Accretion Theory of “Spontaneous” Rotation in Toroidal Plasmas

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Abstract. The accretion theory of spontaneous toroidal rotation connects this phenomenon to the energy and particle transport properties of the plasma column and to the relevant collective modes. The consequent prediction that an inversion of the velocity direction in the transition from the H to the L regime should occur has been verified by the experiments. The theory is consistent also with the observation that the velocity is depressed when a peaked density profile appears as a result of a transport barrier. The fact that the rotation velocity increases with the total energy content is explained by the fact that the inflow of angular momentum, whose source is at the edge of the plasma column, results from the excitation of modes driven by the plasma pressure gradient. A quasi-linear derivation of the angular momentum transport produced by these modes, whose novel feature is the inflow, is given and a model of the relevant (transport) equation is solved. Fluctuations at the edge of the plasma column are considered responsible for the scattering, out of confinement, of particles that transfer, to the surrounding material wall, angular momentum in the same direction as that of the phase velocity of the prevalent modes. Thus the fact that the plasma rotates in the direction of the ion diamagnetic velocity in the H-regime, when the prevalent modes are expected to have phase velocity in the direction of the electron diamagnetic velocity, can be explained and the rate of rotation decrease, as the plasma current is increased, can be justified.

1. Link Between Spontaneous Rotation and Transport

The accretion theory [1] of the “spontaneous rotation” phenomenon [2] observed in axisymmetric toroidal plasma experiments has tied this phenomenon to the transport properties of the plasma column and to the modes responsible for them. In particular, the consequent prediction that the rotation velocity should invert when a drastic change in the confinement characteristics is induced as in the transition from the so-called L-regime to the H-regime, has been confirmed by the experiments [3,4]. The more recent observation [5] of a significant change in the velocity of rotation corresponding to the onset of a transport barrier in the main body of the plasma column is another confirmation of the same feature. A key experimental finding is that spontaneous rotation is not associated with the injection of RF waves [3] as this is seen both in the H-regime and in the L-regime when only Ohmic heating is present (Fig.1). Thus the influence of Ion Cyclotron heating on rotation is indirect in that the plasma thermal energy and consequently the ion pressure gradient is increased by this additional heating. In fact the accretion theory which associates the rotation with this gradient is consistent with the observation that the rate of rotation $\Delta v_{\phi} \propto W_{th}/I_p$ where W_{th} is the total plasma thermal energy and I_p is the plasma current.

2. Flow of Angular Momentum

In the context of the accretion theory, the source of the angular momentum acquired by the central part of the plasma column is at the outer edge of it [6]. Therefore a process for angular momentum inflow (“pinch”) has to be introduced and associated with collective modes that are driven by the plasma pressure gradient. The transfer of opposite angular momentum to the material wall surrounding the plasma is considered to balance that acquired by the plasma column. In particular, particles interacting with modes localized at the edge of this can be scattered to the wall transferring, to it, angular momentum in the same direction as the mode phase velocity.

When the plasma is in the H regime a strong particle density gradient is formed at the edge of the plasma column and it is reasonable to assume that ion temperature gradient driven modes, which have a phase velocity in the direction of the ion diamagnetic velocity, are not excited. The strong density gradient on the other hand favors the onset of radially localized modes whose frequency is proportional to the electron drift mode frequency that is proportional to the density gradient and decreases with the poloidal field $B_p \propto I_p/a$, a being the plasma minor radius. Referring to the simplest, large aspect ratio toroidal configuration, the relevant electric field fluctuations can be represented as $\hat{E} \simeq \tilde{E}(r,\vartheta)\cos\,(\omega - m^0\vartheta + n^0\phi)$ where ϑ and ϕ are the poloidal and toroidal angle, respectively. The toroidal phase velocity is $\omega R_0/n^0$, R_0 being the torus major radius. In particular, $\omega R_0 / n_0 \propto cT_e /(eB_p) d \ln n / dr \equiv \mathrm{v}_{*\mathrm{p}}$. Thus, when the plasma is in the H-regime, the main body of it should acquire angular momentum in the direction opposite to that of $\mathrm{v}_{*\mathrm{p}}$, that is in the direction of the ion diamagnetic velocity, a feature that is clearly seen in the experiments. We note that, the existence of an active region with relatively strong fluctuations [7] near the outer edge of the plasma column, and the presence of a steep density gradient, has been verified in experiments by the Alcator C-Mod machine, in the H-mode regime. In particular a mode whose phase velocity has a direction and value consistent with v_{*p} has been identified [8]. We also note that the observation that $\Delta \mathrm{v}_\phi$ decreases as B_p increases is consistent with the fact that the radial excursion of the single particle orbits decreases as B_p increases as well as with the fact that the mode amplitude can be expected to be correlated with the value of v_{*p}.

