Recent Progress in MHD Stability Calculations of Compact Stellarators

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Abstract

A key issue for compact stellarators is the stability of beta-limiting MHD modes, such as external kink modes driven by bootstrap current and pressure gradient. We report here recent progress in MHD stability studies for low-aspect-ratio Quasi-Axisymmetric Stellarators (QAS) and Quasi-Omnigeneous Stellarators (QOS). We find that the $N = 0$ periodicity-preserving vertical mode is significantly more stable in stellarators than in tokamaks because of the externally generated rotational transform. It is shown that both low-n external kink modes and high-n ballooning modes can be stabilized at high beta by appropriate 3D shaping without a conducting wall. The stabilization mechanism for external kink modes in QAS appears to be an enhancement of local magnetic shear due to 3D shaping. The stabilization of ballooning mode in QOS is related to a shortening of the normal curvature connection length.

1 Introduction

Substantial progress has been made recently in the design of compact stellarators with optimized particle confinement and MHD stability[1, 2]. Two examples are Quasi-Axisymmetric Stellarators (QAS)[3] and Quasi-Omnigeneous Stellarators (QOS)[4]. In the United States, the main design effort is devoted to the proposed National Compact Stellarator Experiment(NCSX)[5] which is based on quasi-axisymmetry and is characterized by moderate size ($R \sim 1.5m$, $B \sim 2T$), low aspect ratio ($R/\langle a\rangle = 3 \sim 4$), $\beta \sim 4\%$ and monotonally increasing iota profile. In addition, efforts are underway to design a smaller compact QOS[6].

A key issue for high-beta compact stellarators is the stability of MHD modes such as vertical mode, and external kink modes and ballooning modes. Unlike traditional stellarators, these compact stellarators usually carry substantial bootstrap current at high beta. This is especially true for QAS since their bootstrap currents are comparable to that of tokamaks due to quasi-symmetry. A sizable bootstrap current can help to generate rotational transform necessary for particle confinement, but it also introduces current-driven MHD instabilities such as external kink modes and vertical modes. Therefore, it is important to investigate these current-driven instabilities as well as pressure-driven ballooning modes. To this end, we use a fully 3D ideal MHD model to determine the MHD beta limit and its dependence on parameters and profiles.

We showed previously that the external kink modes in high-beta QAS can be stabilized by combination of edge magnetic shear and appropriate 3D shaping while maintaining good quasi-symmetry[7]. In this work, we investigate the stability of vertical modes, ballooning modes as well as external kink modes[8, 9, 10]. In particular, we study physical mechanisms responsible for stabilization of these modes.

2 3D Ideal MHD Stability Calculations: Mode Family, Phase Dependence and Convergence Studies

The three dimensional ideal MHD stability code Terpsichore [11] is used to calculate the stability of global MHD modes. The code determines the eigenvalues of the ideal MHD equations by minimizing the plasma potential energy as defined in the energy principle[12],

$$\omega^2 \delta W_k = \delta W_p + \delta W_{vac} \tag{1}$$

where $\omega^2 \delta W_k$ is the kinetic energy, δW_{vac} is the magnetic energy in the vacuum region between plasma and conducting wall, and δW_p is the plasma potential energy written as

$$\begin{aligned}\delta W_p = \frac{1}{2}\int d^3x[\delta \mathbf{B}_\perp{}^2 + (\delta \mathbf{B}_\parallel - \mathbf{B}\frac{\xi \cdot \nabla p}{B^2})^2 \\ + \mathbf{j}_\parallel \cdot \xi \times \delta \mathbf{B} - 2\xi \cdot \nabla p \; \xi \cdot \kappa]\end{aligned} \tag{2}$$

where $\delta\mathbf{B}$ is the perturbed magnetic field, $j_\parallel$ is the parallel equilibrium current along the field line, κ is the magnetic curvature, and ξ is the plasma displacement vector written as

$$\xi = \sqrt{g}\xi^s \nabla\theta \times \nabla\phi + \eta \frac{\mathbf{B} \times \nabla s}{B^2} \tag{3}$$

where s, θ, and ϕ are radial, poloidal and toroidal variables in Boozer coordinates. In Eq. (2), the first and second terms in the integrand are the stabilizing field line bending energy and the field compression energy respectively, the third term is destabilizing due to parallel current and is responsible for kink instabilities. Lastly, the fourth term is usually destabilizing due to unfavorable curvature and pressure gradient. The Terpsichore code takes as input full 3D numerical equilibria obtained by the VMEC[13] code. The radial and surface component of the plasma displacement vector are represented by

