_1\xi(\alpha - 1), \quad s = ST_m,$$

where S is the variable of Laplace transformation. When the RWM is stabilized, all the coefficients in (4) are positive. This would make negative the real parts of the roots of equation (4). Therefore the necessary conditions for RWM stabilization can be fulfilled if

$$\xi(\alpha - 1)k_3 > \gamma_0 - \gamma_{cr}, \quad k_2 > \gamma_0, \quad k_1 > 0, \quad \gamma_{cr} = \alpha\xi. \tag{5}$$

In principle, in this case (double wall) stabilization of RWM could be possible without the term d^2B_r/dt^2 in (3) ($k_3 = 0$), if the instability would not be too strong ($\gamma_0 < \gamma_{cr}$) [1]. According to (5), with proper k_3 this restriction on γ_0 is eliminated.

In ITER cylindrical model the critical value of RWM instability growth rate, Γ_{cr}, corresponds to $\Gamma_{cr}T_1 = m\gamma_{cr} \approx 4$ for $m = 2, 3$. Therefore, moderately unstable RWMs having $\Gamma T_1 < 4$ are expected to be stabilised in ITER without a signal proportional to d^2B_r/dt^2. In order to achieve $Q \geq 5$ at steady state operation, the value of β_N should be somewhere between β_N(*no wall*) and 0.5[β_N(*no wall*) + β_N(*ideal wall*)]. (Here β_N(*no wall*) ≈ 2.6 and β_N(*ideal wall*) ≈ 3.6 are corresponding limits imposed by kink modes on the value of β_N in the cases without conducting wall and with ideally conducting 1st shell [3].) This range of β_N corresponds to moderate unstable RWMs with $\Gamma T_1 < 4$.

In the case of a **single wall** ($\sigma_2 = 0$, $\alpha \to \infty$), assuming $k_3 = 0$, the characteristic equation (4) is reduced to

$$s^2 + p_1 s + p_0 = 0, \quad p_1 = k_2 - \gamma_0, \quad p_0 = k_1. \tag{6}$$

The RWM is stabilized when $k_2 > \gamma_0$ and $k_1 > 0$. These conditions for k_2 and k_1 are the same as those in (5). However, in this case the term with d^2B_r/dt^2 in (3) is not needed even for a very unstable RWM (high value of γ_0).

6. Feedback Control with Poloidal Field Sensors

In this model, a feedback system with sensors measuring the poloidal component B_ϑ of the total perturbed magnetic field on the inner side of the 1st shell cannot be much different from that using the radial sensors because $B_\vartheta = (1 + 2\gamma_0)B_r$ [4]. Using B_ϑ instead of B_r in (3), we obtain the same B_f as in the previous case, if we reduce all k_i by a factor of $1+2\gamma_0$. Therefore, in this case the stabilization of RWMs in the **double wall** tokamak will be achieved with gains k_i smaller than those in (5) for the feedback with radial sensors:

$$\xi(\alpha-1)k_3^\theta > \frac{\gamma_0-\gamma_{cr}}{1+2\gamma_0},\quad k_2^\theta > \frac{\gamma_0}{1+2\gamma_0},\quad k_1^\theta > 0. \tag{7}$$

Similar to the feedback with radial field sensors, a signal proportional to d^2B_ϑ/dt^2 is required when the instability is strong ($\gamma_0 > \gamma_{cr}$). In the case of a **single wall** we obtain the same conditions for k_2^θ and k_1^θ, but the term with d^2B_ϑ/dt^2 is not required even for a very unstable RWM. Note that, in contrast to the case of the feedback with radial field sensors, the required gain k_2^θ does not increase proportional to γ_0. Therefore, with poloidal sensors, any $k_2^\theta > 0.5$ is sufficient for stabilizing the RWM with arbitrary growth rate.

Smaller gains are better, but that cannot be the only reason of the dramatic difference between the feedback with radial and poloidal sensors [5]. The argument above is valid for ideal feedback coils producing a single mode magnetic field. However, the conventional array of saddle feedback coils generates, in addition to the necessary (*m, n*) harmonic, some side-band harmonics that are not needed for RWM stabilization, but affects the probe measurements. The signals measured by the "radial" sensors in the equatorial plane are affected much stronger than the "poloidal" sensors [6]. In some cases the "radial" signal can become zero while the mode is not yet suppressed. That is why the conventional feedback system with radial probes can fail. At the same time, a similar system with poloidal sensors can be quite efficient in suppressing RWM [6].

