Improved Theory of Forced Magnetic Reconnection due to Error Field and its Application to Seed Island Formation for NTM

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Abstract. A seed island is required for destabilizing the neo-classical tearing mode (NTM), which degrades confinement in long sustained, high-confinement, high beta plasmas. The seed island formation due to an MHD event, such as a sawtooth crash, is investigated by applying the improved boundary layer theory of forced magnetic reconnection. This improved theory introduces the non-constant-ψ matching and reveals the complicated feature of the reconnection described by two reconnected fluxes. In the initial evolution, these reconnected fluxes grow on the time scale including the ideal time scale, typical time scale of the MHD event and the time scale of resistive kink mode. The surface current is negative, $\Delta'(t) < 0$, to be consistent with the NTM theory. The theory also yields an integral equation which includes the typical time scale of the resistive kink mode, and allows us to investigate the time evolution of the seed island at $t \approx \tau_A S^{1/3}$.

1 Introduction

A neo-classical tearing mode (NTM) degrades confinement in long sustained, high- confinement, high beta plasmas[1]. Time evolution of the NTM is described by the extended Rutherford theory of non-linear tearing mode[2, 3]. NTM do not grow from a linear mode instability, because the current-driven tearing mode[4] is stable, $\Delta'_0 < 0$[2]. Thus a seed island formation is required for destabilizing NTM like the linear tearing mode evolution preceding the Rutherford theory. The excitation of NTM is dominated by the width of the seed island and by the time scale of the seed island formation [5, 6, 7]. The time scale is characterized by $S = \tau_R/\tau_A$ number as shown by experiments[5]. The seed island around a rational surface is produced by a forced magnetic reconnection[8, 9, 10, 11] due to a boundary perturbation or an MHD event at another rational surface, such as a sawtooth crash, via toroidal coupling. For example, a (2,2) sawtooth precursor is observed before the appearance of (3,2) NTM[5].

In this paper, we apply the improved boundary layer theory of the forced reconnection [11] to the analysis of the seed island formation. The theory adopts the non-constant-ψ asymptotic matching and leads to two reconnected fluxes which describe the complicated time evolution of seed island. In addition, we will present a new method to determine the time evolution of the reconnected fluxes by use of an integral equation. This integral equation can yield the time evolution of the reconnected fluxes at $t \approx \tau_A S^{1/3}$.

2 Basic model

We study the response of plasma to an applied external source as an initial value problem. Even if a magnetic equilibrium is stable for current-driven tearing mode, this external source gives rise to forced reconnection, because poloidal harmonics are coupled in a torus. This reconnection process is calculated by use of the boundary layer theory. First, we obtain quasi-static equilibrium coupled to the external source, following Hegna et.al.[6].

The response of the magnetic perturbation, $\psi_1(r,t) = \psi_{1m}(r,t)\exp(im\theta - in\zeta)$, which has a rational surface at $r = r_m$ where $q(r_m) = m/n$, to a given external source, ψ_s, is governed by the ideal MHD equilibrium equation at the outer region[6]. The perturbation is governed by

$$\Lambda_m \psi_{1m}(r,t) = -\Lambda_s^m \psi_s(r,t), \tag{1}$$

where Λ_m is the cylindrical exterior kink operator and Λ_s^m is an operator describing the geometric coupling between ψ_s and ψ_{1m}. These operators represent second order radial differential operator. The explicit forms of these operators for m and $m+1$ coupling are given in Ref.[12, 13] The solution to Eq. (1) is written as

$$\begin{aligned} \psi_{1m}(r,t) &= \psi_1^+(r_m,t) f_m^+(r) + \psi_s(t/\tau_s) g_m^+(r), \qquad r_m < r, \\ &= \psi_1^-(r_m,t) f_m^-(r) + \psi_s(t/\tau_s) g_m^-(r), \qquad r < r_m, \end{aligned} \tag{2}$$

where $f_m^\pm(r)$ satisfy $\Lambda_m f_m^\pm(r) = 0$ and $g_m^\pm(r)$ satisfy Eq. (1). These functions are subject to the boundary conditions $f_m^\pm(r_m) = 1$, $f^-(0) = 0$, $f^+(a) = 0$ and $g_m^\pm(r_m) = 0$, $g^-(0) = 0$, $g^+(a) = 1$[6, 9].

The time evolution of this quasi-static equilibrium is determined by the outer reconnected flux, $\psi_1^\pm(r_m,t)$, because the external source function, $\psi_s(t/\tau_s)$, is a given function. We consider the time evolution of the quasi-static equilibrium as an initial-value problem, by applying the Laplace transform, $\tilde{F}(r,s) = \int_0^\infty F(r,t)e^{-st}dt$, to the outer-solution, Eq. (2).