$$\xi^s(s,\theta,\phi) = \sum_l \xi_l(s)\sin(m_l\theta - n_l\phi + \Delta) \tag{4}$$

$$\eta(s,\theta,\phi) = \sum_l \eta_l(s)\cos(m_l\theta - n_l\phi + \Delta) \tag{5}$$

where Δ is a phase factor. For stellarators with field period N_p, Fourier harmonics with toroidal mode number n are coupled to $n + kN_p$, where k is an arbitrary integer. There are $N_p/2 + 1$ families for even N_p and $(N_p - 1)/2 + 1$ families for odd N_p. For example, there are two families ($N = 0$, $N = 1$) for both $N_p = 2$ and $N_p = 3$ and there are three families ($N = 0$, $N = 1$ and $N = 2$) for $N_p = 4$. The vertical mode in stellarators belongs to the $N = 0$ family which preserves the stellarator periodicity. However, an eigenmode of $N = 0$ family can have characteristics of a kink or ballooning mode when the dominating toroidal mode number is not zero.

Assuming stellarator symmetry for the underlying equilibrium, only two values of the phase, $\Delta = 0$ (sin phase) and $\Delta = \pi/2$ (cos phase), are meaningful because the modes with sin phase are decoupled from the modes with cos phase. It can be shown that the stability of $N \neq 0$ families does not depend on the phase except for the $N = N_p/2$ family when N_p is even. The phase should be zero for the $N = 0$ family because it breaks the stellarator symmetry.

The Terpsichore code has been benchmarked extensively. Recently we have compared Terpsichore code with the CAS3D code[14] with respect to the kink beta limit of a $N_p = 3$ QAS configuration and found that the calculated beta limits of the two codes agree to within 5%[15].

3 The Stability of the Vertical Mode

It is known that strongly elongated tokamaks suffer from lack of vertical stability which could result in disruptions in the absence of feedback stabilization.

We find that the vertical mode ($N = 0$ family) can be much more stable in QAS than in tokamaks. The configuration c82[3], which is an earlier candidate for NCSX, is calculated to be robustly stable to the vertical mode. The stability has been extended and confirmed by CAS3D calculations with an infinite vacuum region[10]. In order to understand the physics, we have evaluated stability for a series of equilibria by varying the degree of nonaxisymmetric shape of c82. Figure 1(a) shows the eigenvalue of the vertical mode as function of the fraction

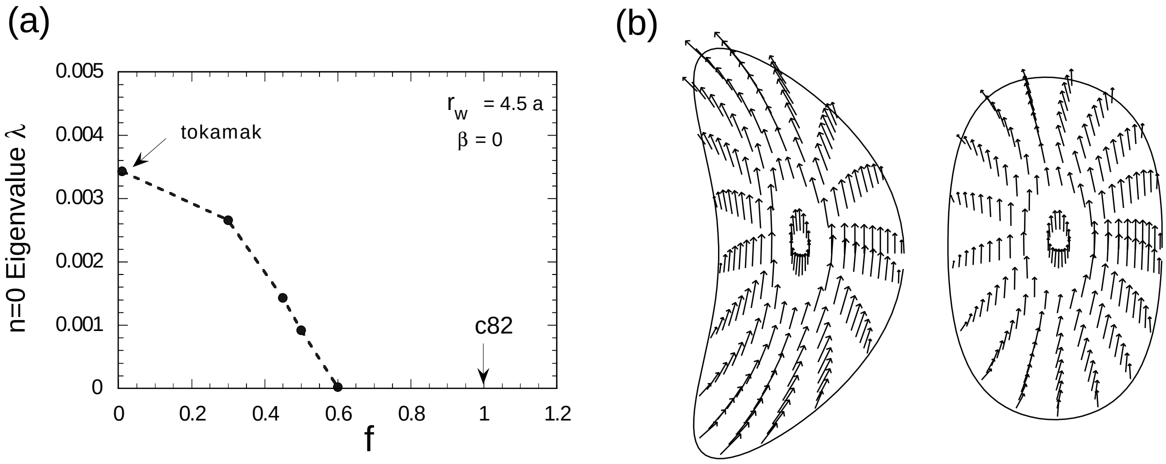

Figure 1: *(a) The eigenvalue of the vertical mode versus fraction of c82's nonaxisymmetric shape; (b) The displacement vector of the vertical mode at $f = 0.45$ for the two symmetric cross-sections.*

