Poloidal Plasma Rotation in the Presence of RF Waves in Tokamaks

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Abstract

It is well known that one of the consequences of strong RF heating is the deformation of the equilibrium distribution function that induces a change in plasma transport and plasma rotation. The poloidal plasma rotation during RF wave heating in tokamaks is investigated using a moment approach. A set of closed, self-consistent transport and rotation equations is derived and reduced to a single equation for the poloidal particle flux. The formulas are sufficiently general to apply to heating schemes that can be represented by a quasilinear operator.

1. Introduction

In the past years one of the most important achievements in magnetically confined fusion research was the discovery of the critical role played by the $\vec{E}\times\vec{B}$ sheared flow which, by damping out the turbulence, leads to an improvement of the plasma confinement [1]. This discovery also permitted to develop alternative methods to actively induce improved modes in tokamak plasmas. Ignoring the ion inertia term, the $\vec{E}\times\vec{B}$ flow can be represented by a radial electric field $E_r = (z_i e n_i)^{-1}\nabla P_i - v_{\theta i}B_\phi + v_{\phi i}B_\theta$. There are thus many sources that can be act on to change the $\vec{E}\times\vec{B}$ flow, eventually leading to different improved confinement modes in tokamak plasmas. The above equation also provides a link between the radial electric field E_r and the plasma rotations (either poloidal or toroidal). This shows that a change in plasma rotation has generally a strong impact on confinement.

The neoclassical transport theory gives a self-consistent description of the plasma rotation [2-5], but generally only considers the contribution from collisions. There are also many papers devoted to the study of active production mechanism of plasma rotation by means of external rf waves [6-14]. In present tokamaks, the external non-ohmic heatings play a dominant role. Some of these may be able to significantly distort the local equilibrium particle distribution away from a Maxwellian distribution(see e.g. Ref.[15-17]). It then results a change in the neoclassical plasma rotation. It is therefore realistic to reexamine the plasma rotation taking into account the strong non-ohmic heatings. In this paper, we develop a general neoclassical theory of poloidal rotation in presence of rf waves in tokamaks using the moment approach given in [4].

2. Basic equations

The moment method starts with the conventional kinetic equation in tokamak plasmas:

$$\frac{\partial f^\alpha}{\partial t} + \vec{v}\cdot\nabla f^\alpha + \vec{F}\cdot\nabla_v f^\alpha = C(f^\alpha) \tag{1}$$

where the standard notations are used. The superscript α represents the species of the particles. The right hand side of this equation is a sum of a collision term $C^\alpha(f) = \sum_{\beta=e,i} C^{\alpha\beta}\left(f^\alpha, f^\beta\right)$. Using an expansion of the distribution function in Hermite polynomials, then Eq.(1) leads to a set of poloidal flux equations [4]:

$$\vec{\omega}^{\alpha} + \vec{\vec{\sigma}}_g^{\alpha} \cdot \vec{g}_{//}^{\alpha} = \vec{\vec{\sigma}}_s^{\alpha} \cdot \vec{s}_{//}^{\alpha} \tag{2}$$

with

$$\vec{\omega}^{e} = \left\{ \alpha\omega_1^i - \omega_1^e, \omega_3^e, \omega_5^e \right\}, \qquad \vec{\omega}^{i} = \left\{ 0, \omega_3^i, \omega_5^i \right\},$$

$$\vec{g}_{//}^{\alpha} = \left\{ \left\langle \frac{B}{B_0} \bar{g}_{//}^{\alpha(1)} \right\rangle, \left\langle \frac{B}{B_0} \bar{g}_{//}^{\alpha(3)} \right\rangle, \left\langle \frac{B}{B_0} \bar{g}_{//}^{\alpha(5)} \right\rangle \right\}, \qquad \vec{s}_{//}^{\alpha} = \left\{ \left\langle g_\rho^{(1)A} \right\rangle, \left\langle g_\rho^{(1)P} \right\rangle, \left\langle g_\rho^{\alpha(1)} \right\rangle \right\},$$

