

# Theoretical Understanding of Tokamak Pressure Limits

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**Abstract** A self-consistent theory for the role of the polarisation current in magnetic island evolution is presented, which suggests that it can, under certain circumstances, provide a threshold island width for growth of neoclassical tearing modes. However, at high  $\beta$  coupling to an unstable resistive wall mode (RWM) removes this threshold, and would limit the achievable plasma pressure. Techniques for stabilising the RWM using a rotating ‘shell’ are described, thus providing the possibility of high  $\beta$  operation.

## 1. Introduction

There is much evidence from existing tokamak data to suggest that the effective pressure limit in tokamaks is set not by ideal magneto-hydrodynamic (MHD) instabilities, but by more slowly growing instabilities such as the resistive wall mode (RWM) and the neoclassical tearing mode (NTM). This paper describes a theoretical investigation to develop further our understanding of the onset conditions for these instabilities, as well as their interaction. In the following section we consider whether or not the polarisation current generated by a small-scale ‘seed’ magnetic island can provide a threshold for the growth of the neoclassical tearing mode. Within the theoretical model which we develop, we find that there is a parameter regime for which the polarisation current stabilises small magnetic islands, and this appears to be consistent with experimental observations. In Section 3 we address the coupling between a RWM and a NTM. We find that the effect of the RWM is to remove the threshold for growth of the NTM, suggesting that such instabilities may be prevalent in high  $\beta$ , steady state tokamaks unless the RWM can be controlled. In Section 4 we describe two possible techniques for stabilising the RWM by faking a rotating shell surrounding the plasma, using a system of magnetic coils or a flowing lithium blanket, for example. We close in Section 5 with a summary.

## 2. Neoclassical Tearing Mode Threshold

Experimentally it is observed that a neoclassical tearing mode grows provided an initial ‘seed’ island (eg from another MHD event) has a width larger than some threshold value. Such a threshold is predicted from the so-called polarisation model which reproduces several of the experimentally observed scalings for the threshold (eg with Larmor radius, collision frequency, etc) [1]. The model is based upon theories which predict that the polarisation current associated with small scale magnetic islands is stabilising. However, it has recently been noted that these theories omit a potentially important destabilising contribution to the polarisation current, which originates from a narrow layer in the vicinity of the island separatrix [2]. Here we perform calculations which self-consistently include this ‘layer’ contribution, and identify the conditions for which the polarisation current is stabilising, in which case it can provide a threshold to neoclassical tearing modes.

We illustrate the essential features of the physics by first considering a fluid model, valid in the limit that the ratio of ion and electron temperatures  $T_i/T_e \ll 1$ . We shall find that the separatrix layer is of width  $\sim \rho_s$ , which is the ion Larmor radius calculated with the electron

temperature, so that the finite ion Larmor radius effects can be treated perturbatively. The system of equations which we then solve are the electron continuity equation, charge conservation ( $\nabla \cdot \mathbf{J}=0$ ), Ohm's law and energy balance:

$$\frac{Dn}{Dt} = \frac{1}{e} \nabla_{\parallel} J_{\parallel} + D \nabla_{\perp}^2 n \quad (1)$$

$$\nabla_{\parallel} J_{\parallel} = \frac{c^2}{4\pi v_A^2} \left[ \frac{DU}{Dt} - \mu \nabla_{\perp}^2 U \right] \quad (2)$$

$$E_{\parallel} + \frac{\nabla_{\parallel} p}{ne} = \frac{J_{\parallel}}{\sigma_{\parallel}} - k \frac{\nabla_{\parallel} T}{e} \quad (3)$$

$$\frac{3}{2} n \frac{DT}{Dt} = \nabla_{\parallel} (\kappa_{\parallel} \nabla_{\parallel} T) + \kappa_{\perp} \nabla_{\perp}^2 T + (1+k) \nabla_{\parallel} \left( \frac{J_{\parallel} T}{e} \right) \quad (4)$$