of nonaxisymmetric shape, f, at fixed current profile and zero beta. Here $f = 1$ corresponds to the full c82 shape and $f = 0$ corresponds a tokamak with the axisymmetric shape of c82. Equilibria are obtained by linear interpolation of the tokamak shape and the c82 shape (i.e., $R_{m,n}(f) = fR_{m,n}$, $Z_{m,n}(f) = fZ_{m,n}$ for $n \neq 0$, where $R_{m,n}$ and $Z_{m,n}$ are Fourier coefficients of the c82 plasma boundary). Figure 1(b) plots the displacement vector of the unstable vertical mode at $f = 0.45$. It clearly shows the expected vertical motion. We observe that there is a large stability margin for the vertical mode in c82 with the marginal point at $f = 0.6$. The results of Fig. 1 are obtained with zero beta because of equilibrium convergence problem at finite beta due to low ι at small f. At finite f, the effects of beta are found to be stabilizing. Thus, an even larger margin is expected for the full plasma.

We have derived an analytic stability criterion for vertical mode in a large aspect ratio stellarator with constant current density and constant external rotational transform[9]. The external rotational transform needed for stability is given by:

$$F_i = \frac{\kappa^2 - \kappa}{\kappa^2 + 1} \tag{6}$$

where $F_i = \iota_{ext}/\iota_{total}$ is the fraction of external rotational transform and κ is the axisymmetric elongation. This criterion has been confirmed by the Terpsichore code. Physically, the external transform is stabilizing because the external poloidal flux enhances the field line bending energy relative to the current-driven term for the vertical instability.

4 The Stability of External Kink Modes

Previous work has shown that the external kink modes in QAS can be stabilized by edge magnetic shear (i.e., $d\iota/ds$ near the edge) generated by 3D shaping[7]. Here we demonstrate that pure 3D shaping effects, which does not modify iota profile, can also contribute to the stabilization of external kink modes. This allows a complete stabilization of external kink modes at moderate edge magnetic shear for the configuration c82. In order to find the optimal shaping for kink stability, we have incorporated the Terpsichore code into a configuration optimizer which includes kink stability as well as quasisymmetry in its objective function. The optimizer is used to determine the necessary 3D shaping for kink stabilization. Figure 2 shows plasma cross-sections

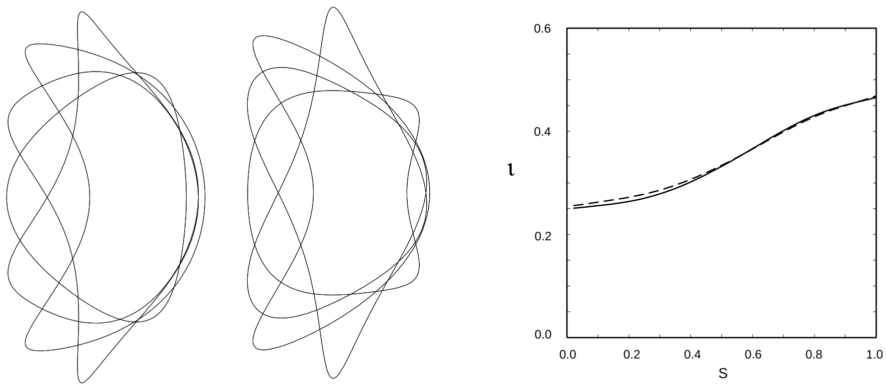

Figure 2: *Plasma cross-sections of a three field period QAS before optimization (left) and after optimization (right). On the far right are the corresponding iota profile before (solid line) and after (dashed line) optimization.*

of a three field period $R/\langle a\rangle = 3.5$ QAS before (left) and after (right) the stability optimization. The initial configuration (called c3m) is unstable to an n=1 kink with eigenvalue of $\lambda = -1.8 \times 10^{-3}$. Figure 3 plots the perturbed pressure contour of the corresponding eigenmode at the two

