Plasma Confinement with a Transport Barrier

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Abstract. Plasma confinement with a transport barrier such as an H-mode or an internal transport barrier mode (ITB-mode) is examined under the constraint due to the conservation of total angular momentum. The results are tested against actual experimental data and general characteristics of the plasma confinement with a transport barrier are well understood from this constraint. This implies that the confinement of tokamak plasmas can be determined by the decay rate of the total angular momentum. It also suggests that the confinement with a transport barrier is good since electrostatic fluctuations cannot affect this constraint, but that electromagnetic fluctuations such as ELM's can can cause the confinement to deteriorate.

1. Introduction

It has been well demonstrated that a single specie plasma such as an electron plasma or an ion plasma confined in a Penning trap has a very long lifetime. This long confinement is understood by the constancy of the total canonical angular momentum in a perfect cylindrical symmetric system [1, 2]. The total canonical angular momentum is given by $P_\theta = \sum_j \left[m_j u_\theta(r_j) r_j + e_j A_\theta(r_j) r_j \right]$, where the quantity $m_j u_\theta(r_j) r_j$ is the mechanical part of the angular momentum for the j-th particle, and the quantity $e_j A_\theta(r_j) r_j$ is the vector potential part. For a typical single specie plasma, the vector potential part dominates and the total angular momentum becomes $P_\theta = \frac{eB}{2} \sum_j r_j^2$ for a uniform magnetic field, where e is the charge of the single specie particle. Therefore, the constancy of the angular momentum places a strong constraint on the radial positions of the particles for a single specie plasma, and this is the essential physics of the long confinement.

On the other hand, in a case of a neutral plasma, pairs of ions and electrons at the same locations can escape together from the system without changing the vector potential term of the total angular momentum. This is the reason why the confinement of neutral plasmas is considered to be poor. However, the situation changes when the mechanical part of the total angular momentum dominates over the vector potential term. Then, the total angular momentum reduces to $P_\theta = \sum_j m_j u_\theta(r_j) r_j$, and the constancy of the total canonical angular momentum in a neutral plasma places a constraint similar to that for the single specie plasma.

The constraint due to the conservation of the total angular momentum in tokamaks will be examined in this paper. Section 2 will describe the method to reduce it to a variation problem that can be treated analytically and the analytical results will also be shown. Then, the results will be tested against actual DIII-D data in Section 3. It will be clearly demonstrated that this constraint plays an important role in plasma confinement with a transport barrier. In Section 4, implications of this constraint on plasma confinement for tokamak plasmas will be discussed.

2. Conservation of the Total Toroidal Angular Momentum in Tokamaks

In a toroidally symmetric system such as a tokamak, the total toroidal angular momentum P_ϕ is conserved. Let us consider a tokamak whose flux surfaces have a concentric circular cross-section. Taking an average over the flux surfaces, one may write

$$P_\phi = 4\pi^2 R_0 \int_0^a nmu_\phi R_0 rdr, \tag{1}$$

where u_ϕ is the toroidal component of the flow velocity u and R_0 is the major radius of the plasma. The total energy ε may be given by

$$\varepsilon = 2\pi R_0 \int_0^a \left(e\Phi\Delta n(r) + \frac{3}{2}\left(p_i(r) + p_e(r)\right) + \frac{1}{2} mnu^2 \right) 2\pi rdr. \tag{2}$$

Since the electric potential energy $e\Phi\Delta n(r)$ is negligible because of the small density difference $\Delta n = n_i - n_e$, the maximum plasma pressure corresponds to the minimum total flow energy

$$W_R = 4\pi^2 R_0 \int_0^a \frac{1}{2} nmu^2 rdr \tag{3}$$

Defining $u_{//}$ and $u_\perp$ as the flows parallel and perpendicular to the magnetic field **B**, we may write

$$u^2 = u_{//}{}^2 + u_\perp{}^2, \tag{4}$$

and

$$u_\phi = u_{//} \frac{B_\phi}{B} - u_\perp \frac{B_\theta}{B}. \tag{5}$$

Thus, the problem of finding the maximum plasma pressure is reduced to finding the minimum value for W_R for a given total number of particles

$$N = 4\pi^2 R_0 \int_0^a nrdr, \tag{6}$$

and for a given total angular momentum P_ϕ. Considering the density to be a function of the flow, i.e. $n(u_{//},\ u_\perp)$, this variation problem can be solved by using the Lagrangian undetermined multiplier method

