Secondary Instability in Drift Wave Turbulence as a Mechanism for Avalanche Formation

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Abstract. We report on recent developments in the theory of secondary instability in drift-ITG turbulence. Specifically, we explore secondary instability as a mechanism for avalanche formation. A theory of radially extended streamer cell formation and self-regulation is presented. Aspects of streamer structure and dynamics are used to estimate the variance of the drift-wave induced flux. The relation between streamer cell structures and the avalanche concept is discussed, as are the implications of our results for transport modeling.

1. Introduction

Traditionally, the problem of predicting the turbulent transport in magnetically confined plasmas has been approached from the perspective of *mean field theory*, namely by proceeding from the assumption that the transport dynamics are well described by average fluxes and local transport coefficients, such as effective diffusivities, etc. This mean field/local transport perspective is the underpinning of the oft used 'mixing length rule' $D = \gamma_L / k_r^2$ which tacitly presumes that a single time $\left(\gamma_L^{-1}\right)$ and space $\left(k_r^{-1}\right)$ scale are sufficient to characterize the turbulent transport process. Indeed, such mixing length guesstimates are crucial to all of the "predictive transport models" (such as the I.F.S. - P.P.P.L. model and various imitations thereof) currently used in the M.F.E. community. However, the arrival of ideas originating from self-organized criticality (SOC) theory[1], which proposes that a scale-invariant spectrum of 'transport events' or 'avalanches' are at work in the dynamics of transport has stimulated a series of experimental and computational investigations which have cast considerable doubt upon the traditional mean field picture. In particular, experimental studies have yielded:

a.) the direct observation and visualization of avalanche type structures on the DIII-D tokamak[2],

b.) the observation that the pdf (probability distribution function) of the transport flux is quasi-Gaussian on the scale of the turbulence correlation length but *strongly non-Gaussian* on larger scales, which is indicative of the formation of structures akin to avalanches[3], c.) the observation that the pdf of the transport flux exhibits finite size scaling, i.e. $P(\Gamma) = (1/L)\,P(\Gamma/L)$, where Γ is flux and L is a scaling parameter, related to the turbulence intensity[4]. This scaling is observed over a broad range of transport event (i.e. avalanche) sizes,

d.) a host of indirect evidence for avalanche and SOC-type phenomena, such as the observation of multi-fractality in turbulence[5], measurements of $1/f$-type spectra of the turbulent flux[6], etc.

Also, both continuum and particle simulations of familiar turbulence models (such as ITG, resistive ballooning, etc.) have noted that:

a.) extended, mesoscale transport events or avalanches are observable and prominent near marginality[7],

b.) anisotropic (radially extended but poloidally narrow) eddys, called streamers, are observable and are clearly related to transport events or bursts. The observed streamers are *nonlinear structures*, involving many n numbers and evolving on time scales distinct from that of the linear growth rate[8],

c.) contribution to the total flux from large events diverges, so that scale-independent transport (i.e. ‘Bohm’) is manifested in models which naively appear to be linked to small scales (‘Gyro-Bohm’)[9].

It should be noted that great care must be taken in designing computational experiments to study avalanches. In particular, experience indicates that global simulations are more accurate than ‘flux tube codes’, which impose unphysical constraints on mescoscale structures[10,11]. Similarly, a fixed flux boundary condition, rather than the traditional, convenient but unphysical assumption of a frozen gradient, reveals considerably richer avalanche dynamics[12]. All told, there is clearly sufficient, compelling evidence to warrant a detailed study of the dynamics of avalanches.