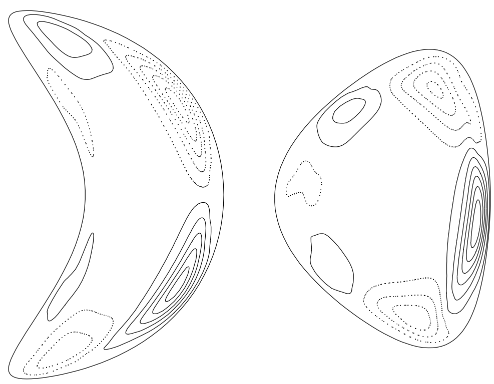

Figure 3: *contours of perturbed pressure at the two symmetric cross-sections for c3m.*

symmetric cross-sections (at $\phi = 0$ and $\phi = \pi/3$). The unstable mode peaks on the outboard side of the plasma (i.e., ballooning). The final configuration after optimization (called c82) is marginally unstable with eigenvalue of $\lambda = -2.6 \times 10^{-5}$ at $\beta = 3.9\%$. We note that the change in the iota profile from c3m's to c82's is minimal (see Fig. 2) and the two order of magnitude reduction in kink eigenvalue can only be attributed to the change in 3D shape. The major change in shape from c3m to c82 is an indentation of plasma boundary on the outboard side at the half-period cross section which is found to be most effective for stabilization.

Table 1: The breakdown of stabilizing and destabilizing terms in the plasma potential energy normalized by the vacuum energy for the most unstable $n = 1$ external kink mode in c3m and c82.

	vacuum	line bending	kink	ballooning
c3m	1.00	4.05	−3.98	−1.72
c82	1.00	4.51	−3.87	−1.64

The physical mechanisms for the stability of the external kink mode are investigated by examining individual contributions to the plasma potential energy and the effects of 3D shaping on local magnetic shear and normal curvature. For this purpose, we have modified the Terpsichore code so that the stabilizing and destabilizing terms can be computed separately. Table I compares the relative contributions of these terms normalized by the vacuum magnetic energy for both c3m and c82. We note that the kink term (i.e., the parallel current-driven term) contributes about 70% of the total destabilizing sum for both cases and is thus the main destabilizing mechanism for the $n = 1$ external kink modes, in accordance with usual expectation. However, the ballooning term (i.e., the curvature-pressure-gradient term) also contributes significantly to the instability. This is the reason the mode exhibits the strong ballooning feature as shown in Fig. 3. For c3m, the configuration would be stable if the ballooning term is neglected since the sum of the destabilizing term is only about 13% higher than the sum of stabilizing terms. Thus, the unstable mode should be called kink-ballooning mode. The pressure also contributes indirectly to the kink term through the parallel Pfirsch-Schlüter current. For both cases, the Pfirsch-Schlüter current contributes about 57% of the kink term, exceeding the contribution from the the volume-averaged parallel current.

We now discuss why the 3D shaping change from c3m to c82 stabilizes the external kink mode. We observe from Table I that the main difference between c3m and c82 is the field line bending term. This suggests that the effect of shaping on *local* magnetic shear plays a significant role. Figure 4 shows the contours of local magnetic shear of c82 (upper plot) on the $s = 0.5$ flux surface. Here, the local magnetic shear $\hat{S}$ is defined by $\hat{S} = -(\sqrt{g}/\Psi'^2)\mathbf{h} \cdot \nabla \times \mathbf{h}$ with $\sqrt{g}$ being the Jacobian and $\mathbf{h} = \mathbf{B} \times \nabla s/|\nabla s|^2$. Note that the global magnetic shear dq/ds is a surface average of $\hat{S}$ where $q = 1/\iota$. The value of local shear ranges from -87.2 to 37.2, while the global shear is $dq/ds = -2.66$. This shows that the local magnetic shear is dominated by helical variation and is much larger than the global shear. We have compared the local magnetic shear of c3m with that of c82 on the outboard side. Indeed we find that the local shear of c82 is substantially larger than that of c3m on the outboard side. This indicates that the local shear change is responsible for the difference in the field line bending energy and the stability between these two configurations.

Recently we have also investigated the kink stability in QOS. Initial results show that the external kink modes are stable for a $N_p = 3$, $R/\langle a \rangle = 3.6$, $\beta = 3.7\%$ QOS with self-consistent bootstrap current. The bootstrap current is negative and decreases the rotational transform at edge by about 15%. When beta is increased to 5%, the plasma is marginally unstable to an interchange-like mode with weak ballooning feature. The bootstrap current contribution to this mode is very small (about 10% of the pressure contribution) and the mode is mainly internal. We have examined the effects of bootstrap current direction on kink stability by artificially changing the sign of the current in the δW_p while keeping all other equilibrium quantities fixed. When the sign of the current is changed to positive, the QOS configuration becomes unstable to a mode with clear external kink features (i.e., the mode is external with significant drive from the kink term). This result shows the importance of the sign of bootstrap current, even for relatively small current magnitude. This indicates that the kink stability can be controlled in a QOS plasma by varying the bootstrap current.