Here we have defined: the plasma density,  $n$  (equal for ions and electrons); the electron temperature,  $T$ ; the cross-field ambipolar particle diffusion coefficient,  $D$ ; the Alfvén velocity,  $v_A$ ; the vorticity  $U = \nabla_{\perp}^2 \phi$ , where  $\phi$  is the electrostatic potential; the perpendicular viscosity  $\mu$ ; the parallel electric field,  $E_{\parallel}$ ; the pressure,  $p$ ; the electron charge,  $e$ ; the parallel electrical conductivity,  $\sigma_{\parallel}$ ; and the parallel and perpendicular thermal heat diffusivities,  $\kappa_{\parallel}$  and  $\kappa_{\perp}$ , respectively. Finally, the convective time derivative,  $D/Dt$ , includes the  $\mathbf{E} \times \mathbf{B}$  flow and  $k=0.71$  is a coefficient describing the effect of the thermal force.

Neglecting the dissipation terms (ie those associated with viscosity, radial diffusion, resistivity, etc) and working in a sheared slab approximation to the true tokamak geometry, we can simplify Eqs (1)-(4) analytically, working in the frame of reference where the island is at rest. First, the parallel heat conduction dominates in Eq (4), so we have  $T=T(\Omega)$ , where  $\Omega$  is a flux surface quantity, defined in terms of the radial coordinate  $x=(r-r_s)$  ( $r$  is the minor radius,  $r=r_s$  is the position of the rational surface) and the helical angle  $\xi=\theta-(n/m)\zeta$  ( $\theta$  and  $\zeta$  are the poloidal and toroidal angles, respectively, and  $m$  and  $n$  are the poloidal and toroidal mode numbers):

$$\Omega = 2 \frac{x^2}{w^2} - \cos \xi \quad (5)$$

with  $w$  the island half-width. Ohm's law can then be simplified and integrated to yield an expression for the density perturbation  $\delta n$  caused by the island:

$$\delta n = \varphi + H(\Omega) \quad (6)$$

where  $H(\Omega)$  is a free function arising from the integration along field lines and  $\varphi$  is the dimensionless electrostatic potential,  $\varphi=e\phi/T$ . Finally, we eliminate  $J_{\parallel}$  from Eqs (1) and (2), and integrate along lines of constant  $\varphi$ , using Eq (6) to derive:

$$\rho_s^2 \nabla_{\perp}^2 \varphi - K(\varphi) = H(\Omega) \quad (7)$$

where  $K(\varphi)$  is an arbitrary function of the integration, related to the flow profile. To close the set of equations, we integrate the charge conservation equation (2) to determine the polarisation current in terms of  $\varphi$ , and then use this result in Ampère's law to derive how the polarisation current affects the island evolution. Thus we find that we can write

$$\frac{dw}{dt} \propto \Delta'_s + \Delta_{\text{pol}} \quad (8)$$

where  $\Delta'_s$  represents the free energy available in the equilibrium current profile, and  $\Delta_{\text{pol}}$  is the contribution to the free energy from the polarisation current:

$$\Delta_{\text{pol}} = 4g \frac{L_s^2}{w^3} \frac{\omega(\omega - \omega_{*i})}{k_{\theta}^2 v_A^2} \quad (9)$$

Here  $\omega$  is the mode frequency in the  $\mathbf{E} \times \mathbf{B}$  rest frame,  $\omega_{*i}$  is the ion diamagnetic drift frequency (negligible in this cold ion model but, as we shall see later, it will be important),  $k_\theta$  is the poloidal wavenumber, and the coefficient  $g$  is:

$$g = -\frac{4}{\pi} \left( \frac{k_\theta c_s}{\omega} \right)^2 \oint d\xi \int_{-\infty}^{\infty} \frac{dx}{w} \frac{dH}{d\Omega} \varphi \left( \cos \xi - \frac{\langle \cos \xi \rangle}{\langle 1 \rangle} \right) \quad (10)$$

Angled brackets represent the average over flux surfaces which annihilates the  $\nabla_{\parallel}$  operator.