Theoretical paradigms for avalanche phenomena have been limited to approximate solutions of discrete (cellular automata) models of sandpiles[13] and to the analysis of highly simplified, reduced 1D models[14,15]. In particular, there is definitely a gap between these rather idealized systems and even comparatively simple continuum models of turbulence in confined plasmas. In this paper, we present a theory of avalanche dynamics for a simple, 2-field model of ITG turbulence. We conceive of the avalanche as a radially extended, poloidally asymmetric convective cell, called a streamer, and tackle the calculation of its evolution with the methods of modulational stability theory. This approach is thus an extension of the theory of convective cell formation developed by Sagdeev[16], et al., Dawson[17], et al., and Taniuti[18], et al., in the 70’s. However, we extend the aforementioned paradigm by considering:

a.) the role of magnetic curvature, the pressure advection nonlinearity and proximity to marginal stability in streamer formation,

b.) the self-regulation of streamer cells by the feedback of their poloidally sheared radial flows upon the underlying ITG instabilities which support them, and by Kelvin-Helmholtz type instability, which causes their break-up, c.) the effect of streamers on the statistics of the flux. In particular, we demonstrate the proportionality of the variance of the flux to the streamer intensity level. This simple argument thus bridges the gap between the statistical theory of self-organized criticality and the continuum dynamics of familiar plasma turbulence models.

To avoid possible confusion, we emphasize here that our nomenclature *'streamer' denotes a nonlinear structure, not the linear ballooning mode cells seen in simulations during the linear growth phase*. The relevance of the latter to the time asymptotic turbulence dynamics is dubious.

ii. Theory of Streamer Generation

We consider the simplest possible model of curvature-driven ITG turbulence. The basic equations are[19]

(1) a) $$\left(\partial_t - \underline{\nabla}\phi \times \underline{z} \cdot \nabla\right)\left(1-\nabla^2\right)\phi + \mathrm{v}_{*_\mathrm{n}}\partial_y \phi + \mathrm{v}_\mathrm{B}\partial_y p + \nu_0 \nabla^2\nabla^2\phi = 0\,,$$

b) $$\left(\partial_t - \underline{\nabla}\phi \times \underline{z} \cdot \nabla\right)p - \chi_0 \nabla^2 p = \mathrm{v}_{*_\mathrm{p}}\partial_y \phi\,.$$

Here the equations are de-dimensionalized by $k\rho \to k$, $\Omega_i t \to t$, $e\phi/T \to \phi$, $p/p_0 \to p$, so that $\mathrm{v}_{*_\mathrm{p}} = \rho/L_p$, $\mathrm{v}_{*_\mathrm{n}} = \rho/L_n$ and $\mathrm{v}_\mathrm{B} = \rho/L_B$. Also, the highly simplified diffusive dampings ν_0 and χ_0 ensure the presence of small scale dissipation to maintain regularity. For inviscid scales, linear perturbation theory gives:

2) $$\omega = \frac{\omega_*}{2\left(1+k_\perp^2\right)}\left(1 \pm \left(1 - 4\left(1+k_\perp^2\right)\frac{\mathrm{v}_B \mathrm{v}_{*p}}{\mathrm{v}_{*_\mathrm{n}}^2}\right)^{1/2}\right).$$

Note that a threshold exists $\left(\text{at } \mathrm{v}_{*_\mathrm{n}}^2/\mathrm{v}_\mathrm{B}\mathrm{v}_{*p} = 4\right)$, that instability requires $k_\perp^2 > 1/4\left(\frac{\mathrm{v}_{*_\mathrm{n}}^2}{\mathrm{v}_\mathrm{B}\mathrm{v}_{*_\mathrm{p}}}\right) - 1$, and that small scale modes grow faster (until the small scale dissipation cut-off is encountered). Thus, it is clearly meaningful to speak of large scale (secondary) streamer cells populated by the interaction of small scale primary modes. This is similar to avalanches in CA models produced by topplings of adjacent lattice sites.

In order to study cell dynamics, it is convenient to define the low-pass filtered field $\langle A \rangle_<$ by

3) $$\langle A \rangle_< = \int\limits_{k_y < k_{y\mathrm{min}}} dk_y \int\limits_{k_x < k_{x\mathrm{min}}} dk_x \, \mathrm{A_k} \, e^{i\underline{k}\cdot\underline{x}}\,.$$

Here, the bandpass effectively filters out the high-$\underline{k}$ components, which hereafter are treated as background turbulence intensity, thus allowing us to focus on the large scale cell components. Note that the filter is, in general, anisotropic. The low pass filtered equations are:

4) a) $$\partial_t \langle (\phi - \nabla^2 \phi) \rangle_< + \mathrm{v}_* \partial_y \langle \phi \rangle_< + \mathrm{v}_B \partial_y \langle p \rangle_< = S_\phi$$

b) $$\partial_t \langle p \rangle_< - \mathrm{v}_{*p} \partial_y \langle \phi \rangle_< = S_p$$

where

c) $$S_\phi = -\langle \partial_x \left[(\partial_y \phi) \nabla^2 \phi \right] \rangle_< + \langle \partial_y \left[(\partial_x \phi) \nabla^2 \phi \right] \rangle_<$$

d) $$S_p = \langle \partial_x \left[(\partial_y \phi) * p \right] \rangle_< - \langle \partial_y \left[(\partial_x \phi) p \right] \rangle_<$$

are the sources for the streamer cells, and represent drive by beat interaction of small scales. Observe that the self-interactions of the large scales are neglected. Straightforward calculation then allows us to write the sources as:

5) a) $$S_\phi = \partial_x \partial_y \left(\langle (\partial_y \phi)^2 \rangle_< - \langle (\partial_x \phi)^2 \rangle_< \right) + 2(\partial_y^2 - \partial_x^2) \langle (\partial_x \phi)(\partial_y \phi) \rangle_<$$

b) $$S_p = \partial_x \langle (\partial_y \phi) p \rangle_< - \partial_y \langle (\partial_x \phi) p \rangle_< .$$

Here ∂_x and ∂_y acting on quantities within brackets probe only the un-filtered (large) scales. S_ϕ and S_p represent the effects of turbulent Reynolds stresses and ion thermal flux, respectively. Observe that S_ϕ is clearly quite sensitive to anisotropy of the spectrum of small scales. S_p may be further simplified by using the (broadened) quasilinear response of p to ϕ to write

6) a) $$S_p = \partial_y \langle (\partial_x \phi) R (\partial_y \phi) \rangle_< \mathrm{v}_{*p} - \partial_x \langle (\partial_y \phi) R (\partial_y \phi) \rangle_< \mathrm{v}_{*p}$$

where

b) $$R_{\underline{k}} = \Delta \omega_{\underline{k}} / (\omega_{\underline{k}}^2 + \Delta \omega_{\underline{k}}^2).$$

It is interesting to examine the structure of S_ϕ and S_p in different limits. For simplicity, we will consider isotropic turbulence, so that $\langle (\partial_y \phi)^2 \rangle_< = \langle (\partial_x \phi)^2 \rangle_<$. In the 'streamer limit' $\partial_y >> \partial_x$, so that potential and pressure perturbations of the streamer remain coupled. Note that both S_ϕ and S_p are both ultimately proportional to the turbulence Reynolds stress. In the opposite, 'zonal flow limit' where $\partial_x >> \partial_y$, $\langle \phi \rangle_<$ and $\langle p \rangle_<$

decouple, so that $\langle\phi\rangle_<$ is driven by momentum transport alone (Reynolds stress again!) while $\langle p\rangle_<$ is driven by thermal transport alone. (N.B.: Strictly speaking, the $\langle\phi\rangle_<$ equation must be modified in the pure zonal flow limit to reflect the fact that the electrons are not adiabatic for $k_y = k_{\|} = 0$. This change amounts to taking $1 + k_x^2\rho^2 \to k_x^2\rho^2$ for zonal flows). Finally, observe that isotropic cells are not pumped unless the small scale turbulence is anisotropic.