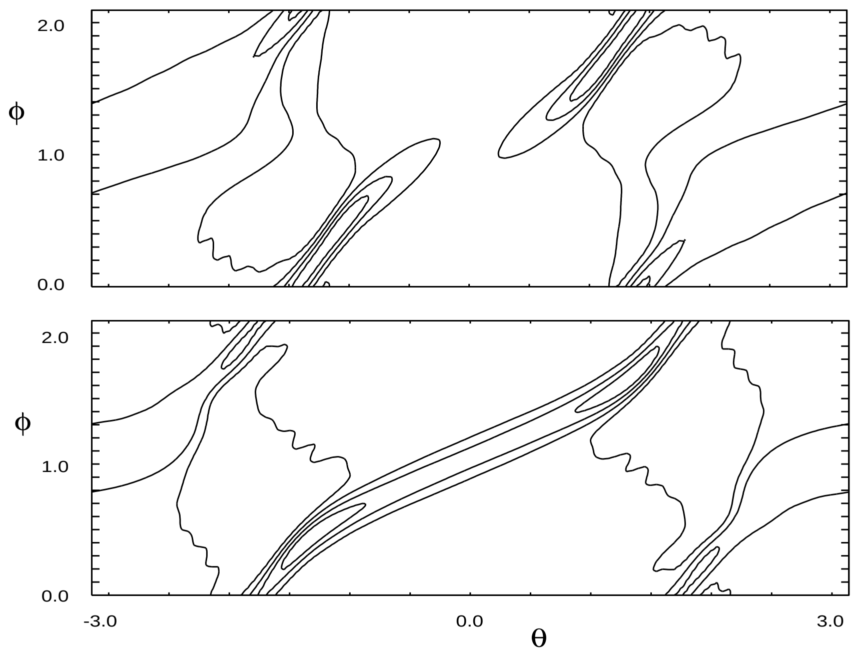

Figure 4: *The contour plot of the local magnetic shear of c82 and the NCSX configuration on the* $s = 0.5$ *flux surface for one field period* ($0 < \phi < 2\pi/3$). *The local shear ranges from* -87.2 *to* 37.2 *in c82 and from* -32.0 *to* 11.8 *in NCSX.*

5 The Stability of Ballooning Modes

In the preceding section, we studied the low-n global kink modes with main toroidal mode number in range of $n = 1 \sim 4$. Here we investigate the stability of high-n ballooning modes by using a local ballooning mode equation (i.e., infinite n limit) and the global Terpsichore code. Previous results showed that the ballooning modes can be stabilized in QAS by axisymmetric shaping. Here, we show that 3D shaping can also stabilize ballooning modes. In particular, the ballooning stability in QOS is studied using the ballooning code COBRA[16]. The code solves a 1D ballooning mode equation in non-straight magnetic coordinates which are much more efficient than standard codes. This code is used to optimize the ballooning beta limit while preserving good transport properties. In one example, the beta limit is increased from 2% to 4% as a result of optimization. The stabilization is related to a shortening of the normal curvature characteristic length along the magnetic field lines.

We have also used the global stability code Terpsichore to examine the stability of finite-n ballooning modes for toroidal mode in range of $n = -5 \sim 25$. In particular, we assess the dependence of beta limit on the toroidal mode number. Figure 5 plots the growth rates of the ballooning mode as function of plasma beta for several mode tables denoted by their maximum toroidal mode number. The results are obtained for the previous NCSX configuration c82 at the fixed plasma current. Each mode table consists of 182 pairs of (m, n) for the perturbation with seven toroidal mode numbers and 26 poloidal modes for each n. The distribution of the poloidal mode numbers in the table is centered around the resonant harmonic $m_0 = n * \iota_0$ with $\iota_0 = 0.417$. The location of ι_0 corresponding to the most unstable surface inferred from an infinite-n ballooning stability calculation (which yields a beta limit of about 3.8%). From Fig. 5, we see that the beta limits of the finite-n modes are significantly higher than the infinite-n limit. This implies Finite Larmor Radius(FLR) corrections should be considered in order to determine a realistic ballooning beta limit.