$$\delta(W_R - \lambda_1 N - \lambda_2 P_\phi) = 0, \tag{7}$$

where λ_1 and λ_2 are constant. Equation (7) is written as

$$\left[(u^2 - \lambda_1 - \lambda_2 u_\phi) \frac{\partial n}{\partial u_{//}} + (2u_{//} - \lambda_2 \frac{\partial u_\phi}{\partial u_{//}}) n \right] \delta u_{//} + \left[(u^2 - \lambda_1 - \lambda_2 u_\phi) \frac{\partial n}{\partial u_\perp} + (2u_{//} - \lambda_2 \frac{\partial u_\phi}{\partial u_\perp}) n \right] \delta u_\perp = 0. \tag{8}$$

Accordingly, the solution for the density that satisfies Eq. (8) is given by

$$n = \frac{c}{u^2 - \lambda_1 - \lambda_2 u_\phi}, \tag{9}$$

where c is constant. Therefore, the density is determined when the flow $(u_{//},\ u_\perp)$ is given except for 3 adjustable parameters. Since the force balance equation must satisfy

$$\nabla p_i = -en(E_r + u_\perp B), \tag{10}$$

the ion pressure and, thus, the ion temperature can also be calculated by integrating Eq. (10) once the flow, the safety factor (or poloidal magnetic field) and the radial electric field are known.

In fusion experiments we try to maximize not only the plasma pressure, but also the plasma density. Therefore, the problem reduces to finding the maximum density and the minimum rotation energy for a given total toroidal angular momentum. That is, the following two equations must be satisfied simultaneously:

$$\delta(N - \lambda_3 P_\phi) = 0, \tag{11}$$

and

$$\delta(W_R - \lambda_4 P_\phi) = 0\,, \tag{12}$$

where λ_3 and λ_4 are constant. This is a very strong constraint that both $u_{//}$ and $u_\perp$ can no longer vary independently and one of them must be constant. A solution that satisfies experimental conditions is

$$u_{//} = u_0 = u_\phi(0)\ . \tag{13}$$

Then, Eq. (11) and Eq. (12) are written as

$$(1 - \lambda_3 u_\phi)\frac{\partial n}{\partial u_\perp} + \lambda_3 \frac{B_\theta}{B} n = 0, \tag{14}$$

and

$$(1 - \lambda_4 u^2)\frac{\partial n}{\partial u_\perp} + 2\lambda_4 u_\perp n = 0\,. \tag{15}$$

Eliminating $\dfrac{\partial n}{\partial u_\perp}$ and n, one obtains

$$\lambda_4(1+2\lambda_3)\left(\frac{B_\theta}{B}u_\perp\right)^2 - 2\lambda_4\left(\frac{B_\theta}{B}u_\perp\right) - (1-\lambda_4 u_0{}^2)\left(\frac{B_\theta}{B}\right)^2 = 0\,. \tag{16}$$

The solution for Eq. (16) that satisfies $u_\perp(0) = 0$ is given by

$$\frac{B_\theta}{B}u_\perp = u_1\left(-1+\sqrt{1+\lambda\frac{B_\theta{}^2}{B^2}}\right), \tag{17}$$

where λ and $u_\perp$ are constant. Using Eq. (5), one can rewrite the solution as

$$u_\phi = \frac{B_\phi}{B}u_0 - u_1\left(-1+\sqrt{1+\lambda\frac{B_\theta{}^2}{B^2}}\right)\ , \tag{18}$$

and the solution for the density is given by

$$n = \frac{1-\lambda_3 u_0}{1-\lambda_3 u_\phi}n(0)\,. \tag{19}$$

Since $B_\phi \approx B$, the solutions (18) and (19) can be approximated as

$$u_\phi \approx u_0 - u_1\left(-1+\sqrt{1+\lambda\frac{B_\theta{}^2}{B^2}}\right), \tag{20}$$

and

$$n = \frac{n(0)}{\sqrt{1+\lambda\dfrac{B_\theta{}^2}{B^2}}}\ , \tag{21}$$

or

$$n = \frac{n(0)}{1+\alpha\left(1-\dfrac{u_\phi}{u_o}\right)}\,. \tag{22}$$

This means that profiles of the plasma density and the safety factor q can be determined once the flow is given, or vice versa. Therefore, as indicated earlier, the profile of the ion temperature can also be calculated by integrating Eq. (10) once the flow is given assuming that the radial electric field is given or negligible.