Hereafter, we will focus on the extreme streamer cell limit, where $\partial_y\langle\phi\rangle_< >> \partial_x\langle\phi\rangle_< \to 0$. In order to examine the stability of large scales, the modulational response of the Reynolds stress and thermal flux to streamer potential perturbations must be extracted. Thus, we write:

7) a) $$\langle(\partial_x\phi)(\partial_y\phi)\rangle_< = \Sigma \cong \left(\frac{\delta\Sigma}{\delta\langle\phi\rangle_<}\right)\langle\phi\rangle_<$$

b) $$\langle(\partial_x\phi)R(\partial_y\phi)\rangle_< = \Sigma' \cong \left(\frac{\delta\Sigma'}{\delta\langle\phi\rangle_<}\right)\langle\phi\rangle_< .$$

For notational convenience, from now on we write $\langle\phi\rangle_<$ as $\bar{\phi}$. Fourier analyzing $\bar{\phi}$ as $\bar{\phi} = \sum_{q,\Omega}\bar{\phi}_{q,\Omega}e^{i(qy-\Omega t)}$, we then obtain the nonlinear dispersion relation and eigenfrequency for streamer cells. These are:

8) a) $$\Omega^2(1+q^2) - \Omega(q\,\mathrm{v}_{*n}) + \mathrm{v}_B\mathrm{v}_{*p}q^2\left(1-\delta\Sigma'/\delta\bar{\phi}\right) + 2iq^2\frac{\delta\Sigma}{\delta\bar{\phi}}\Omega = 0,$$

b) $$\Omega = \frac{q\,\mathrm{v}_*}{2} - iq^2\frac{\delta\Sigma}{\delta\bar{\phi}} \pm \frac{1}{2}\left(\delta\omega^2 + \left(4i(q\,\mathrm{v}_*)q^2\frac{\delta\Sigma}{\delta\bar{\phi}} + 4q^2\mathrm{v}_{*\rho}\mathrm{v}_{*B}\frac{\delta\Sigma'}{\delta\bar{\phi}}\right)\right)^{1/2} .$$

In Eqn. (8b), $\delta\omega^2 = q^2\left(\mathrm{v}_*^2 - 4\mathrm{v}_{*p}\mathrm{v}_{*B}\right)$ is a measure of the deviation from marginality. We have dropped the term $\left(\delta\Sigma/\delta\bar{\phi}\right)^2$, which is $0(e\phi/T)^4$, and have taken $1+q^2 \cong 1$. We use adiabatic theory to determine $\delta\Sigma/\delta\bar{\phi}$ and $\delta\Sigma'/\delta\bar{\phi}$. For ITG turbulence, which is quite similar in its dynamics to Rayleigh-Benard convection, the adiabatic invariant is the Wigner function[20]:

9) $$N(\underline{k}) = (1+k^2)^2\sum_{\underline{q}}\phi_{\underline{k}+\underline{q}}\phi_{\underline{q}-\underline{k}}e^{2i\underline{q}\cdot\underline{x}} .$$

Obviously here $k_x >> k_{x,min}$ and $k_y >> k_{y,min}$. Note that N is essentially the potential enstrophy of the underlying ITG mode vortices, which is a measure of the effective ‘roton’ density of the turbulence. N is conserved, up to dissipation and buoyancy drive. Thus, we can write

10) a) $$\Sigma = \sum_{\underline{k}>\underline{k}_{\min}} \frac{k_x k_y}{\left(1+k^2\right)^2}\; N(\underline{k}),$$

b) $$\Sigma' = \sum_{\underline{k}>\underline{k}_{\min}} \frac{k_x k_y R_{\underline{k}}}{\left(1+k^2\right)^2}\; N(\underline{k}),$$

so that $\delta\Sigma/\delta\overline{\phi}$ and $\delta\Sigma'/\delta\overline{\phi}$ are now easily determined using the linearized wave kinetic equation (W.K.E.). For streamer cells, the linearized W.K.E. is:

11) a) $$\frac{\partial\hat{N}}{\partial t} + \mathrm{v}_{g,y}\frac{\partial\hat{N}}{\partial y} + \gamma_{\underline{k}}\hat{N} = \frac{\partial}{\partial y}\left(k_x\overline{V}_x\right)\frac{\partial\langle N\rangle}{\partial k_y}.$$

(N.B.: Here $\overline{V}_x = -\partial y\overline{\phi}$, the $E\times B$ velocity of the streamer cell) and the response $\hat{N}$ is thus:

b) $$\hat{N}_{q,\Omega} = \frac{i\,q^2 k_x \overline{\phi}_{q,\Omega}}{\Omega - q\,\mathrm{v}_{g,y} + i\,\gamma_{\underline{k}}}\,\partial\langle N\rangle/\partial k_y.$$

