Effects of electric field and Coriolis force on electrohydrodynamic stability of poorly conducting couple stress parallel fluid flow in a channel

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ABSTRACT

The linear stability of electrohydrodynamic poorly conducting couple stress viscous parallel fluid flow in a channel is studied in the presence of a non-uniform transverse electric field and Coriolis force using energy method and supplemented with Galerkin Technique. The sufficient condition for stability is obtained for sufficiently small values of the Reynolds number, R_e. From this condition we show that strengthening or weakening of the stability criterion is dictated by the values of the strength of electric field, the coefficient of couple stress fluid and independent of Taylor number. In particular, it is shown that the interaction of electric field with couple stress is more effective in stabilizing the poorly conducting couple stress fluid compared to that in an ordinary Newtonian viscous fluid.

Key words: stability, couple stress, electrohydrodynamic, non-uniform transverse electric field

1. INTRODUCTION

The effective functioning of microfluidic devices in electronics, electrical and mechanical engineering involving fluids, particularly those having vibrations and petroleum products containing organic, inorganic and other microfluidics, require the understanding and control of stability of parallel fluid flows. These substances, dissolving in the fluid, make the fluid poorly conducting. The electrical conductivity, σ, of such poorly conducting fluidics, increases with the temperature and the concentration of freely suspended particles. These freely suspended particles in fluid spin producing microrotation, forming micropolar fluid. According to Eringen (1966) the micropolar fluids may be regarded as non-Newtonian fluids like fluid suspensions. The presence of dust in the atmospheric fluid, the cholesterols, RBC, WBC and so on in the physiological fluid, the Hylauronic acid and nutrients in synovial fluid in synovial joints, the presence of Deuterium - Tritium (DT) in inertial fusion target may also be modeled using micropolar fluid theory of Eringen (1966). This theory takes care of the inertial characteristics of the substructure particles which are allowed to spin and thus undergo microrotation (Peddicson and McNitt 1970, Ariman et al. 1973, Lukaszewicz 1999 and Eringen 2001). A particular case of micropolar fluid theory, when the microrotation balances the natural vorticity of a poorly conducting fluidics in the presence of an electric field, is called 'electrohydrodynamic couple stress fluid' (EHDCF) (Rudraiah 1998, 2003).

We however note that the hydrogen isotopes DT are freely suspended in IFT which will spin and executing microroation. This microroation in a fluid forms micropolar fluid whose theory has been developed by Eringen (1996). According to Eringen, a particular case of micropolar fluid, when the microroation balances with the natural vorticity of the fluid there will be an antisymetric stress known as couple stress and forming couple stress fluid (CSF). So far, in the literature on IFE, to our knowledge, no work is available on the Electrohydrodynamic stability of flows using CSF. These EHDCFs exhibit a variation of electrical conductivity, $\nabla \sigma$, increasing with temperature and concentration of freely suspended particles, releases the charges from the nuclei forming distribution of charge density, ρ_e. These charges induce an electric field, $\vec{E}_i$. If need be, we can apply an electric field, $\vec{E}_a$, by embedding electrodes of different potentials at the boundaries. The total electric field, $\vec{E} = \vec{E}_i + \vec{E}_a$, produces a current density, $\vec{J} = \sigma \vec{E}$, according to Ohm's law and also produces an electric force, $\vec{F}_e = \rho_e \vec{E}$. This $\vec{J}$ acts as sensing and the force, $\vec{F}_e$, acts as actuation. These two properties make the poorly conducting couple stress fluid to act as a smart material (Rudraiah 2003). This smart couple stress poorly conducting fluid plays a significant role in controlling stability of parallel flows which is essential for an effective function of machineries that are used in practical problems mentioned above. This poorly conducting CSF in the presence of an electric filed, is called Electrorheological Fluid (ERF)/EHDCF introduced by Winslow (1949) and Rajagopal and Wineman (1995). These ERFs have a number of possible technical applications in the various areas of microelectronics, industrial applications in addition to increase the efficiency of the extraction of IFE as discussed above. Because of this importance considerable interest has been evinced, during the last decade, in the study of experimental and theoretical aspects of the effects of couple stress fluid (see K. Rajagopal and A.S. Wineman 1995, K. Rajagopal and M. Ruzicka 1996 and M. Ruzicka 1997).