Following the same line of reasoning we argue that when the plasma column is in the L-regime the high values that the thermal diffusivity acquires at the edge of the plasma column are the result of the excitation of ion temperature gradient (ITG) driven modes. The phase velocity of these modes of which there are two kinds, one that can be found in a one dimensional geometry [9,10] and one that depends on the presence of a toroidal curvature [11], is in the direction of the ion diamagnetic velocity $\mathrm{v}_{pi} = -c/(eB_p n) dp_i / dr$. We consider that the effect of these modes is prevalent at the edge of the plasma column and that in this region particles interacting with these modes, are then scattered to the wall. Then the main body of the plasma column will rotate in the opposite direction of v_{pi}, that of the electron diamagnetic velocity. Thus a reversal of the toroidal velocity of the central part of the plasma column should be observed in the transition from the L-regime to the H-regime. This has been clearly seen in experiments with ohmic heating only carried out by the Alcator C-Mod machine [3]. The same reversal has been observed clearly in experiments where ICRH heating is prevalent by the Tore Supra machine [4] in the transition from the L-mode regime to an enhanced confinement regime with peaked density profiles. The values of the velocity in the ohmic H-regime produced by Alcator C-Mod have been in the range 3-8 km/sec.

3. Model Transport Equation

The experiments indicate clearly that the transport of angular momentum is faster than predicted by the collisional transport theory, when the source of heating that maintains the plasma thermal energy is sharply decreased [2]. Another experimental indication is that the toroidal velocity extends to the center of the plasma column with profiles that are peaked [2] or that are flat [5] locally.

Considering that, in the accretion theory, the source of angular momentum is near the edge of the plasma column, a simplified transport model can be adopted, to describe some of the observed features, that is similar to the well known one introduced earlier [12] for the particle transport. The latter reproduces the observed centrally peaked density profiles when a source is present at the edge of the plasma column (e.g., due to gas injection). The simplified model balances an inflow velocity of the specific angular momentum density $m_i nJ$, where $J = R\mathrm{v}_\phi$, and an outward diffusion, in the interior of the plasma column, as exemplified by a flux Γ_J of the form

$$\Gamma_J \simeq -m_i n\left(J_0 \mathrm{v}_J + D_J \frac{\partial J}{\partial r} \right)\mathbf{e}_r \qquad (1)$$

to be used within the conservation equation

$$\frac{\partial m_i nJ}{\partial t} + \nabla \cdot \Gamma_J = S_J \qquad (2)$$

where S_J is a source localized at the edge. The ratio v_J / D_J is taken to be an increasing function of the radius in the main body of the plasma column, as in the case of the particle transport model equation, and in particular

$$\frac{\mathrm{v}_J}{D_J} \simeq 2\frac{r}{a^2}\alpha_J \qquad (3)$$

where $\alpha_J \simeq$ const. near the center of the plasma column. Clearly, when $\alpha_J \sim 1$ the stationary profile resulting from $\Gamma_J \simeq 0$ in this region, $J \simeq J_0(1-\alpha_J r^2/a^2)$, is peaked, while if α_J is small the profile is flat. In order to simulate the kinds of profiles that have been observed experimentally, Eq. (2) has been solved numerically (by I. Dimov and S. Kurebayashi) using an ad hoc source of angular momentum localized at $r = a$ for different values of α_J and models for D_J (Fig.2) considering that transport analyses of well confined plasmas indicate that the relevant thermal coefficients are strongly increasing functions of the plasma radius.