It follows, then, that $\delta\Sigma/\delta\overline{\phi}$ is given by:

12) a) $$\frac{\delta\Sigma}{\delta\overline{\phi}} = \sum_{\underline{k}>\underline{k}_{\min}} q^2 \frac{k_x^2}{\left(1+k^2\right)^2} R\left(\Omega - q\,\mathrm{v}_{g,y}\right) k_y \frac{\partial\langle N\rangle}{\partial k_y},$$

b) $$R\left(\Omega - q\,\mathrm{v}_{g,y}\right) = \gamma_{\underline{k}}\Big/\left(\Omega - q\mathrm{v}_{g,y}\right)^2 + \gamma_{\underline{k}}^2.$$

$R\left(\Omega - q\,\mathrm{v}_{g,y}\right)$ is the broadened resonance function for interaction between the streamer phase velocity and the ITG mode group velocity. Observe that, in contrast to the case of zero-frequency zonal flows, an unambiguous quasilinear limit of R clearly exists, i.e. as $\gamma_k \to 0,\ R \to \pi\delta\left(\Omega - q\,\mathrm{v}_{g,y}\right)$. $\delta\Sigma'/\delta\overline{\phi}$ follows similarly.

It is clear that $\delta\Sigma/\delta\overline{\phi} < 0$ and $\delta\Sigma'/\delta\overline{\phi} < 0$ for $\partial N/\partial k_y < 0$, which is virtually *always* the case in drift wave turbulence. Thus, streamer cells will be *nonlinearly excited* in the absence of a population inversion. Note that drive occurs via both Reynolds stress and pressure advection coupling, and that proximity to linear marginality clearly has an important effect upon streamer evolution. To clarify this, we consider two limits. For streamers that are strongly linearly stable, $\Omega \cong \left(q\,\mathrm{v}_* \pm \delta\omega\right)/2 - i\,q^2\delta\Sigma/\delta\overline{\phi}$. Thus, the streamer growth rate $\gamma_q = \mathrm{Im}\,\Omega = -q^2\delta\Sigma/\delta\overline{\phi}$ is due to Reynolds stress coupling, and is quadratic in small scale fluctuation intensity, i.e. $\delta\Sigma/\delta\overline{\phi} \sim \langle N\rangle \sim \left(e\phi/T\right)^2$. Finally, using dimensional units, $\gamma_q \sim (q\rho)^4$, with $q\rho < 1$. However, for scales which are linearly marginal (so that $\delta\omega^2 = 0$), $\Omega = q\mathrm{v}_*/2 - iq^2\delta\Sigma/\delta\bar{\phi} \pm \frac{1}{2}\left(4i\left(q\mathrm{v}_*\right)q^2\delta\Sigma/\delta\bar{\phi} + 4q^2\mathrm{v}_{*p}\mathrm{v}_{*B}\delta\Sigma'/\delta\bar{\phi}\right)^{1/2}$. Since $q\rho < 1$, this may be simplified to $\Omega \cong q\mathrm{v}_*/2 - iq^2\delta\Sigma/\delta\bar{\phi} \pm \left(q^2\mathrm{v}_{*p}\mathrm{v}_{*B}\delta\Sigma'/\delta\bar{\phi}\right)^{1/2}$, so that $\gamma_q \sim \left(q^2\mathrm{v}_{*p}\mathrm{v}_B\right)^{1/2}(e\phi/T)$. Note that near marginality, γ_q scales directly with curvature, is controlled by modulation of pressure advection, and scales with $(e\phi/T)$. Thus, we can conclude that there is a regime of fast streamer drive $(\sim e\phi/T)$ near marginal stability and a regime of slow drive $\sim (e\phi/T)^2$ when the large scales are stable. The cross-over between these two limits occurs when $\delta\omega^2 \sim \max\left(iq\mathrm{v}_* q^2\delta\Sigma/\delta\bar{\phi},\ q^2\mathrm{v}_{*p}\mathrm{v}_{*B}\delta\Sigma'/\delta\bar{\phi}\right)$. Note that the streamer always has a real frequency $\sim q\mathrm{v}_*$.