Clearly, the effect of the polarisation current on magnetic island evolution depends only on  $g$ , which in turn depends on the solution of Eq (7) for  $\varphi$ . Thus, to proceed we require forms for  $H(\Omega)$  and  $K(\varphi)$ , and these are derived from transport equations. These transport equations are constraint equations which are determined from the higher order dissipation terms in Eqs (1)-(4). We do not present a detailed derivation of these here, but merely quote the results, and refer the interested reader to Ref [3]. The mode frequency  $\omega$  is determined in [3], and depends on the parameter  $\eta_e$ , which is the ratio of density to temperature gradient length scales; it is therefore appropriate at this level of discussion to treat  $\omega$  as a free parameter. A constraint equation which determines  $H(\Omega)$  has its origins in the electron continuity equation; this leads us to adopt the form

$$H(\Omega) = \left( 1 - \frac{\omega}{\omega_{*e}} \right) \frac{w}{L_n} \frac{\sqrt{\Omega} - 1}{\sqrt{2}} \Theta(\Omega - 1) \quad (11)$$

where  $\Theta$  is the step function. The ion continuity equation arises from eliminating  $J_{\parallel}$  from Eqs (1) and (2). The leading order terms can then be annihilated by averaging along lines of constant  $\varphi$  to give the result:

$$\mu \rho_s^2 \langle \nabla_{\perp}^4 \varphi \rangle_{\varphi} = D \langle \nabla_{\perp}^2 \varphi + \nabla_{\perp}^2 H \rangle_{\varphi} \quad (12)$$

where

$$\langle \dots \rangle_{\varphi} = \oint \dots \left( \frac{\partial \varphi}{\partial x} \right)^{-1} d\xi \quad (13)$$

with the integral taken at constant  $\varphi$ . Thus our aim is to solve Eqs (7) and (12) simultaneously to derive  $K(\varphi)$  and  $\varphi$ , and then to use the result in Eq (10) to calculate  $g$  and so determine the role of the polarisation current in tearing mode evolution. Before we describe the solutions, there are two points we should like to draw attention to regarding this system of equations. First, consider Eq (7) far from the island, where  $K(\varphi)$  is matched to its linear form; thus  $K = (1 - \omega_{*e}/\omega)\varphi$ , representing the sum of the electron adiabatic response and long wavelength electron drift wave ion response. For  $0 < \omega < \omega_{*e}$  the solutions for  $\varphi$  are then oscillatory due to a strong coupling to the electron drift wave: such solutions would be subject to ‘shear’ damping, and would be relatively extended over the long radial length scale associated with the ion Landau resonance position, rather than the much shorter island width scale. For other propagation frequency regimes the solutions are localised on the length scale associated with  $w$ ; we concentrate on these. A second interesting feature can be illustrated by considering Eq (12). In the presence of viscosity, one might expect the viscosity to remove perturbations with short radial wavelengths so that no localised isolated magnetic island solutions could exist. However, Eq (12) shows that it is not just the viscous forces which determine the profile of  $\varphi$  (or, equivalently, the flow profile), but also the friction forces associated with electron-ion collisions (embodied in the ambipolar diffusion coefficient,  $D$ ) are important. One can then show that the solutions for  $\varphi$  are localised around the island on a length scale  $\sim \sqrt{(\mu/D)\rho_s}$  ( $\gg \rho_i$ , justifying our fluid treatment).

In Fig 1a we show the form for  $g$  which arises from the solution to Eqs (7), (10) and (12). In this calculation, which treats small magnetic islands more self-consistently than any previous