4. Quasi-linear Theory

Here we demonstrate that electrostatic modes driven by the ion pressure gradient can produce an inflow velocity of angular momentum and find an expression for Γ_J from quasi-linear theory that can lend support to Eq. (2). For the sake of simplicity we refer to a one dimensional plane geometry where the magnetic field is along the $\mathbf{e}_z$ direction. A flow velocity $\mathrm{v}_\parallel(x)\mathbf{e}_z$, along the magnetic field is present and is smaller that the ion thermal velocity v_{thi}. We note that, in the case of deuterons, $\mathrm{v}_{thi} \simeq 310\ \mathrm{km/sec} \times (T_i / 1\ \mathrm{keV})^{1/2}$. Since the maximum observed flow velocity in the Alcator C-Mod experiments does not exceed 100 km/sec in the center of the plasma column where T_i is higher than 1 keV, this is well below v_{thi}. We consider electrostatic modes, represented by $\hat{\phi} \simeq \tilde{\phi}(x_0)\exp\left(-i\omega t + ik_y y + ik_\parallel z\right)$, that are localized around a surface $x = x_0$ and define the Doppler shifted frequency $\varpi \equiv \omega - k_\parallel \mathrm{v}_\parallel(x_0)$. For $\varpi > k_\parallel \mathrm{v}_{thi}$ the perturbed longitudinal momentum conservation equation is $m_i n\left(-i\varpi\tilde{\mathrm{v}}_\parallel + \tilde{\mathrm{v}}_{Ex}\, d\mathrm{v}_\parallel / dx\right) = -ik_\parallel \tilde{p}_i - ik_\parallel e n\tilde{\phi}$, where $\tilde{\mathrm{v}}_{Ex} = \tilde{E}_y c / B = -ik_y c\tilde{\phi}/B$ and $i\varpi\tilde{p}_i \simeq -\tilde{\mathrm{v}}_{Ex}\, dp_i / dx$. In particular, we consider $\left(\varpi/k_\parallel\right) d\mathrm{v}_\parallel/dx \sim (dp_i/dx)/(m_i n)$ and $|dp_i/dx| c/(eBn) > |\varpi/k_y|$. Then

$$\tilde{\mathrm{v}}_\parallel \simeq -\frac{k_y}{\varpi}\frac{c}{B}\tilde{\phi}\left[\frac{d\mathrm{v}_\parallel}{dx} + \frac{k_\parallel}{\varpi}\frac{1}{m_i n}\frac{dp_i}{dx}\right]. \qquad (4)$$

We note that the velocity gradient term $d\mathrm{v}_\parallel / dx$ is a destabilizing factor for modes in the frequency range $k_\parallel \mathrm{v}_{thi} < \varpi < k_\parallel \mathrm{v}_{the}$ when [13,14] $k_y k_\parallel\, d\mathrm{v}_\parallel / dx > 0$ and $k_y / k_\parallel < 0$ is the sign that we consider referring to the case where $d\mathrm{v}_\parallel / dx < 0$ for $\mathrm{v}_\parallel > 0$ as this applies to the nearly stationary velocity profiles established within the plasma column. In particular, if we refer to modes for which $\mathrm{sgn}\,\varpi = \mathrm{sgn}\,k_y$ such as $\varpi \propto k_y \mathrm{v}_{*e}$ where $\mathrm{v}_{*e} \equiv -cT_e/(eB) d\ln n/dx$, we see that the term proportional to dp_i/dx in Eq. (4) is of opposite sign to that of $d\mathrm{v}_\parallel/dx$. To illustrate this point further we note that the perturbed ion guiding center conservation equation is $\partial\hat{n}_i/\partial t + \mathrm{v}_{\mathrm{Ex}}\, dn/dx + ik_\parallel(n\hat{\mathrm{v}}_\parallel + \hat{n}_i \mathrm{v}_\parallel) = 0$ and we include [13,14] a very small dissipative term ν_D in Eq. (4) representing, for instance, the effects of finite longitudinal viscosity. Then if we take $\hat{n}_e \simeq \hat{n}_i$ and, since $\varpi < k_\parallel \mathrm{v}_{the}$, $n_e \simeq ne\hat{\phi}(1 - i\varepsilon_k)/T_e$ where $\varepsilon_k << 1$ takes into account, generically, different factors including nonlinear interactions that can drive the considered mode unstable. The resulting dispersion relation is

$$\varpi(1 - i\varepsilon_k) \simeq k_y \mathrm{v}_{*e} - \frac{cT_e}{eB}\frac{\varpi - i\nu_D}{\varpi^2} k_\parallel k_y \left[\frac{d\mathrm{v}_\parallel}{\mathrm{dx}} + \frac{k_\parallel}{\varpi}\frac{1}{m_i n}\frac{dp_i}{dx}\right] \tag{5}$$

and we argue that condition $k_y k_\parallel\, d\mathrm{v}_\parallel/dx > 0$ is the one for which the mode is easier to excite.