$$\vec{\vec{\sigma}}_g^{e} = \begin{pmatrix} \tilde{\sigma}_{//} & -\tilde{\alpha}_{//} & -\tilde{\gamma}_{//} \\ \tilde{\alpha}_{//} & -\tilde{\kappa}_{//}^{e} & -\tilde{\delta}_{//}^{e} \\ \tilde{\gamma}_{//} & -\tilde{\delta}_{//}^{e} & -\tilde{\varepsilon}_{//}^{e} \end{pmatrix}, \qquad \vec{\vec{\sigma}}_g^{i} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\tilde{\kappa}_{//}^{i} & -\tilde{\delta}_{//}^{i} \\ 0 & -\tilde{\delta}_{//}^{i} & -\tilde{\varepsilon}_{//}^{i} \end{pmatrix}$$

$$\vec{\vec{\sigma}}_s^{e} = \begin{pmatrix} \tilde{\sigma}_{//} & -K_e & 0 \\ \tilde{\alpha}_{//} & 0 & -K_e \\ \tilde{\gamma}_{//} & 0 & 0 \end{pmatrix}, \qquad \vec{\vec{\sigma}}_s^{i} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -K_i \\ 0 & 0 & 0 \end{pmatrix}$$

and ω_n denotes the poloidal fluxes, $\omega_n^\alpha = h_\theta^{\alpha(n)} B_0 \ell_p \ell_\varsigma / R_0 B_p$ and $h_\theta^{\alpha(1)} = \sqrt{m_\alpha / T_\alpha} \int d\vec{v}\, v_\theta f^\alpha / n_\alpha$, n=1,3,5. The coefficients with tilde are the classical transport coefficients (see e.g. Ref.[4]). Finally $\alpha = \sqrt{m_e T_i / m_i T_e}$ and $K_\alpha = I\ell_\rho / R_0 B_P \Omega_{\alpha 0} \tau_\alpha$. Obviously Eq.(2), together with the solubility constraint

$$< B\bar{g}_{//}^{e(1)} / B_0 > + \alpha A < B\bar{g}_{//}^{i(1)} / B_0 > = 0 \tag{3}$$

where $A = \tau_e \Omega_e / \tau_i \Omega_i$, forms a self-consistent set of equations that determines the poloidal flux ω_n^α.

In order to solve Eq.(2) and (3) we derive an expression for the so-called “addition source terms” $\bar{g}_{//}^{\alpha(n)}$ in terms of the classical thermodynamic sources $g_\rho^{\alpha(n)}$. This is done by solving the drift kinetic equation [4]. In the framework of quasilinear wave theory, the drift kinetic equation is as follows:

$$\frac{\partial f}{\partial t} + (\vec{v}_{//} + \vec{v}_d) \cdot \nabla f + \vec{E} \cdot \nabla_v f = C(f) + Q(f) \tag{4}$$

where Q(f) is the quasilinear rf diffusion operator. The term proportional to the electric field accounts for internally produced fields. The procedure which we use to solve the kinetic equation is by now a standard one (see e.g. Ref. [4]) but requires to be adapted in order to apply to a non-ohmic plasma.

3. Perturbed distribution function

The non-equilibrium distribution function f^α is assumed not to be too different from the equilibrium distribution function f_{eq}, which in the presence of a strong heating is distorted from Maxwellian distribution f_M^α. We thus have

$$f^\alpha = f_{eq}^\alpha (1 + \chi_C^\alpha) = f_M^\alpha (1 + \chi_Q^\alpha)(1 + \chi_C^\alpha)$$

where we assume the ordering $\chi_Q^\alpha << 1, \chi_C^\alpha << 1$. The distribution functions are expanded according to the usual mixed neoclassical orderings in the banana region:

$$f^\alpha = f_0^0 + f_1^0 + f_0^1 + f_1^1 + \ldots \approx f_M (1 + \chi_Q + \chi_C) \tag{5}$$

Since the rf wave only affects a specific part of the particle population, the resonant particles, the detailed solution of the kinetic equation strongly depends on the heating mechanism. The local equilibrium distribution function can be obtained from the lowest order bounce-averaged equation:

$$\left\langle C(f_{eq}^{\alpha}) + Q(f_{eq}^{\alpha}) \right\rangle_b = 0 \tag{6}$$

The deviation to the equilibrium state is obtained by solving the next order contribution to the drift kinetic equation. For the first order distribution function, the collision operator is[4]

$$C(f_1^0) = \frac{2\nu_\alpha}{B}\sqrt{1-\lambda B}\frac{\partial}{\partial\lambda}\lambda\sqrt{1-\lambda B}\frac{\partial f_1^0}{\partial\lambda} + v_{//}N_C(f_0)$$