While this simple study employs only local analysis, it is nevertheless possible to deduce certain aspects of streamer physics in toroidal geometry from the results obtained here. Streamers always have a finite q_y which translates to finite-*n* $(k_\theta = nq(r)/r)$ in a torus. Except for extremely low *n*'s, then, streamer structure should be compatible with the ballooning mode representation. Indeed, since the streamer drive $\Sigma \sim |\phi(\theta)|^2$ is proportional to the intensity (envelope) of the underlying ballooning-ITG modes, streamers can also be expected to extend along magnetic field lines and exhibit the other structural features of ballooning modes in a torus. Recent simulations[21] indicate that streamers indeed do exhibit such characteristics of ballooning structure, albeit with many *n*-modes participating. This vitiates the oft-stated assumption that magnetic shear, toroidicity, etc. will inhibit convective cell and streamer formation.

iii. Self-Regulation Mechanisms for Streamers

It is important to realize that the turbulent state with streamer cells is dynamic, rather than static. In particular, while cells are pumped by small scales, they also feedback on the underlying ITG modes by shearing, as well as via gradient relaxation. Here, it is important to note that, in contrast to zonal flows, "shearing" refers to *poloidal shearing of radial streamer flows*, rather than the usual process of radial shearing of poloidal flows, i.e. see Fig. (1). The shearing process is a stochastic one, whereby ensemble of streamer cells induces a random walk of the ITG mode k_y, which ultimately couples ITG-driven spectral energy to high-k_y damping. Stochastic methodology is applicable if the underlying ITG rays are chaotic (i.e. have a positive Lyapunov exponent). In that case, standard quasilinear theory allows us to write the W.K.E. for $\langle N\rangle$ as:

13) a) $$\frac{\partial\langle N\rangle}{\partial t} = \frac{\partial}{\partial k_y} D_{k_y} \frac{\partial\langle N\rangle}{\partial k_y} + \gamma_{\underline{k}}\langle N\rangle - \Delta\omega_{\underline{k}}\frac{\langle N\rangle^2}{N_0}$$

b) $$D_{k_y}=\sum_q q^2 k_x^2 \left|\bar{\phi}_q\right|^2 R\left(\Omega_q - q\,\mathrm{v}_{g,y}\right).$$

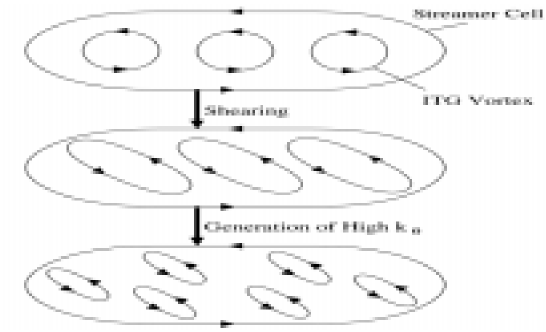

FIG. 1. Poloidal shear of radial streamer flows strains and enhances the decorrelation of ITG vortices.

Here, D_{k_y} represents the stochastic refraction of ITG eddys by poloidally sheared streamer flows[22]. Note however, that in contrast to the case of its analogue for zonal flow feedback shearing, $\Omega \neq 0$ for streamers. Hence, $\langle N \rangle$ evolution can saturate by *plateau formation* at $\underline{k}$ such that $\mathrm{v}_{g,y}(\underline{k}) = \Omega_q/q$. While growth and local spectral interactions can be expected to perturb the flattened $\langle N \rangle$, observation of such a plateau formation trend in simulations would be one indicator of the presence and activity of this important feedback mechanism.