This paper deals with the Electrohydrodynamic stability of a poorly conducting parallel viscous couple stress fluid flow in a channel in the presence of non-uniform electric field and coriolis force using linear stability analysis. In hydromagnetics, the linear stability of an incompressible non-dissipative ordinary fluid of variable density without buoyancy force has been investigated (see Taylor 1931). He concludes that the coriolis forces has a stabilizing influence and he further concludes that it brings about stability in the configuration, when it is thoroughly unstable without them. Barcilon and Pedlosky (1967) have investigated the hydrodynamic steady motions in a rotating stratified fluid using the linear theory and have shown that the rotating stratified fluid persists into the non-linear range. In particular, they have shown that even in the non-linear region the diffusive processes are very important throughout the fluid region. The stability of heterogeneous fluids has been extensively investigated (see Shivakumara and Venkatachalappa, M 2004, Miles1961 and Howard and Gupta 1962). These results have been extended to include the stability of cylindrical masses of fluid but mostly for axisymmetric disturbances. This has been extended to investigate the stability for non-axisymmetric disturbances of homogeneous, incompressible fluid having a solid body rotation (see Ludwieg1961). Later stability of heterogeneous flows to non-axisymmetric disturbances has been studied (see Rudraiah and Narayana 1972). So far, to our knowledge, much work has not been done on the study of Electrohydrodynamic stability using smart property of couple stress poorly conducting fluid.

The study of it is the main objective of this paper. In this paper we consider the combined effect of coriolis force and Electric force on electrohydrodynamic stability of a homogeneous viscous couple stress fluid flow in a channel. To achieve this objective, this paper is planned as follows. The required basic equations, corresponding boundary conditions are given in section 2 on mathematical formulation. The basic state and the stability equations are given in section 3. The stability analysis and Numerical solution is given respectively in section 4 and 5. Discussion in the section 5 and conclusions in the final section 6.

2. MATHEMATICAL FORMULATION

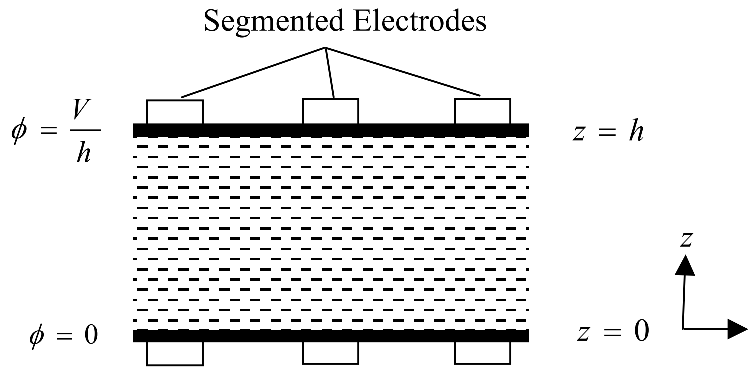

Fig 1: Physical Configuration

We consider a horizontal poorly conducting couple stress fluid flow in a channel bounded on both sides by electro-conducting impermeable rigid plates embedded with segmented electrodes located at $z = 0$ and $z = h$ having different electric potentials $\phi = 0$ at $z = 0$ and $\phi = \left(\frac{V}{h}\right)$ at $z = h$ as shown in Fig 1.

For the sake of clarity, we first give the general form of modified basic equations for a poorly conducting couple stress incompressible fluid in a channel, modification in the sense of addition of the couple stress, electric force and coriolis force to be obtained from the general form of Maxwell equations.