Similarly, the quasilinear rf operator can also be written in the form of the pitch-angle scattering term plus a reminder

$$Q(f_1^0) = \frac{m_\alpha}{BT}\frac{\sqrt{1-\lambda B}}{x}\frac{\partial}{\partial\lambda}\frac{2\lambda D_\perp \delta(\omega - n\Omega - k_{//}v_{//})}{\sqrt{1-\lambda B}}\frac{\partial f_1^0}{\partial\lambda} + v_{//}N_Q(f_0)$$

From the first order equation, it is easy to get the first order distribution function in the form

$$f_1^0 = -v_{//}\frac{J(\rho)}{R_0 B_P}\frac{1}{\Omega}L_Q f_M + G \equiv \left(v_{//}\, G_0(E,\rho) + G_1\right) f_M \tag{7}$$

Here $L_Q \equiv \left(1+\chi_Q\right) f_M^{'} / f_M + \chi_Q^{'}$ accounts for the contribution of the rf heating to the classical source terms $g_\rho^{\alpha(n)}$ and

$$f_M^{'} \equiv \frac{\partial f_M}{\partial\rho} = -\frac{\ell_\rho}{\tau_\alpha}\left(\frac{m_\alpha}{T_\alpha}\right)^{1/2}\left[\left(g_\rho^{\alpha(1)} - g_\rho^{\alpha(1)A}\right) + \sqrt{\frac{2}{5}}(x-\frac{5}{2})g_\rho^{\alpha(3)}\right]f_M$$

As usual we have $G_1 = 0$ for trapped particles and

$$G_1 = \frac{1}{m_\alpha}\int dM\left\{\frac{\left\langle B\,\tilde{L}(E,M,\rho)G_0\right\rangle_B}{\left\langle \tilde{L}(E,M,\rho)\,v_{//}\right\rangle_B} - \frac{1}{v_\alpha}\frac{M^{-1}\int dM\left\langle B\left(N_C + N_Q\right)\right\rangle_B}{\left\langle \tilde{L}(E,M,\rho)\,v_{//}\right\rangle_B}\right\} \tag{8}$$

for passing particles. This contribution includes two different effects from the heating. First $N_C + N_Q$ corresponds to the part of the collision and the heating operator which are not of the form of a pitch angle operator. They are considered here as source terms to be determined by some self-consistent constraints. Second, there is a direct influence of the heating through the factor

$$\tilde{L} \equiv 1 + \frac{m_\alpha}{T_\alpha}\frac{D_\perp}{v_\alpha}\frac{\delta(\omega - n\Omega - k_{//}v_{//})}{x\left(1-\lambda B\right)} = 1 + \frac{2D_\perp}{v_\alpha}\frac{\delta(\omega - n\Omega - k_{//}v_{//})}{v_{//}^2}$$

Here $\{\lambda, x\} = \left\{\frac{M}{E - e_\alpha\Phi}, \frac{E - e_\alpha\Phi}{T_\alpha}\right\}$ are velocity related variables.

4. Expansion of the perturbed distribution function in Lagurre-Soine polynomials

One of the advantages of the moment approach given in Ref.[4] is that we can develop the transport analysis without knowing the explicit forms of $N_C + N_Q$. These quantities are replaced by approximate expressions expanded in Laguerre-Sonine polynomials. Performing the pitch-angle average of the perturbed part of the distribution function, we get

$$\hat{\chi}(x) = \sum_\sigma \frac{B}{4}\int_0^{1/B} d\lambda\left\{\chi_Q + G_0 + G_1 + G_{12}\right\} = \hat{\chi}_Q + \hat{\chi}_0 + \hat{\chi}_1 + \hat{\chi}_{12} \tag{9}$$

The quantity $\hat{\chi}$ is then expanded in series of Laguerre-Sonine polynomials

$$\hat{\chi}(x,\rho) = \sqrt{x}\sum_{n=0}^{\infty} a^n L_n^{3/2}(x) = \sqrt{x}\left[a^0\, L_0^{3/2}(x) + a^1\, L_1^{3/2}(x) + a^2\, L_2^{3/2}(x) + \ldots\right] \tag{10}$$

where $a^n = \frac{2}{\sqrt{\pi}}\int_0^\infty dx\, x\, e^{-x}\hat{\chi}(x,\rho)L_n^{3/2}(x)$ and $L_n^{3/2}(x)$ is the Laguerre-Sonine polynomials.