3 Improved Boundary Layer Theory

Next, we present the improved boundary layer analysis and the non-constant-ψ asymptotic matching. In order to obtain the outer reconnected flux we should investigate the dynamics in the vicinity of the resonant surface, where the resistivity and the inertia of the plasma are important. In the inner layer, we adopt the reduced MHD equations. The asymptotic matching of the inner solution to the outer solution yields the reconnected flux.

The forced reconnection process is described by two reconnected fluxes[11]. In the previous works, the outer reconnected flux is not distinguished from the inner reconnected flux. In the improved boundary layer analysis of forced reconnection, the non-constant-ψ matching revealed the difference of these reconnected fluxes: outer reconnected flux, $\tilde{\psi}_{1m}(r_m,s) \equiv \tilde{\psi}_{1m}^+(r_m,s) = \tilde{\psi}_{1m}^-(r_m,s)$, and inner reconnected flux, $\tilde{\psi}_{\rm in}(r - r_m/\varepsilon r_m = 0, \hat{s})$, where $\hat{s} = \tau_c s$, $\tau_c = \tau_A/\varepsilon k_m r_m$, $\varepsilon = (S k_m r_m)^{-1/3}$, $k_m \equiv m s(r_m)/r_m$, $s(r_m) = r_m q'(r_m)/q(r_m)$. The difference between these reconnected flux is caused by the non-constant-ψ nature of the inner layer. The non-constant-ψ matching, yield the outer reconnected flux, and the inner reconnected flux as,[11]

$$\tilde{\psi}_{1m}(r_m,s) = \frac{\Delta_s' \tilde{\psi}_s(s)}{\Delta_{\rm in}'(s) - \Delta_0'}, \tag{3}$$

$$\tilde{\psi}_{\rm in}(\frac{r-r_m}{\varepsilon r_m}=0,\hat{s})=\left\{1-\frac{\hat{s}^{3/2}}{\hat{s}^{3/2}-1}F(1,-1/2,\hat{s}^{3/2}/4+3/4,1/2)\right\}\tilde{\psi}_{1m}(r_m,s), \qquad (4)$$

where

$$\Delta'_{\rm in}(s)=\frac{-\pi\hat{s}^{5/4}}{8\varepsilon r_m}\frac{\Gamma(\hat{s}^{3/2}/4-1/4)}{\Gamma(\hat{s}^{3/2}/4+5/4)},\quad \Delta'_0=\left[\frac{df^{\pm}_m}{dr}\right]^{r_m+0}_{r_m-0},\quad \Delta'_s=\left[\frac{dg^{\pm}_m}{dr}\right]^{r_m+0}_{r_m-0}. \qquad (5)$$

The parameter Δ'_s is a discontinuity in the radial derivative of $g^{\pm}_m$ due to a external source and is positive. The tearing mode stability parameter, Δ'_0, is a discontinuity in the radial derivative of $f^{\pm}_m$. Since the original static equilibrium is supposed to be stable for the current-driven tearing mode, Δ'_0 is negative.

In the next section, we introduce a new method to calculate the inverse of the Laplace transformed reconnected flux. This method yields an integral equation which enables one to investigate the time evolution of the seed island at $t\sim\tau_A S^{1/3}$.

4 Evolution equation for outer reconnected flux

In this section we propose a new method to determine the time evolution of the outer reconnected flux. The inversion of the Laplace transform of Eq. (3) gives the following inhomogeneous second kind Volterra equation:

$$\psi_{1m}(r_m,t)+\int_0^t\psi_{1m}(r_m,\tau)G(t-\tau)d\tau=\frac{-\Delta'_s}{\Delta'_0}\psi_s(t), \qquad (6)$$

where the kernel $G(t)$ is the Bromwich integral of $\Delta'_{in}(s)$ which consists of the sum of residues at the poles and the integral along the branch cut in the complex s plane. It is written as

$$G(t)=\frac{1}{\tau_\alpha}\left\{-\frac{2}{3\sqrt{\pi}}\exp(\frac{t}{\tau_c})-\frac{4}{3\pi}\sum_{n=1}^{\infty}\frac{\sqrt{n-1/4}}{n!}\Gamma(n-1/2)\exp(\frac{-t}{2\tau_n})\sin(\frac{\sqrt{3}}{2}\frac{t}{\tau_n})\right.$$
$$\left.+\frac{1}{3\pi^2}\int_0^{\infty}\sqrt{x}|\Gamma(ix-1/4)|^2\exp(-(4x)^{2/3}t/\tau_c-\pi x)dx\right\},$$

where

$$|\Gamma(ix-1/4)|^2=|\Gamma(-1/4)|^2\prod_{n=0}^{\infty}\frac{(n-1/4)^2}{x^2+(n-1/4)^2},\qquad \tau_n=\frac{\tau_c}{(4n-1)^{2/3}}, \qquad (7)$$

where

$$\tau_\alpha=\frac{-\Delta'_0\tau_A}{\pi k_m},\qquad \tau_c=\frac{\tau_A S^{1/3}}{(k_m r_m)^{2/3}}$$

denote the ideal time scale and the time scale of resistive kink mode, respectively. Note that the amplitude of the tearing mode stability parameter, Δ'_0, affects to the ideal time scale, τ_α. The right-hand side of Eq. (6) represents the external source signal of MHD event, because the external source is represented by $\psi_s(t/\tau_s)$, where τ_s is the typical time scale of the external source. At $t=0$ the integral part vanishes and $\psi_s(0)=0$, and thus the reconnected flux vanishes at $t=0$ to satisfy the initial condition.