7. Feedback Control with Voltage Saturation

For a given γ_0, the value of feedback gains providing a desired quality of RWM control can be easily found from characteristic equations (4) or (6). For example, in the case of a **single wall**, feedback control with the radial field sensors will have a critically damped regime with a settling time T_{set} when $k_2 = \gamma_0 + 2/\tau_{set}$, $k_1 = 1/\tau_{set}^2$, $\tau_{set} = T_{set}/T_m$. The poloidal field sensors ensures the critically damped regime when these gains are reduced by a factor of $1+2\gamma_0$.

Even with the appropriate choice of feedback gains, the RWM control can fail, when the voltage requested by the controller (3) is higher than the limit, V_{max}, established by the power supply. For a given value of V_{max}, we can estimate the level of the perturbation B_r when the control of the mode is impossible because of the voltage limitation. Consider a mode growing till $t = 0$ without control. At $t = 0$ the feedback control is switched on. If $B_r(0) = \tilde{B}_0$ is high enough, the voltage requested by the controller via (3) and (2) is higher than the capability of power supply V_{max}, and only the constant voltage will be applied to the feedback circuit. In this case, the RWM evolution is described by (1) and (2) with $V_f = -V_{max}$. Neglecting the term proportional to T_m/T_f in (2), the solution is expressed by the following formula:

$$B_r = \left(\tilde{B}_0 - B_{cr}\right)e^{\gamma\tau} + \left(\frac{\gamma}{\lambda}\right)^2 B_{cr}e^{-\lambda\tau} + \frac{(\gamma+\lambda)(\gamma\lambda\tau - \gamma + \lambda)}{\lambda^2}B_{cr},\quad B_{cr} = \frac{\lambda b_{m,n}T_m}{\gamma(\gamma+\lambda)\gamma_0 L_f}V_{\max}.$$

The evolution of RWM will follow one of the two scenarios. If $\tilde{B}_0 < B_{cr}$, B_r will sooner or later reduce to the level when the required feedback voltage becomes lower then V_{max}. This makes possible feedback stabilization of the RWM. In the opposite case, when $\tilde{B}_0 \geq B_{cr}$, the

RWM will grow unlimited. It should be noted that, in practice, the value of the critical field B_{cr} is reduced by a technical limit imposed on the current in the feedback circuit.

In ITER, the error field correction coils (CCs) will be used for the feedforward and feedback stabilisation of RWM. The system comprises 6 top CCs, 6 side CCs, and 6 bottom CCs. Toroidally opposite coils are connected to produce the magnetic field with $n = 1$ and have a common power supply. The feedforward stabilisation is achieved by error field correction using all CCs. The feedback stabilisation of RWM will be provided by the voltage applied to the side CCs according to a signal from the magnetic probes located between the plasma and vacuum vessel inner shell. A rough estimate for ITER with the design parameters $V_0 = 40$ V/turn, $b = 0.1$ T/MA•turn, $L_f = 50$ μH/turn^2 gives B_{cr} about 1 mT. Assuming that an RWM with the amplitude $B_r \approx 1$ mT can be detected, the instability can be stabilized with the voltages available for the ITER side CCs.

8. Conclusions

The study based on the cylindrical model [1] has shown that ITER-like double wall vacuum vessel does not significantly reduces RWM instability growth rate ($\gamma \approx \gamma_0$), but deteriorates feedback control. There is a critical value of instability growth rate, Γ_{cr}, above which the feedback voltage should have a term proportional to d^2B/dt^2 in addition to the conventional term proportional to dB/dt. In ITER steady state operation with $Q \geq 5$, moderately unstable RWMs having $\Gamma T_1 < 4$ is expected. These instabilities have growth rates less than the estimated value of Γ_{cr} and therefore their stabilization can be achieved without knowledge of d^2B/dt^2. The voltage of 40 V/turn in side CCs seems reasonable for control of the RWM having the amplitude less than about 1 mT. These analytical results are also supported by numerical calculations [2].

The poloidal field magnetic sensors located inside the vacuum vessel inner shell are preferable for feedback control. For example, in the case of a single wall and ideal feedback coils, RWM stabilization can be achieved with the "radial" sensors when the gain k_2 is proportional to the instability growth rate γ_0, whereas with the "poloidal" sensors, it can be achieved with k_2 independent on γ_0, if $k > 1/2$.

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