3. Comparison with Experimental Data

The theoretical results have been tested against the DIII-D plasma profiles in its peak performance [3]. The red solid curves in Fig. 1 show the DIII-D profiles of the toroidal angular velocity u_ϕ / R_0, the plasma density n, the safety factor q, and the ion temperature T_i with an internal transport barrier (ITB) produced by a counter neutral beam injection with 5 beam sources. The profiles nearly correspond to its peak performance with the 5 beam sources at 1109 ms (shot number 99848). The blue dashed curves show the theoretical profiles generated from the experimental toroidal angular velocity by using Eqs. (20) - (22). The ion temperature curve is generated based on the following equation since the radial electric fields are usually calculated from Eq. (10).

$$T_i(r) = \frac{1}{kn(r)}\left[p_i(r)_{EX} + \int_0^r \{(enu_\perp B)_{EX} - (enu_\perp B)_{TH}\} dr \right], \qquad (23)$$

where k is the Boltzmann constant. The agreement between the theoretically generated profiles and the experimental profiles are remarkable except near the edge.

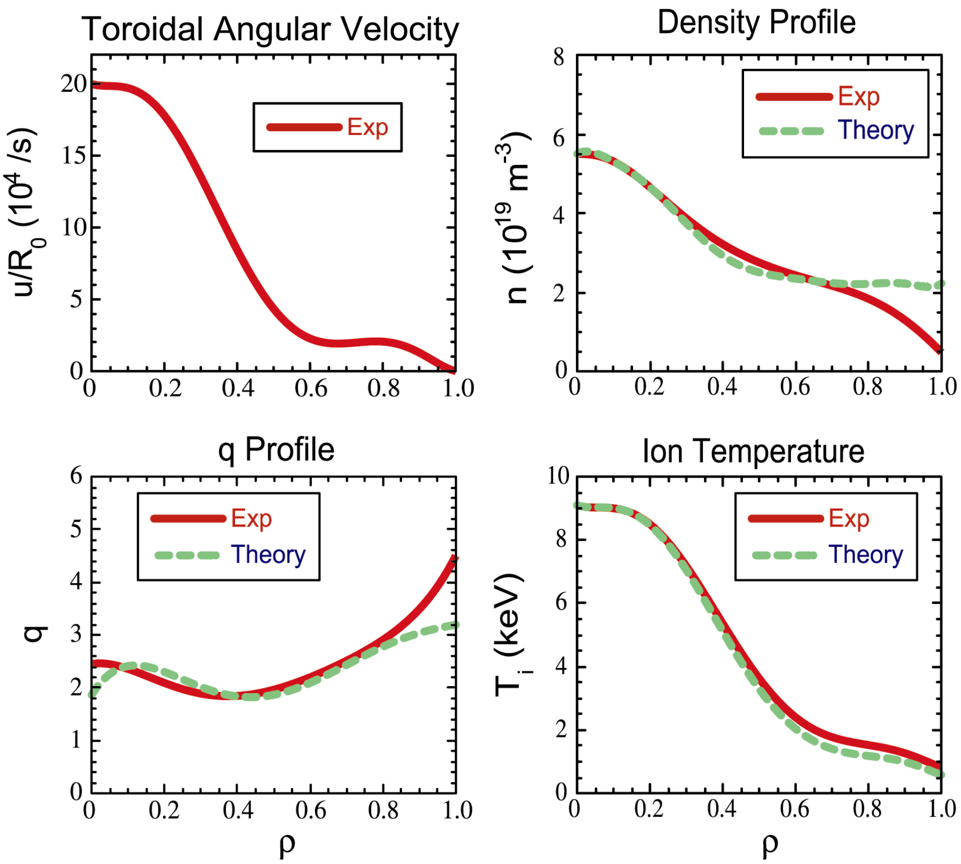

Fig. 1 Comparison of the DIII-D profiles for a counter NBI with 5 beam sources and the theoretical profiles generated from the experimental profile of the safety facter q.

Figure 2 shows the experimental (red) and theoretical (blue) profiles for a neutral beam injection with 6 beam sources at 1109 ms (shot number 99847). The ITB starts deteriorating with the higher power (6 sources) [3]. However, the agreements between the two profiles is still very good.

This agreement clearly demonstrates that the constraint due to the conservation of the total toroidal angular momentum plays an important role in the confinement of the DIII-D plasmas and that the general ITB characteristics of the best performing DIII-D plasmas are explained as the consequence of this constraint. The physical meaning of this constraint and its implications for the plasma confinement will be discussed in the next section.