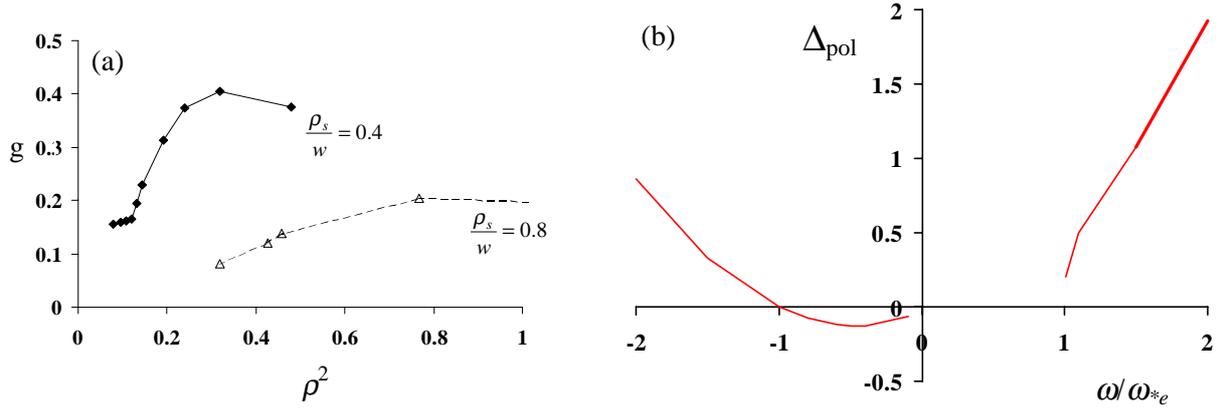


FIGURE 1. (a) Plot of  $g$  versus  $\rho^2 = \omega\rho_s^2/(\omega - \omega_{*e})$  for  $\mu=D$  and two values of  $\rho_s/w$  for the cold ion fluid model. (b) Plot of  $\Delta_{\text{pol}}$  versus  $\omega$  for the gyro-kinetic model with  $T_e=T_i$ .

calculation, we see that  $g > 0$  for cold ions. From Eq (9) we therefore expect the polarisation current to be stabilising provided the island propagates in the ion direction,  $\omega_{*i} < \omega < 0$ . This frequency regime is only significant for finite ion temperature, and the fluid model we have described here cannot be used to calculate  $g$  in that case (ie, when  $\rho_s \sim \rho_i$ ): a full gyro-kinetic model for the ions is required. We have explored this using a model in which the response to the magnetic perturbations is fully nonlinear, but we approximate the response to the electrostatic perturbations using linear gyro-kinetic theory. We again find that  $g > 0$ , and the resulting calculation of  $\Delta_{\text{pol}}$  is shown in Fig 1b.

While it is possible that the results presented here may be sensitive to the magnetic geometry and the assumptions associated with the perturbative treatment of the radial diffusion processes, there is some experimental evidence that ‘seed’ magnetic islands are indeed ‘born’ in the frequency range  $\omega_{*i} < \omega < 0$  when our model predicts a stabilising polarisation current [4]. We therefore take the polarisation current model as providing the threshold to neoclassical tearing modes in a tokamak, which then evolve according to [1]:

$$\tau_r \frac{dw}{dt} = \Delta'_s + \frac{\beta}{w} \left( 1 - \frac{w_c^2}{w^2} \right) \quad (14)$$

where  $\beta$  (typically  $\sim 1$ ) is proportional to the pressure in the plasma, and characterises the bootstrap current drive for NTMs in a tokamak,  $\tau_r$  is proportional to the resistive diffusion time and  $w_c$  is the threshold island width associated with the polarisation current threshold. We now explore the consequences of Eq (14) for pressure-limiting phenomena in tokamaks.

### 3. NTMs in High $\beta$ Plasmas.

Equation (14) describes the essential features of NTMs: the ‘Rutherford’ term proportional to  $\Delta'_s$ , the bootstrap current drive proportional to  $\beta$ , and a threshold associated with the polarisation current,  $w_c$ . Taking  $\Delta'_s$  to be constant, and negative, we plot  $dw/dt$  versus  $w$  in Fig 2a; note that an initial ‘seed’ island must exceed a critical threshold width to excite the NTM, which then grows to a large saturated amplitude. Now let us suppose that we are at sufficiently high  $\beta$  that the plasma would be unstable to ideal MHD modes in the absence of a perfectly conducting wall. The presence of a partially conducting wall improves the situation if positioned sufficiently close to the plasma, but nevertheless results in a slowly growing resistive wall mode. It is then natural to ask whether or not this RWM could provide the ‘seed’ to trigger the NTM. We address this question in this section, employing a simple plasma model, which neglects the plasma flow.