The time scale in the exponential function in the kernel $G(t)$ is the same as the typical time scale of the inner layer, i.e. the resistive kink mode, $\tau_c \propto \tau_A S^{1/3}$, because the kernel represents the response of the inner layer to the applied external source. The first term in the kernel corresponds to the pole of the resistive kink mode.

The integral equation gives the time scale of the subsequent evolution toward a fully reconnected state, which can be observed in experiments. Since the integral equation gives the time evolution at $t \sim \tau_A S^{1/3}$, it also yields a criterion for the transition from a linear to a nonlinear stage without any assumption such as the constant-ψ approximation or the requirement of helicity conservation. If the surface current decays sufficiently, the constant-ψ approximation is applicable. On the other hand, if the surface current does not decay, the helicity conservation can be applied to the non-linear evolution.

Although the reconnection process at the neutral surface in the inner layer is described by the inner reconnected flux, we derive the integral equation only for the outer reconnected flux in this work. The integral equation for the inner reconnected flux, $\psi_{in}(0,t)$, is deduced in the same way as the above method.

5 Time Evolution of Seed Island and $\Delta'(t)$

The time evolution of the seed island is characterized by the two reconnected fluxes. The outer reconnected flux determines the time evolution of the quasi-static equilibrium, Eq. (2), and is defined at $r = r_m$. On the other hand, the inner reconnected flux describes the reconnection process at the neutral surface in the inner layer, $(r - r_m)/\varepsilon r_m = 0$. The inertia of the plasma makes the inner-layer reconnected flux, $\tilde{\psi}_{\rm in}(0, \hat{s})$, deviate from the reconnected flux, $\tilde{\psi}_{1m}(r_m, s)$. In the initial evolution, the outer reconnected flux increases on the time scale including the ideal time scale, τ_α, and the time scale of external source, τ_s, such as the sawtooth crash. The inner reconnected flux grows on the time scale including the resistive kink time scale, $\tau_c \propto \tau_A S^{1/3}$, and the time scale of the source signal, τ_s[11]. Note that these initial evolution is valid for $\tau_A << \tau_\alpha$ which corresponds to the ideal marginal stability, $\Delta'_0 \to -\infty$, to be precise.

A surface current induced on the resonant surface is represented by the total current in the inner layer. It is equivalent to the finite jump of the θ-component of the perturbed part of magnetic field at the resonant surface, $[d\tilde{\psi}_{1m}(r,t)/dr]_{r_m-0}^{r_m+0}$, which is related to $\Delta'(t) \equiv [d\tilde{\psi}_{1m}(r,t)/dr]_{r_m-0}^{r_m+0}/\psi_{1m}(r_m,t) = \Delta'_0 + \Delta'_s\psi_s(t)/\psi_{1m}(r_m,t)$. It increases with increasing amplitude of the external source, $\psi_s(t)$, while it decreases with increasing outer reconnected flux, $\psi_{1m}(r_m,t)$, because $\Delta'_0 < 0$ for the stable equilibrium and $\Delta'_s > 0$. The surface current is in such a direction as to oppose the progress of the reconnection, $\Delta'(t) < 0$ in the initial evolution[11]. This negative sign of $\Delta'(t)$ confirms both the analysis of experiment[1][5] and the assumption in the NTM theory.

6 Summary

An improved boundary layer theory of forced magnetic reconnection[11] is applied to the study of the seed island formation. The theory takes into account correctly the effect of plasma inertia in the inner layer by introducing the non-constant-ψ matching[11]. By use of this theory, we found the following results. The seed island formation is described by the two reconnected fluxes. In the initial evolution, the outer reconnected flux grows on the time scale including the time scale of the source signal due to the MHD event and the ideal time scale for $\tau_A << \tau_\alpha$ which corresponds to the ideal marginal stability, $\Delta_0' \to -\infty$. The inner reconnected flux grows on the time scale including the time scale of the source signal and resistive kink mode. The sign of the surface current induced on a resonant surface is negative, $\Delta'(t) < 0$. We have also derived the integral equation which allows us to investigate the evolution of the seed island at $t \sim \tau_A S^{1/3}$. The integral equation can give the criterion for a transition to a non-linear evolution of the seed island formation.

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