The relevant quasilinear flux $\Gamma_k = m_i n \langle \hat{\mathrm{v}}_\parallel^{(k)} \hat{\mathrm{v}}_{Ex}^{(-k)} + \hat{\mathrm{v}}_\parallel^{(-k)} \hat{\mathrm{v}}_{Ex}^{(k)} \rangle$ when using, for simplicity, Eq. (4) is

$$\Gamma_k \simeq -\frac{2\gamma_k}{\varpi_k^2}\langle \left|\hat{\mathrm{v}}_{\mathrm{Ex}}^{(\mathrm{k})}\right|^2 \rangle \left[m_i n \frac{d\mathrm{v}_\parallel}{dx} + 2\frac{k_\parallel}{\varpi_k}\frac{dp_i}{dx}\right]. \tag{6}$$

Here $\gamma_k = \mathrm{Im}\,\varpi_k$ is the mode growth rate and note that $D_k \equiv (2\gamma_k/\varpi_k^2)\langle\left|\mathrm{v}_{Ex}^{(k)}\right|^2\rangle$ is the familiar quasilinear "diffusion" coefficient. We see that the term dp_i/dx is responsible for the relevant inflow. Since, in the case we consider, the source of the momentum is at the edge of the plasma column, we argue that dp_i/dx drives the velocity gradient $d\mathrm{v}_\parallel/dx$ and keeps Γ_k relatively small. We consider the factor responsible for maintaining the mode amplitude to be the plasma pressure gradient which, in addition, drives the relevant thermal conductivity. Thus both γ_k and the mode amplitude $\tilde{\mathrm{v}}_{Ex}^{(k)}$ depend on this factor. Note that the inflow term $k_\parallel/(\varpi n)\, dp_i/dx$ is about $(k_\parallel/k_y)(eB/c)[T_i/T_e + (dT_i/dx)/(T_e\, d\ln n/dx)]$. Thus, in the process, $d\,\mathrm{v}_\parallel/dx$ should increase as the ion temperature gradient is increased. This is consistent also with the experimental observations that the rotation velocity increases when, at constant density, the plasma total thermal energy is increased. The experimental observation [5] that the rotation velocity decreases as a peaked density profile is produced in the central part of the plasma column, when a transport barrier is formed that does not change the ion temperature nor its profile, is also consistent with the fact that the inflow term becomes depressed when $d\ln n/dx$ increases.

Considering the limit where $\eta_i \equiv d\ln T_i/d\ln n \gtrsim \omega^2/(k_\parallel \mathrm{v}_{thi})^2 > 1$ the dispersion relation (5) yields also the ion temperature gradient driven mode, modified by the presence of $d\mathrm{v}_\parallel/dx$. The contribution of this to the spectrum of modes that can be excited is important. In particular, in the range of wavelengths where the mode frequency is nearly real and $(\varpi/k_\parallel)(d\mathrm{v}_\parallel/dx) \approx (dp_i/dx)/(m_i n)$, (taking $k_\parallel > 0$ so that $\omega/k_\parallel < 0$ while $k_y > 0$), the effect of ν_D is sufficient to maintain the mode unstable. An estimate of the effective diffusion coefficient D_{eff} for the angular momentum can be deduced from experiments in which the injected heating is turned off, thus forcing the rotation velocity to decrease. The decay time reported in

Ref. [2] for Alcator-C Mod is $\tau_D \simeq 70$ m/sec and for a rough estimate we may take $D_{eff} \simeq a^2\kappa/(4\tau_D)$ where κ is the plasma elongation. Thus, if we take $a \simeq 0.2$ m and $\kappa \simeq 1.6$, $D_{eff} \simeq 0.23$ m^2/sec and the density fluctuation level that can produce $D_k \sim D_{eff}$ is relatively low.