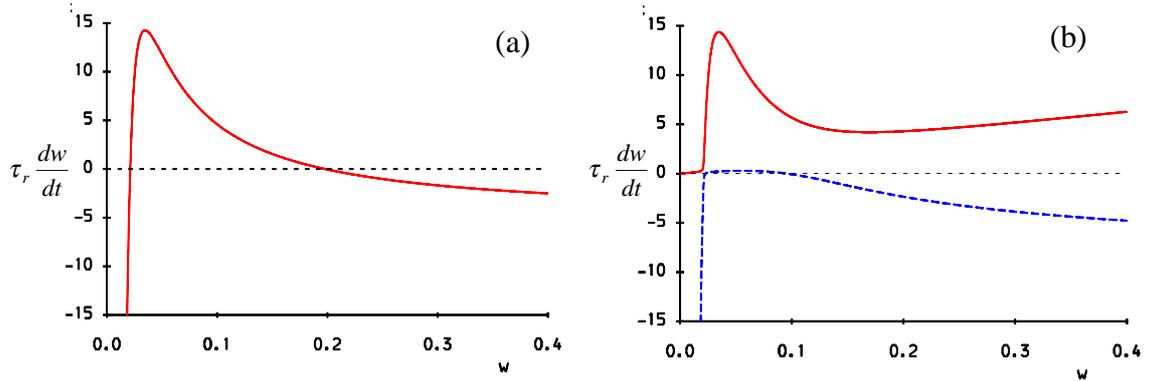


FIGURE 2. Plot of  $dw/dt$  for the neoclassical tearing mode, assuming a threshold caused by the polarisation current (a); note a finite ‘seed’ island size must be achieved to excite the mode ( $\beta=1$ ,  $\Delta'_s=-5$ ,  $w_c=0.02$ ). In (b) we consider the same case, but now the plasma is unstable to an RWM; note how the coupling to the RWM removes the threshold ( $\beta=1$ ,  $\delta=5$ ,  $\varepsilon=0.1$ ,  $\tau=0.01$  and  $w_c=0.02$ ).

The RWM stability properties can be incorporated through the parameter  $\Delta'_s$ . Consider a situation where there is a resonant surface in the plasma, with an associated jump in the radial derivative of the logarithm of magnetic potential, ie,  $\Delta'_s$ . At the resistive wall there will be a second jump in this quantity, which we denote by  $\Delta'_w$ . Now these two quantities are described by the equations of ideal MHD, and are thus simply related by an equation of the form [5]:

$$\Delta'_s = \frac{1 + c_1 \Delta'_w}{c_2 + c_3 \Delta'_w} \quad (15)$$

where the coefficients  $c_i$  depend on the specific properties of the tokamak equilibrium. As we are interested in an RWM, we require the plasma to be unstable in the absence of a wall, ie when  $\Delta'_w \rightarrow 0$ . In this limit we expect an inertial response at the rational surface,  $r=r_s$ ,  $\Delta'_s \sim -1/(p\tau_A)$ , where  $\tau_A$  is the Alfvén time and here  $p$  would be the *ideal* MHD growth rate, in the absence of any wall. We parameterise this growth rate, normalised to  $\tau_A$ , by  $\varepsilon$ , ie we write  $c_2 = -\varepsilon$ , where  $\varepsilon$  is considered to be small and positive close to marginal stability. Thus, a convenient, simple form for  $\Delta'_s$ , which has the features of a pressure-driven resistive wall mode in a tokamak, is

$$\Delta'_s = \frac{1 - \delta\gamma\tau_w}{\gamma\tau_w - \varepsilon} \quad (16)$$

where we have taken  $\Delta'_w = \gamma\tau_w$ , with  $\gamma(t)$  the instantaneous non-linear growth rate (not exponential growth in general) and  $\tau_w$  the resistive wall time, and  $\delta$  parameterises the stability of the equilibrium in the presence of a perfectly conducting wall ( $\delta < 0$  would be tearing-unstable). We now make the approximation that the time-dependence of the magnetic flux at the rational surface is the same as that at the wall, and express the rate of growth of the magnetic island at the rational surface in the form:

$$\gamma(t) = \frac{1}{w} \frac{dw}{dt} \quad (17)$$

(a factor 2 has been absorbed into  $\delta$  and  $\varepsilon$ ). Substituting these expressions into the island evolution equation gives us the following equation, describing coupling between the RWM and NTM:

$$\gamma_w = \frac{1 - \delta\gamma\tau}{\gamma\tau - \varepsilon} + \frac{\beta}{w} \left( 1 - \frac{w_c^2}{w^2} \right) \quad (18)$$