Another possible feedback mechanism which may limit streamer growth is Kelvin-Helmholtz type instability of the streamer flow[23]. In contrast to the case of zonal flows, Kelvin-Helmholtz type modes for streamers are simple and robust. This is a consequence of the fact that a KH instability is basically an interchange of two vortices across the midpoint of the shear layer. In the case of zonal flows, this interchange is a radial one, which forces the vortex tubes involved to rotate, so as to align with the local (sheared) magnetic field. Thus, KH instabilities will be severely inhibited by magnetic shear, Landau damping, etc. For streamers, the interchange is azimuthal (at roughly constant radius) so no vortex tube rotation is required. Also, since plasma free energy is stored in radial gradients, streamer KH modes are driven by flow shear, only. Thus, well known results from hydrodynamics are applicable. In the case of long, thin streamers, which can be crudely approximated as tangential discontinuities for the case of $q_x \Delta y < 1$ (here q_x is the wavenumber of the KH mode and $\Delta y \sim 1/q$ is the poloidal width of the streamer), the KH growth rate will scale as $\gamma_{q_x,KH} = q_x \overline{\mathrm{V}}/2$, where $\overline{\mathrm{V}}$ is the streamer flow velocity.

All told, the coupled system of streamers and ITG vortices is self-regulating, and clearly of the ‘predator-prey’ form. It can be described by the equations:

14) a) $$\frac{\partial}{\partial t}\left|\bar{\phi}_q\right|^2 = \gamma_q \left|\bar{\phi}_q\right|^2 - \gamma_{KH}\left|\bar{\phi}_q\right|^2 - \gamma_{d,q}\left|\bar{\phi}_q\right|^2$$

b) $$\frac{\partial\langle N\rangle}{\partial t}=\frac{\partial}{\partial k_y}D_{k_y}\frac{\partial\langle N\rangle}{\partial k_y}+\gamma_{\underline{k}}\langle N\rangle-\Delta\omega_{\underline{k}}\frac{\langle N\rangle^2}{N_0}.$$

together with the transport equation for mean pressure. Here γ_q is the modulation instability pumping rate, γ_{KH} is the Kelvin-Helmholtz instability growth rate, (both given above), and $\gamma_{d,q}$ refers to residual linear Landau and collisional damping, etc. The $\langle N\rangle$ equation is the same as Eqn. (19).

It is interesting to compare the efficacy of the two nonlinear self-regulation mechanisms, namely KH instability $\left(\text{note } \gamma_{KH}=\gamma_{KH}(\bar{\phi})\right)$ and random shearing. Crudely put, D_{k_y} states that random shearing will quench streamer drive at a rate $\gamma_{sh}\sim\overline{\left(\partial y\overline{\mathrm{V}}\right)^2}\tau_{ac}$, where $\overline{\left(\partial y\overline{\mathrm{V}}\right)^2}$ is the mean square poloidal shearing rate of the streamer flow field and τ_{ac} is the autocorrelation time of the streamer pattern. Estimating $\overline{\left(\partial y\overline{\mathrm{V}}\right)^2}\tau_{ac}$ as $\alpha\left(\partial y\overline{\mathrm{V}}\right)$, where α is a factor $\leqq 1$, it follows that $\gamma_{KH}/\gamma_{sh}\sim q_x/\alpha q_y$. Thus, it seems that both processes will be significant, and that detailed quantitative studies will be required for further elucidation of their relative strength. At this point, however, it does seem fair to say that the conventional wisdom which states that 'streamers break up via Kelvin-Helmholtz instabilities' seems little more than convention.

iv. Streamers and the Statistics of Transport

A quantitative theory of transport must account for and predict avalanche phenomena. As avalanches are intrinsically bursty and intermittent, such a theory must necessarily be *statistical*, i.e. *designed to predict the pdf of the transport flux*, and not merely its mean value. While even approximate calculations of turbulence pdfs remain elusive[24] (though recent applications of instanton methods to very simple models such as 1D Burgers turbulence hold promise in this regard[25]), the modulational theory of streamer generation does allow us to estimate the *variance* of the turbulent flux. The flux variance is directly related to the streamer intensity, which can (in principle) be calculated using Eqns. (14a,b). Thus, some insight into the variance of the heat flux pdf and its dependencies can be obtained.