We notice that in Eq.(9) the first three terms of the right-hand side can be evaluated explicitly whereas the last term, arising from the non-pitch-angle part of the collision and heating operators, is yet to be determined. Using the expansion $\hat{\chi}_{12}(x,\rho) = \sqrt{x}\sum_{n=0}^{\infty} c^n L_n^{3/2}(x)$ and using the relation: $h_{//}^{\alpha(2n+1)} = \sqrt{3}\, a^n$, we get the coefficient c^n in terms of the poloidal flux

$$c^n = \mu_{n1}\omega_1 + \mu_{n2}\omega_3 + \mu_{n3}\omega_5 + \mu_{n4}(g_\rho^{\alpha(1)} - g_\rho^{\alpha(1)A}) + \mu_{n5}\, g_\rho^{\alpha(3)} + \mu_{n0} \tag{11}$$

Up to now we have successfully expressed the perturbed distribution function in terms of the poloidal flux and the plasma parameters.

5. Calculation of generalized stress tensor

It now remains to use the explicit form of the non-equilibrium distribution function in the evaluation of the addition source terms. We have

$$< \frac{B}{B_0} g_{//}^{\alpha(2i-1)} > = -\frac{4\pi}{\sqrt{5}}\frac{1}{n_\alpha B_0}(\frac{T}{m})^{3/2} < \int_0^\infty dx \int_0^{1/B} d\lambda\, B^2\, x\, F_{2i}(x)\,[C(f)+Q(f)] >, \quad i = 1,2,3 \tag{12}$$

where $F_2(x) = 1$, $F_4(x) = \sqrt{2}(x - 5/2)$, $F_6(x) = \sqrt{2/7}\,(x^2 - 7x + 35/4)$. The calculation is straightforward and the result of the evaluation can, similarly to Eq.(2), be cast in matrix form:

$$\vec{g}_{//}^{\alpha} = \vec{\vec{\eta}}^{\alpha}\cdot\vec{\omega}^{\alpha} + \vec{\vec{\eta}}^{\alpha s}\cdot\vec{s}^{\alpha} + \omega_1^i\,\vec{\zeta}^{\alpha} \tag{13}$$

where

$$\vec{s}^{\alpha} = \left\{\left\langle g_\rho^{\alpha(1)} - g_\rho^{\alpha(1)A}\right\rangle, \left\langle g_\rho^{\alpha(3)}\right\rangle, 1\right\} \tag{14}$$

$$\vec{\zeta}^{\alpha} = \left\{-\alpha\xi_{11}^{\alpha}, -\alpha\xi_{21}^{\alpha}, -\alpha\xi_{31}^{\alpha}\right\} \tag{15}$$

$$\vec{\vec{\eta}}^{e} = -\begin{pmatrix} -\xi_{11}^e & \xi_{12}^e & \xi_{13}^e \\ -\xi_{21}^e & \xi_{22}^e & \xi_{23}^e \\ -\xi_{31}^e & \xi_{32}^e & \xi_{33}^e \end{pmatrix};\ \vec{\vec{\eta}}^{i} = -\begin{pmatrix} 0 & \xi_{12}^i & \xi_{13}^i \\ 0 & \xi_{22}^i & \xi_{23}^i \\ 0 & \xi_{32}^i & \xi_{33}^i \end{pmatrix};\ \vec{\vec{\eta}}^{\alpha s} = -\begin{pmatrix} \xi_{14}^\alpha & \xi_{15}^\alpha & \xi_{10}^\alpha \\ \xi_{24}^\alpha & \xi_{25}^\alpha & \xi_{20}^\alpha \\ \xi_{34}^\alpha & \xi_{35}^\alpha & \xi_{30}^\alpha \end{pmatrix} \tag{16}$$

It is worth to remark that for the particle species not interacting with rf wave we have $\vec{\vec{\eta}}^{\alpha(s)} = 0$ and $\vec{\vec{\eta}}^{\alpha}$ reduces to the expression known for the purely ohmic-heating case.