to be solved for  $\gamma$ , which has been normalised to the plasma resistive time,  $\tau_r$ , and we have introduced  $\tau = \tau_w/\tau_r$ . We show the results from the solution to this equation in Fig 2b: note that the RWM is now coupled to the NTM, and there is no threshold. Initially, for very small island widths, the mode is essentially a slowly growing RWM. Once the width reaches the critical width,  $w_c$ , the NTM is excited, and the mode grows rapidly to a large amplitude. This calculation, although simplistic in that it neglects issues associated with the plasma flow and torque balance, suggests that the most important mode to stabilise in the high  $\beta$  regimes (ie above the no-wall limit) will be the RWM. In the following section we describe two possibilities for stabilising this instability.

#### 4. Resistive Wall Mode Stabilisation

One might envisage that rotating the vessel wall, so that it appears to the plasma to be a perfect conductor, would stabilise the RWM. Unfortunately, unless rotation speeds are very high, the RWM simply Doppler shifts to remain locked to the wall rotation, and then there is no stabilising effect. However, there are two alternative possibilities for stabilising the RWM based on this idea: (i) to introduce a second conducting wall which rotates relative to the first so that the mode cannot lock to both [6], and (ii) to have a single wall, but with different parts of the wall rotating relative to others [7]. We explore these possibilities, concentrating on the linear stability properties of the RWM.

(i) *Secondary rotating wall.* This was proposed as a possible technique for stabilising RWMs in reversed field pinches (RFP) [6]. The tokamak situation is slightly different to that of the RFP because the RWM then has poloidal harmonics which are resonant inside the plasma, and this needs to be taken into account. We consider the situation when there is a single internal resonant surface. Defining the magnetic vector potential,  $\Psi$ , this must satisfy jump conditions at three radial locations: at the resonance  $r=r_s$  and at each of the two walls  $r=r_1, r_2$ . We define three matching parameters,  $\Delta'_i$ :  $\Delta'_1$  represents the stability when the closest fitting wall is a perfect conductor;  $\Delta'_2$  represents the stability when the first wall is absent and the second wall is a perfect conductor, and  $\Delta'_3$  represents the stability in the absence of either wall. We define the jump in the logarithmic derivative of  $\Psi$  at  $r=r_s$  for the actual geometry by  $\Delta'_s$  and use Ampère's law to show that the jump in the logarithmic derivative in  $\Psi$  across either wall is equal to  $p\tau_i$  (where  $p$  is the complex linear growth rate of the mode in the wall rest frame, and  $\tau_i$  is the time constant associated with the resistivity of each wall, labelled by  $i=1, 2$  for the first and second walls, respectively). To derive a dispersion relation for the RWM, we then match across each discontinuity at  $r=r_s, r_1, r_2$  to derive [8]:

$$\Delta'_s(p) = \frac{X^2 Y (\delta + \Delta'_2) - \delta (1 + \varepsilon \Delta'_2) (X + (p - i\Omega_2) \tau_2) p \tau_1 + XY \Delta'_2 (1 - \varepsilon \delta) (p - i\Omega_2) \tau_2}{-\varepsilon X^2 Y (\delta + \Delta'_2) + (1 + \varepsilon \Delta'_2) (X + (p - i\Omega_2) \tau_2) p \tau_1 + XY (1 - \varepsilon \delta) (p - i\Omega_2) \tau_2} \quad (19)$$

where  $\Omega_2$  is the toroidal angular rotation velocity of the outer (second) wall in the frame where the first wall is at rest. We have parameterised  $\Delta'_1$  and  $\Delta'_3$  in terms of  $\varepsilon$  and  $\delta$ , as follows (see Section 3). First, we wish to consider the situation when the plasma would be stable if the first wall were a perfect conductor: this requires  $\Delta'_1 = -\delta$ , where  $\delta$  is positive. Second, we want the plasma to be unstable to an ideal MHD mode when there are no walls present; thus we write  $\Delta'_3 = -1/\varepsilon$ , where  $\varepsilon$  is considered to be small. The two remaining parameters,  $X$  and  $Y$ , are defined by  $X = 2m/(1-Y)$ ,  $Y = (r_1/r_2)^{2m}$  where  $m$  is the poloidal mode number, and  $r_i$  is the radius of the  $i$ th wall.