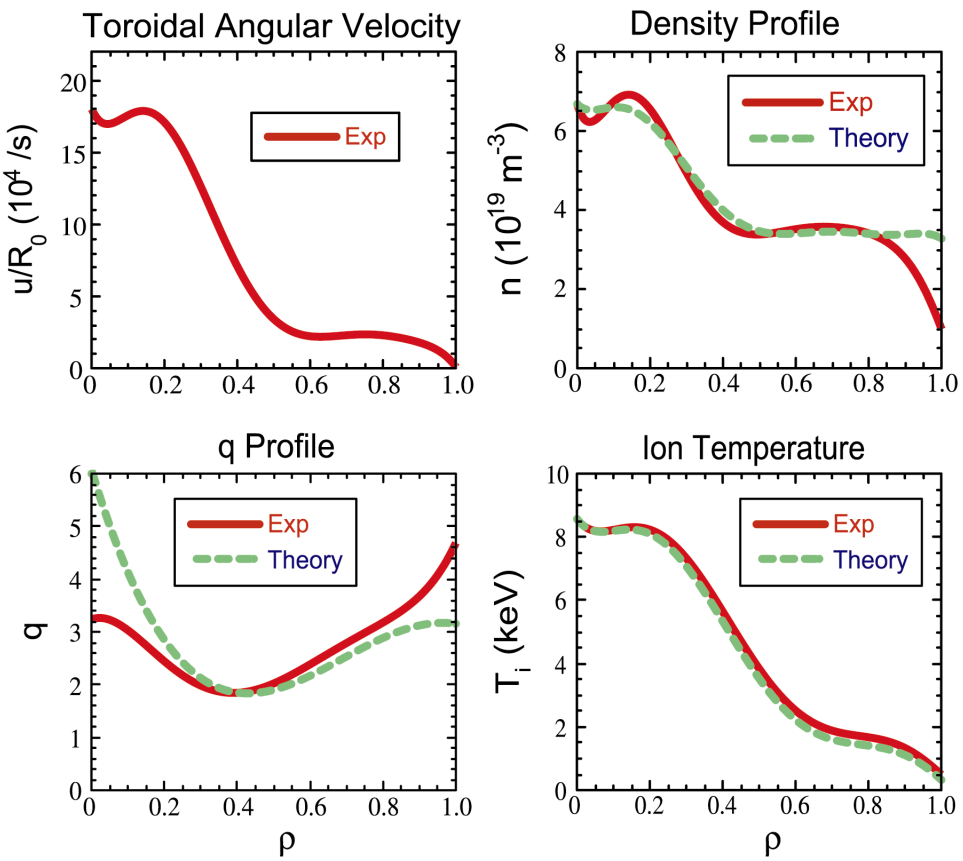

Fig. 2 Comparison of the DIII-D profiles for a counter NBI with 6 beam sources and the theoretical profiles generated from the experimental profile of the safety facter q.

There is another piece of evidence to support the role of this constraint. The theory shows that the density is directly related to the toroidal rotation while in the case of the ion pressure its gradient is associated with the toroidal rotation. Therefore, the location of the sharp density gradient is slightly different from that of the ion temperature, which is often observed in experimental profiles.

4. Confinement with a Transport Barrier

The condition for the minimum flow energy W_R with a given total toroidal angular momentum P_ϕ is equivalent to that for the maximum P_ϕ with a given W_R. This means that neither the plasma density profile nor the plasma pressure profile can change without losing the total toroidal angular momentum once the condition for the maximum P_ϕ is established. In other words, the plasma confinement is determined by the decay rate of the total toroidal angular momentum. Since the total angular momentum is conserved through Coulomb interactions including particle interactions with electrostatic fluctuations, good plasma confinement with the transport barrier is expected. On the other hand, electromagnetic interactions could change the total angular momentum. Therefore, plasma confinement with electromagnetic fluctuations such as ELM's or MHD activities could be poorer. In fact, ELMy H-modes show degraded confinement. The decay of the total angular momentum with the transport barrier without magnetic activities can occur only through interactions between the plasma and the outside world including interactions with neutrals. Therefore, a long confinement can be expected under this condition.