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References

[1] B. Coppi, *2000 International Sherwood Fusion Theory Meeting*, Paper 2C20 (Publ. Univ. California, Los Angeles, CA, March 2000) and earlier presentations.
[2] J.E. Rice *et al.*, *Nuclear Fusion* **39** (1999) 1175.
[3] J.E. Rice, *Bull. Am. Phys. Soc.* **44** (1999) 22.
[4] G. T. Hoang *et al.*, *Nuclear Fusion* **40** (2000) 920.
[5] J.E. Rice *et al.*, in *Fusion Energy 2000*, Paper IAEA-CN-77 EXP 5/24 (IAEA, Vienna, to be published).
[6] B. Coppi, G. Penn and L.E. Sugiyama, in *Fusion Energy 1998* (IAEA, Vienna, 2000), Paper IAEA-F1-CN-69-Th3/7.
[7] J. Snipes, Private Communication
[8] A. Mazurenko *et al.*, *Bull. Am. Phys. Soc.* **44** (1999) 257.
[9] L. I. Rudakov and R.Z. Sagdeev, *Soviet Phys.-Doklady* **6** (1961) 415.
[10] B. Coppi , M.N. Rosenbluth and R.Z. Sagdeev, *Phys. Fluids* **10** (1967) 582.
[11] B. Coppi and F. Pegoraro, *Nucl. Fusion* **17** (1977) 969.
[12] B. Coppi and N. Sharky, *Nucl. Fusion* **21** (1981) 1363.
[13] B. Coppi, M.I.T. Report PTP 89/2, RLE, Cambridge MA (1989), and B. Basu and B.Coppi , *J. Geophys. Res.* **34** (1989) 5314.
[14] B. Coppi, *Phys. Lett.* **A**, **201** (1) (1995) 66.

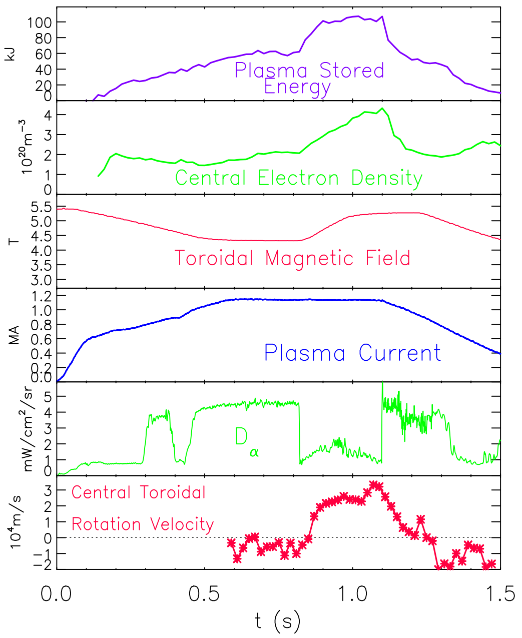

FIGURE 1. Experimental evidence of toroidal rotation in the absence of injected heating (courtesy of J.E. Rice, 2000).

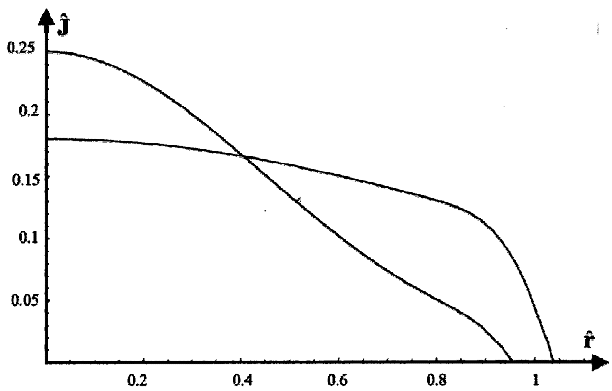

FIGURE 2. Specific angular momentum $\hat{J}$ profiles derived from the solution of Eq. (1) for different values of the inward transport velocity relative to the relevant diffusion coefficient as represented by the parameter α_J introduced in Eq. (3). The peaked velocity profile corresponds to a larger value of α_J.