To complete the dispersion relation we need to specify a model for the plasma response in the layer associated with the rational surface at  $r=r_s$  in the presence of the walls. We choose a 'visco-resistive' response [9], given by

$$\Delta'_s(p) = (p - i\Omega_{pl}) \frac{\tau_A^{1/3} \tau_r^{5/6}}{\tau_V^{1/6}} \quad (20)$$

where  $\tau_R$  and  $\tau_V$  characterise the layer resistive and viscous times and  $\Omega_{pl}$  is the plasma toroidal rotation frequency in the rest frame of the first wall.

We now show numerical solutions of Eqs (19) and (20) for the marginal stability contours as  $\Omega_{pl}$  and  $\Omega_2$  are varied. We fix  $\delta=1$ ,  $\varepsilon=0.1$  and  $\tau_A^{1/3} \tau_r^{5/6} / \tau_V^{1/6} = 1$ . The first situation we consider is  $\Delta'_2 = 5$ , so that the second wall is sufficiently close to the plasma that if it were a perfect conductor it could stabilise the ideal MHD mode (ie that which would exist in the absence of walls), but is too far away to stabilise the resistive mode associated with the rational surface at  $r=r_s$ . The marginal stability contours, shown in Fig 3a, illustrate that a minimum plasma rotation *and* wall rotation must be maintained to stabilise the RWM. The second situation we consider is that where the second wall is moved still further in so that the resistive mode would also be stabilised if the wall were a perfect conductor, ie  $\Delta'_2 = -0.3$ . In this case, Fig 3b shows that for a sufficiently high wall rotation, plasma rotation is not necessary for stability. As the position of the second wall approaches that of the first wall, it requires increasingly more wall rotation to stabilise the RWM. There are two situations when it is practically impossible to stabilise the plasma by a second, rotating wall. The first is when the second wall is so far from the plasma, that it would not be able to stabilise the ideal MHD mode even if it were perfectly conducting; the second is when the first wall is sufficiently close to the plasma that if it were a perfect conductor it could stabilise the ideal MHD mode, but it is not close enough to stabilise the resistive mode.

(ii) *Non-uniform wall rotation.* We have studied two cases where the flow in a single wall has a poloidal variation: (a) a toroidal flow which varies as  $V\sin\theta$ , where  $\theta$  is the poloidal angle, and (b) a poloidal flow, with a step function variation. Expanding the magnetic field perturbation in poloidal Fourier harmonics,  $b_m$ , where  $m$  labels the poloidal harmonics, we find the following difference equation for the case of toroidal flow by taking the radial component of the curl of Ohm's law, and integrating across the wall:

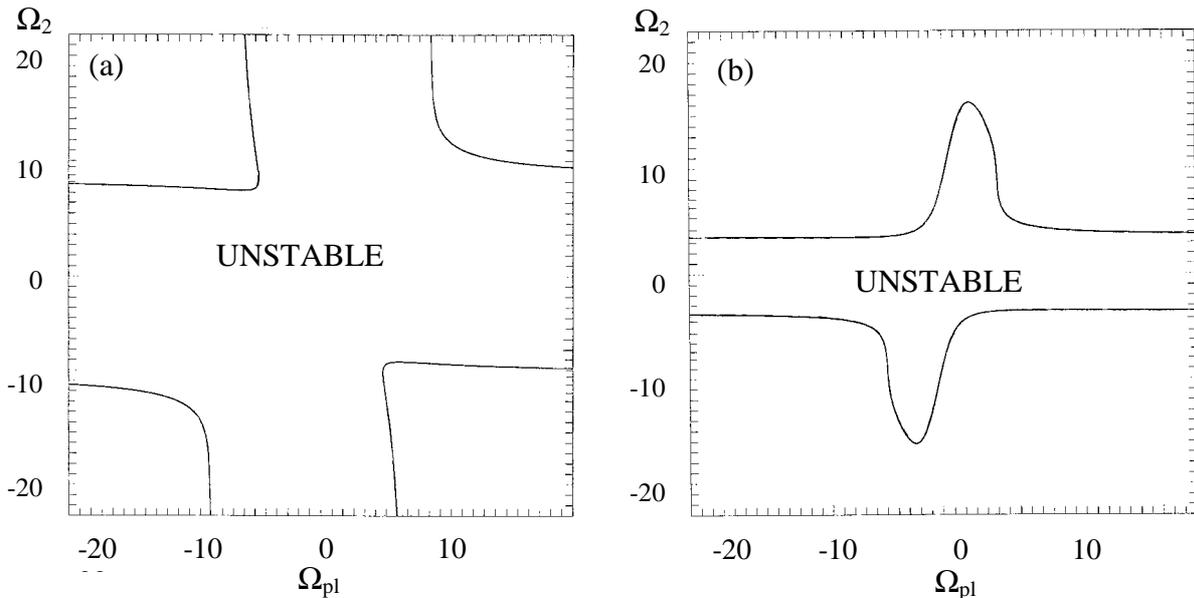


FIGURE 3. Marginal stability contours for a plasma ideal unstable with no wall, but stable if the first wall were a perfect conductor; the two cases are for different positions of the second wall, as described in the text: (a)  $\Delta'_2 = 5$ , (b)  $\Delta'_2 = -0.3$ .  $\Omega_{pl}$  is the plasma toroidal rotation velocity,  $\Omega_2$  is the second wall rotation velocity.

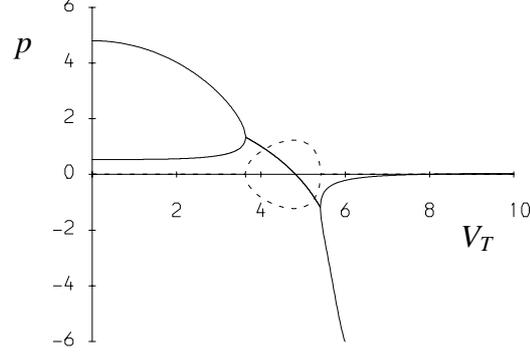


FIGURE 4. Complex growth rate,  $p$ , versus toroidal flow velocity,  $V_T$  of the vessel, having a poloidal variation  $\sim \sin\theta$ ; the full curve is the real part, dashed curves the imaginary part.

$$p b_m + V_T (b_{m-1} - b_{m+1}) = \Delta'_m b_m - \kappa m^2 b_m \quad (21)$$

where  $V_T = nV\tau_w/2R$ ,  $R$  is the major radius,  $p$  is normalised to  $\tau_w$  and  $\kappa = \delta_w/a \ll 1$ , with  $\delta_w$  the wall thickness. We adopt a model  $q$ -profile which has a single,  $m=2$  resonance such that  $\Delta'_3 = 4.89$  and all other harmonics except  $m=2$  are negative. Following the procedure which led to Eq (15) we use our model  $q$ -profile to derive  $\Delta'_2$  for the internal resonance:

$$\Delta'_2 = \frac{1 - 1.722p}{0.017 + 0.218p} \quad (22)$$

and solve Eq (21) numerically. The results for the two most unstable eigenvalues are shown in Fig 4, with a critical flow for stability given by  $V_T \approx 5$ . A second case which we have studied is that of a poloidal flow  $V$  with the top half plane rotating in the opposite direction to the bottom. The relevant dimensionless measure of flow is then  $V_p = mV\tau_w/2a$ , and we again find that the RWM is stabilised when the flow is sufficiently large,  $V_p \approx 1$ . The step function poloidal flow couples more poloidal harmonics  $b_m$ , which has an enhanced stabilising influence.

## 5. Summary

We have shown in this paper that there is a theoretical basis for the so-called polarisation model for the NTM threshold, though this does depend crucially on the island propagation frequency and more work remains to be done for a complete predictive theory. We have also demonstrated that above the no-wall  $\beta$ -limit the RWM can trigger an NTM (ie there is no NTM threshold when an RWM is destabilised). Thus, in high performance plasmas it may be important to control the RWM and we have described two possible techniques for stabilising this mode.

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