Since the profile is dictated, transport coefficients are globally constrained and they cannot be determined by local conditions. This may seem very controversial. However, electrostatic fluctuations could generate radial electric fields that may affect the plasma confinement through an $E \times B$ plasma rotation. On the other hand, in actual experiments $E \times B$ plasma rotation terms are usually smaller than $\nabla p_i \times B$ terms, suggesting that electrostatic fluctuations are not causing a large transport in good tokamak plasmas. Also, some theoretical studies indicate that the plasma condition with an ITB is close to the onset condition for an electrostatic instability such as the ion temperature gradient (ITG) mode. This could be explained in the following way. In an actual tokamak plasma, the angular momentum is driven from outside by some heating mechanisms such as neutral beam injection. Therefore, there is no guarantee that the driven angular momentum profile matches the maximum angular momentum profile. Therefore, it is conceivable that such an electrostatic instability as the ITG mode is needed to redistribute the angular momentum in order to create the maximum condition. In other words, the instability may be helping to trigger a good confinement condition rather than hurting the confinement.

On the other hand, the stability conditions for electromagnetic and/or MHD fluctuations could determine the confinement of tokamak plasmas since they can change the total angular momentum.

5. Remarks

It is amazing that the simple theoretical predictions agree well with the experimental data despite a number of approximations. For example, the DIII-D plasmas have an elongation of about 1.6 and the flux surfaces are not concentric while the theory assumes concentric circular cross-sections. There are some discrepancies between the theoretical density profiles and the experimental profiles near the edge. This may be partly due to the fact that the poloidal rotation u_θ may not be neglected near the edge. The poloidal rotation is expected to be small in tokamaks because of the existence of trapped particles. However, this condition may not hold near the edge because of a higher neutral particle density and possibly disturbed magnetic flux surfaces and toroidal ripple fields.

The theory also assumes the condition that the mechanical term of the angular momentum dominates the vector potential term. Let us now examine the condition:

$$m_i n(r)\left|u_\phi(r)\right| r >> e\frac{Br^2}{2}\left|\Delta n\right|. \qquad (24)$$

Since the u_ϕ value is usually larger than the $E \times B$ rotation

$$\left|u_\phi(r)\right| \geq \left|\frac{E}{B}\right|. \qquad (25)$$

By using Maxwell's equation

$$\varepsilon_o divE_r = e\Delta n, \qquad (26)$$

Inequality (16) is rewritten as

$$\frac{\omega_{pi}^2}{\omega_{ci}^2} >> \frac{a}{\delta_E}, \qquad (27)$$

where ω_{pi} and ω_{ci} are the ion plasma and cyclotron frequency, and the characteristic length of the localized radial electric field is expressed as

$$\delta_E = \frac{E_r}{\frac{dE_r}{dr}}. \qquad (28)$$

That is, the necessary condition is a very large ion plasma dielectric constant $\frac{\omega_{pi}^2}{\omega_{ci}^2}$. In a typical H-mode plasma, ω_{pi} near the edge and ω_{ci} are around 4×10^9 s^{-1} and 4×10^8 s^{-1}, respectively, and δ_E and a are of the order of 1 cm and 50 cm. Therefore, the condition (27) could be critical near the edge.

It should be noted that the actual toroidal rotation velocity represents the carbon impurity velocity measured by the Doppler shift method and that the deuteron rotation velocity is assumed to be the same. This assumption does not hold under certain conditions [4-6]. Interpretation of the carbon toroidal rotation near the edge should also be done with cautions.

6. Conclusions

In conclusion, the constraint due to the constancy of the total angular momentum well describes the general characteristics of the plasma transport with a transport barrier. This implies that the confinement of tokamak plasmas is determined by the decay rate of the total toroidal angular momentum, and thus electrostatic fluctuations may not affect the confinement, but electromagnetic fluctuations could cause the confinement to deteriorate.

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References

[1] J.H. Malmberg, T.M. O'Neil, Phys. Rev. Lett. **39**, 1333 (1977).
[2] D.H.E. Dubin, T.M. O'Neil, Rev. Mod. Phys. **71(1)**, 20 (1999).
[3] J. DeBoo, et al., Bull. Am. Phys. Soc. **44**, **No. 7**, 72 (1999). http://fusion.gat.com/pubs/browse.cgi
[4] D.R. Ernst, et al., Phys. Plasmas **5**, 665 (1998).
[5] Y.B. Kim, et al., Phys. Fluids B **3**, 2050 (1991).
[6] J. Kim, et al., Phys. Rev. Lett., **72**, 2199 (1994).