We note that one of the limitations encountered in the continuum theory is the lack of taking into account the microrotation of freely suspended particles in a fluid. For example, hyaluronic acid (HA) molecules and other nutrients present in synovial fluid, RBC, WBC and so on in blood, DT in Inertial Fusion Target (IFT) in the extraction of Inertial Fusion Energy (IFE) and so on are freely suspended executing spin. In that case, the microrotation of the microelements must be taken into account in deriving the required basic equations where the microelement motions play a significant role. In such situations, the couple stress theory a particular case of micropolar fluid, as explained above, is useful. Then the required basic equations for a couple stress poorly conducting fluid, following Stokes (1968), Rudraiah et al (1998, 2011), are:

The conservation of mass, for an incompressible fluid:

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

The conservation of momentum:

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right) = -\nabla p + \mu \nabla^2 \vec{q} - \lambda \nabla^4 \vec{q} + \rho_e \vec{E} - 2\rho \Omega \times \vec{q} \tag{2}$$

The conservation of energy:

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa \nabla^2 T \tag{3}$$

The conservation of species:

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = \varpi \nabla^2 C \tag{4}$$

where $\vec{q} = (u, w)$ is the velocity, ρ the density, p the pressure, λ the coefficient of couple stress, μ the viscosity of the fluid, Ω the angular velocity, ρ_e the distribution of charge density , T the temperature, κ the thermal conductivity, C the concentration, ϖ the soluble diffusivity

The conservation of charges:

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \vec{J} = 0 \tag{5}$$

$$\vec{J} = \rho_e \vec{q} + \sigma \vec{E} \tag{6}$$

$\vec{J}$ the current density, which is the sum of convective current, $\rho_e \vec{q}$, and conduction current, $\sigma \vec{E}$, σ the electrical conductivity, $\vec{E}$, the electric field. These are supplemented with the Maxwell Field equations for a conducting medium:

Gauss law

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{7}$$

where ε_o is the dielectric constant for free space.

In a poorly conducting fluid, the induced magnetic field is negligible and there is no applied magnetic field, hence the Faradays law becomes

$$\nabla \times \vec{E} = 0 \tag{8}$$

That is, the electric field is conservative, so that

$$\vec{E} = -\nabla \phi \tag{9}$$

where ϕ is the electric potential

Eq. (5), using Eqs. (6) and (1), takes the form

$$\frac{D\rho_e}{Dt} + \nabla \cdot \left(\sigma \vec{E} \right) = 0 \tag{10}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\vec{q} \cdot \nabla \right)$. We note that in a poorly conducting fluid $\sigma << 1$ and hence any perturbation on it is assumed to be negligible and increases with conduction temperature, T_b, and diffusion concentration, C_b, such that

$$\sigma = \sigma_o \left[1 + \alpha_t \left(T_b - T_0 \right) + \alpha_c \left(C_b - C_o \right) \right] \quad . \tag{11}$$

Here σ_0 is that of σ at $T_b = T_0$ and $C_b = C_0$, α_t and α_c are the volumetric expansion coefficients of σ. The expressions for T_b and C_b are obtained in section 3.1.

3. THE LINEAR STABILITY EQUATIONS FOR A COUPLE STRESS POORLY CONDUCTING FLUID

In this section we derive the stability equations subject to infinitesimal disturbances superposed on the basic state given in section 3.1 below.

3.1 Basic State

We consider a basic flow, in a poorly conducting couple stress fluid and assuming it to be fully developed and unidirectional parallel to the plates driven by a constant pressure gradient $\frac{\partial p_b}{\partial x}$. Then the basic flow, u_b, parallel to the boundaries in the x - direction, satisfies the momentum equations

$$0 = -\frac{\partial p_b}{\partial x} + \mu \frac{\partial^2 u_b}{\partial z^2} - \lambda \frac{\partial^4 u_b}{\partial z^4} + \rho_{eb} E_{bx} - 2\Omega w_b \tag{12}$$

$$0 = -\frac{\partial p_b}{\partial z} + \rho_{eb} E_{bz} - 2u_b \Omega$$

where the suffix b represents the basic state quantities.

The boundary conditions are the no-slip conditions

$$u_b = 0 \text{ at } z = 0 \text{ and } h \tag{13}$$

and the couple stress conditions

$$\frac{d^2 u_b}{dz^2} = 0 \text{ at } z = 0 \text{ and } h \tag{14}$$

Further, T_b and C_b in Eq. (11) are the solutions of

$$\frac{d^2T_b}{dz^2}=0 \text{ and } \frac{d^2C_b}{dz^2}=0 \tag{15}$$

satisfying the conditions

$$T_b = T_0 \text{ and } C_b = C_0 \text{ at } z=0 \tag{16a}$$

$$T_b = T_1 \text{ and } C_b = C_1 \text{ at } z=h \tag{16b}$$

Solutions of Eqs. (15), satisfying the conditions (16a, b), are

$$T_b = \frac{\Delta T}{h}z + T_0 \text{ and } C_b = \frac{\Delta C}{h}z + C_0 \tag{17}$$