The ITG-driven heat flux Q is given by

15) $$Q=-\left[\sum_{\underline{k}>\underline{k}_{min}}R_{\underline{k}}\frac{k_y^2}{\left(1+k^2\right)^2}\left(\langle N(\underline{k})\rangle+\hat{N}(\underline{k})\right)\right]\frac{\partial\langle p\rangle}{\partial r}.$$

Here, the streamer flow induced spectral modulations $\hat{N}$, cause fluctuations $\hat{Q}$ in the heat flux about its mean value. Noting that $\hat{N}(k)$ is given by Eqn. (11b), the flux perturbation $\hat{Q}=Q-\langle Q\rangle$ is:

16) $$\hat{Q} = \sum_{\underline{k}>\underline{k}_{\min}} R_{\underline{k}} \frac{k_y^2}{\left(1+k^2\right)^2} \sum_q e^{iqy} q^2 \overline{\phi}_q R\left(\Omega - q\,\mathrm{v}_{g,y}\right) k_x \frac{\partial \langle N(k)\rangle}{\partial k_y} \left(\frac{\partial \langle p\rangle}{\partial r}\right).$$

Since it is not especially illuminating to display the detailed calculation of the normalized flux variance $\left\langle \hat{Q}^2 \right\rangle / \langle Q \rangle^2$ here, we directly proceed to simply write its estimate, which can easily be shown to be

17) $$\frac{\left\langle \hat{Q}^2 \right\rangle}{\langle Q \rangle^2} \approx \sum_q q^4 \left|\overline{\phi}_q\right|^2 \left(\frac{\alpha_1 k_x}{k_y \mathrm{v}_g' \Delta k_y} \right)^2_{res}.$$

Here $\mathrm{v}_g' = \partial \mathrm{v}_{g,y} / \partial k_y$, Δk_y is the turbulence spectral width, and α_1 is the power law index for $\langle N \rangle$ (i.e. $\langle N \rangle \sim \left(k_y\right)^{-\alpha_1}$. Note that since the streamer induced heat flux varies poloidally, $\left(\left\langle \hat{Q}^2 \right\rangle / \langle Q \rangle^2\right)^{1/2}$ is finite while $\left\langle \hat{Q} \right\rangle / \langle Q \rangle$ vanishes. Thus, the RMS normalized heat flux perturbation $\Delta Q = \left(\left\langle \hat{Q}^2 \right\rangle / \langle Q \rangle^2\right)^{1/2}$ is given by

18) $$\Delta Q = \left(\sum_q q^4 \left|\overline{\phi}_q\right|^2 \right)^{1/2} \left| \frac{\alpha_1 k_x}{k_y \mathrm{v}_g' \Delta k} \right|_{res}.$$

Note that ΔQ is directly proportional to $\overline{\phi}$, the streamer fluctuation level. Not surprisingly, then, strong streamer excitation necessarily implies that the heat flux pdf has large variance. Indeed, balancing nonlinear pumping growth with Kelvin-Helmholtz break-up gives $\overline{\mathrm{v}}_q \sim \gamma_q / q_x$, so

19) $$\Delta Q \sim \left(q_y \gamma_q / q_x\right)\left(\alpha_1 k_x / k_y \mathrm{v}_g' \Delta k\right).$$

Several aspects of this estimate of the normalized flux variance are of interest. First, note that $\Delta Q \sim 1/q_x$ where q_x is the wavenumber of the (streamer flow) Kelvin-Helmholtz (KH) instability). Thus, long wavelength KH implies large ΔQ, since the residual vortices (i.e. those produced by KH break-up of the streamer) will be extended (i.e. $q_x \gtrapprox q_x$). Note also that near marginality, $\gamma_q \sim \omega_c (e\phi/T)$ (where ω_c is the curvature frequency and $e\phi/T$ is the ITG fluctuation level). Thus, the combined influences of large γ_q and small $q_x \left(q_x \gtrapprox q_y \right)$ suggests that the flux variance can indeed be significant, i.e. $\Delta Q \sim .2 - .5$ for typical parameters. This estimate suggests that further, quantitative studies of flux statistics are most certainly warranted.

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