6. Poloidal rotation

Substituting Eq(13) into Eq.(2), we have

$$\vec{\omega}^{\alpha} = \vec{\vec{L}}_{\alpha}^{-1}\cdot\left\{\vec{\vec{\sigma}}_s^{\alpha}\cdot\vec{s}_{//}^{\alpha} - \vec{\vec{\sigma}}_g^{\alpha}\cdot(\vec{\vec{\eta}}^{\alpha}\cdot s^{\alpha})\right\} - \vec{\vec{L}}_{\alpha}^{-1}\cdot(\vec{\vec{\sigma}}_g^{\alpha}\cdot\vec{\zeta}^{\alpha})\,\omega_1^i \tag{17}$$

with $L_\alpha = \left(\vec{\vec{1}} + \vec{\vec{\sigma}}_g^{\alpha}\cdot\vec{\vec{\eta}}\right)$. Eq.(17) is obviously of the form $\vec{\omega}^{\alpha} = \vec{H}^{\alpha} - \vec{B}^{\alpha}\,\omega_1^i$ with

$$\vec{H}^{\alpha} = \vec{\vec{L}}_{\alpha}^{-1}\cdot\left\{\vec{\vec{\sigma}}_s^{\alpha}\cdot\vec{s}_{//}^{\alpha} - \vec{\vec{\sigma}}_g^{\alpha}\cdot(\vec{\vec{\eta}}^{\alpha}\cdot s^{\alpha})\right\} = (H_1^{\alpha}, H_2^{\alpha}, H_3^{\alpha})$$

$$\vec{B}^{\alpha} = \vec{\vec{L}}_{\alpha}^{-1}\cdot(\vec{\vec{\sigma}}_g^{\alpha}\cdot\vec{\zeta}^{\alpha}) = (B_1^{\alpha}, B_2^{\alpha}, B_3^{\alpha})$$

Combining Eq.(17) and the solubility constraint Eq.(3), we get the ion poloidal rotation ω_1^i:

$$\omega_1^i = \Delta^{-1}\left(\Delta^i + \Delta^e\right) \tag{18}$$

where

$$\Delta = (\alpha + B_1^e)\xi_{11}^e + B_2^e\,\xi_{12}^e + B_3^e\,\xi_{13}^e + \alpha A(\xi_{11}^i + B_2^i\,\xi_{12}^i + B_3^i\,\xi_{13}^i)$$

$$\Delta^e = H_1^e \xi_{11}^e + H_2^e \xi_{12}^e + H_3^e \xi_{13}^e + \xi_{14}^e < g_\rho^{e(1)} - g_\rho^{e(1)A} > + \xi_{15}^e < g_\rho^{e(3)} > + \xi_{10}^e$$

and

$$\Delta^i = \alpha A (H_2^i \xi_{12}^i + H_3^i \xi_{13}^i + \xi_{14}^i < g_\rho^{i(1)} - g_\rho^{i(1)A} > + \xi_{15}^i < g_\rho^{i(3)} > + \xi_{10}^i$$

Eq.(18) gives the ion poloidal rotation in terms of the classical thermodynamic sources and in terms of the heating power. The electron poloidal rotation is readily obtained from

$$\omega_1^e = \alpha \omega_1^i - (H_1^\alpha - B_1^\alpha \ \omega_1^i) = (\alpha + B_1^\alpha)\ \omega_1^i - H_1^\alpha \tag{19}$$

Eqs.(18) and (19) give a general expression for the plasma rotation during rf wave heating. They are valid whatever type of cyclotron heating scheme is used. A detailed study can only be done by specifying heating scheme. For example during ion cyclotron resonance heating we can evaluate the ion poloidal rotation and compare the result obtained from Eq.(18) [18] and the ohmic result given in Ref.[4]. It is then shown that: a) The poloidal flux depends on both the density gradient and temperature gradient. b) The poloidal flux depends on the temperature gradient in a very complex manner, as can be seen from Eq.(13) which shows the significant differences with that in the pure collision plasmas. c) Some terms are independent of the density or temperature gradients, but also contribute the poloidal flux. The existence of these terms makes the plasma poloidal rotation sensitive to the heating mechanism. A more detailed analysis will be given in another self-contained paper.

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