where $\Delta T = T_1 - T_0$ and $\Delta C = C_1 - C_0$

Substituting the solutions given by Eq. (17) into Eq. (11), we get

$$\sigma_b = \sigma_0(1+\alpha_h z) \tag{18}$$

where $\alpha_h = \alpha_t \dfrac{\Delta t}{h} + \alpha_c \dfrac{\Delta C}{h}$.

In a poorly conducting fluid, the frequency of charge distribution is smaller than the corresponding relaxation frequency of the electric field, so that $\dfrac{D\rho_e}{Dt}$ in Eq. (10) is negligible compared to $\nabla\cdot\left(\sigma \vec{E}\right)$. Then, from Eq. (10) after neglecting $\dfrac{D\rho_e}{Dt}$ and using Eqs. (9) and (18), we get

$$\frac{\partial^2\phi_b}{\partial z^2} + \alpha_h \frac{\partial \phi_b}{\partial z} = 0 \tag{19}$$

subject to the boundary conditions

$$\phi_b = 0 \quad \text{at } z=0 \tag{20a}$$

$$\phi_b = V \quad \text{at } z=h \tag{20b}$$

where V is the applied uniform electric potential.

We make quantities in Eqs. (19) and (20a,b) dimensionless, using

$$\phi_b^{\ *} = \frac{\phi_b}{V}, \rho_{eb}^* = \frac{\rho_{eb}}{\varepsilon_0\left(\dfrac{V}{h^2}\right)}, x^* = \frac{x}{h}, z^* = \frac{z}{h} \tag{21}$$

where the asterisks (*) denote the dimensionless quantities. Substituting Eq.(21) into Eqs.(19) and (20a,b) and for simplicity neglecting the asterisks, we get

$$\frac{\partial^2 \phi_b}{\partial z^2} + \alpha_h \frac{\partial \phi_b}{\partial z} = 0 \tag{22}$$

satisfying the boundary conditions

$$\phi_b = 0 \text{ at } z = 0 \tag{23a}$$

$$\phi_b = 1 \text{ at } z = 1 \tag{23b}$$

The solution of Eq. (22), satisfying the boundary conditions (23a,b), is

$$\phi_b = \frac{\log(1+\alpha_h z)}{\log(1+\alpha_h)} \tag{24}$$

The expression for ρ_{eb} can be obtained, from Eq. (7), using Eq. (24), as

$$\rho_{eb} = \frac{\alpha_h^2}{(1+\alpha_h z)^2 \log(1+\alpha_h)} \tag{25}$$

Eq. (9), using Eq. (24), becomes

$$E_{bx} = 0,\ E_{bz} = -\frac{\alpha_h}{(1+\alpha_h z)\log(1+\alpha_h)} \tag{26}$$

We make Eqs. (12) to (14) dimensionless, using

$$u_b^* = \frac{u_b}{u_0}, p_b^* = \frac{p_b}{\rho u_0^2}, z^* = \frac{z}{h}, x^* = \frac{x}{h}, \rho_{eb}^* = \frac{\rho_{eb}}{\varepsilon_0\left(\frac{V}{h^2}\right)}, E_b^* = \frac{E_b}{\left(\frac{V}{h}\right)} \tag{27}$$

where u_0 is the average velocity. Substituting Eq. (27) into Eqs. (12) to (14), using Eqs. (25) and (26) and simplifying and for simplicity neglecting the asterisks, we get

$$\Lambda_c D^4 u_b - D^2 u_b = c \tag{28}$$

where $\Lambda_c = \frac{1}{a^2}$, $a = \frac{h}{m}$, $m = \sqrt{\frac{\lambda}{\mu}}$ the coefficient of the couple stress fluid, $c = -\mathrm{Re}\,P$ a positive constant, $P = \frac{\partial p_b}{\partial x}$, $\mathrm{R}_e = \frac{u_0 h}{\upsilon}$ the Reynolds number. The required boundary conditions are

$$u_b = D^2 u_b = 0 \text{ at } \mathrm{z} = 0 \text{ and } 1 \tag{29}$$

Solving Eq. (28), using the boundary conditions (29), we get

$$u_b = c\left[\frac{z}{2} - \frac{z^2}{2} - \Lambda_c + \Lambda_c \cosh\left(\frac{1}{2\sqrt{\Lambda_c}} - \frac{z}{\Lambda_c}\right)\sec h\left(\frac{1}{2\sqrt{\Lambda_c}}\right)\right] \tag{30}$$