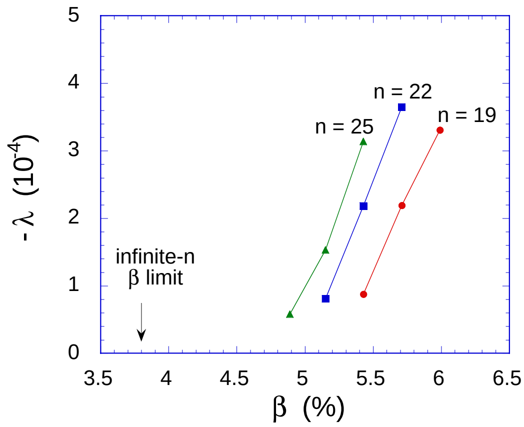

Figure 5: *The ballooning mode eigenvalue as a function of plasma beta.*

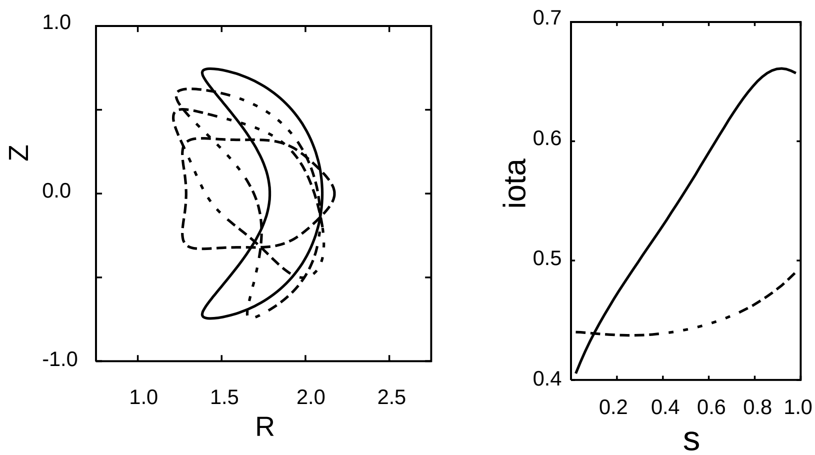

Figure 6: *Cross-sections of the NCSX configuration and its iota profiles for the vacuum (dashed) and the plasma (solid) with* $\beta = 4.2\%$.

6 MHD Stability of the NCSX Configuration

The current version (called $li383$) of the NCSX configuration is a three field period QAS with aspect ratio $R/\langle a\rangle = 4.4$ and plasma beta $\beta = 4.2\%$. The configuration was obtained by numerically minimizing a target function which includes kink eigenvalue, ballooning eigenvalue, Mercier index, and deviation from axi-symmetry. Figure 6 shows its plasma cross-sections (left) and iota profiles in vacuum (dashed) and plasma (solid). Compared with its predecessor (c82), it has higher rotational transform and lower bootstrap current. The plasma is marginally stable to Mercier modes, ballooning modes and external kink modes. The vertical mode is expected to be robustly stable due to high external transform. In contrast to c82, the local magnetic shear in this configuration has *continuous* helical stripes through the outboard side (i.e., near $\theta = 0$), as shown in Fig. 4 (lower plot). This helical structure of local shear is similar to that in a QAS configuration developed by Garabedian and Ku[17] and is thought to be responsible for its favorable kink stability [18] by inhibiting the formation of global kink mode structure.

7 Conclusions

The MHD stability properties of current-carrying compact stellarators are investigated using full 3D calculations. It is shown that 3D shaping (or 3D coils of stellarators) can stabilize MHD modes in three ways. First, 3D shaping generates external rotational transform which enhances the field line bending energy relative to the parallel current-driven kink term. In particular, it is found that the vertical mode is much more stable in quasi-axisymmetric stellarators than in equivalent tokamaks when the external rotational transform is comparable to the current-induced transform. Second, 3D shaping can stabilize MHD modes through appropriate control of iota profiles. It is shown that the external kink modes can be stabilized by enhancing the global shear near the plasma edge in QAS. Third, 3D shaping can be used to affect local quantities such as the local magnetic shear and curvature which are important to MHD stability. It is shown that appropriate 3D shaping can stabilize external kink modes in QAS by enhancing the local magnetic shear on the outboard side. In quasi-omnigeneous stellarators, it is shown that 3D shaping can stabilize external kink modes by controlling the sign and size of bootstrap currents. It is also shown that the ballooning modes can be stabilized by appropriate 3D shaping. In summary, the results found here demonstrate that there exists a new class of compact stellarators which are stable to MHD modes at high beta without a conducting wall.

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