To find *c*

The average velocity is given by $u_0 = \frac{1}{h}\int_0^h u_b dz$. From this we get

$$c = \frac{12}{1 - 12\Lambda_c + 24\Lambda_c^{3/2}\tanh\left(\frac{1}{2\sqrt{\Lambda_c}}\right)}$$

Substitute c in Eq. (30), we get

$$u_b = \frac{-6\left(\left(1+z\right)z + 2\Lambda_c - 2\Lambda_c \cosh\left(\frac{1-2z}{2\sqrt{\Lambda_c}}\right)\sec h\left(\frac{1}{2\sqrt{\Lambda_c}}\right)\right)}{1 - 12\Lambda_c + 24\Lambda_c^{3/2}\tanh\left(\frac{1}{2\sqrt{\Lambda_c}}\right)} \tag{31}$$

3.2 Stability Equations

In a two – dimensional, incompressible, homogeneous poorly conducting couple stress fluid flow, with $q_i = \left(u, w\right)$, the Eqs. (1) to (5) takes the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\nabla^2 u - \frac{\lambda}{\rho}\nabla^4 u + \frac{\rho_e}{\rho}E_x - 2\Omega w \tag{32}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\nabla^2 w - \frac{\lambda}{\rho}\nabla^4 w + \frac{\rho_e}{\rho}E_z + 2\Omega u \tag{33}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{34}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = \kappa\nabla^2 T \tag{35}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + w\frac{\partial C}{\partial z} = \varpi\nabla^2 C \tag{36}$$

$$\frac{\partial \rho_e}{\partial t} + u\frac{\partial \rho_e}{\partial x} + w\frac{\partial \rho_e}{\partial z} + \sigma\left(-\nabla^2\phi - \frac{1}{\sigma}\frac{\partial \phi}{\partial z}\frac{\partial \sigma}{\partial z}\right) = 0 \tag{37}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial z^2} + \frac{\partial^4}{\partial z^4}$

To study the Electrohydrodynamic stability of couple stress poorly conducting fluid in a saturated porous medium as shown in Fig. 2, we superimpose an infinitesimal disturbance, denoted by the primes, over the basic state denoted by suffix *'b'* of the form

$$u = u_b + u', w = w', p = p_b + p', E_x = E_{bx} + E'_x, E_y = E_{bz} + E'_z, \rho_e = \rho_{eb} + \rho_e{}', T = T_b + T', C = C_b + C' \quad (38)$$

Substituting Eq. (38) into Eqs. (32) to (37), linearising by neglecting the product and higher order of prime quantities compared to the basic state, we obtain

$$\frac{\partial u'}{\partial t} + u_b \frac{\partial u'}{\partial x} + w' \frac{\partial u_b}{\partial z} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial z^2}\right) - \frac{\lambda}{\rho}\left(\frac{\partial^4 u'}{\partial x^4} + 2\frac{\partial^4 u'}{\partial x^2 \partial z^2} + \frac{\partial^4 u'}{\partial z^4}\right) - \frac{1}{\rho}\left(\rho_{eb} E'_x + \rho'_e E_{bx}\right) - 2\Omega w' \quad (39)$$

$$\frac{\partial w'}{\partial t} + u_b \frac{\partial w'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial z^2}\right) - \frac{\lambda}{\rho}\left(\frac{\partial^4 w'}{\partial x^4} + 2\frac{\partial^4 w'}{\partial x^2 \partial z^2} + \frac{\partial^4 w'}{\partial z^4}\right) + \frac{1}{\rho}\left(\rho_{eb} E'_z + \rho'_e E_{bz}\right) \quad (40)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \quad (41)$$

$$\frac{\partial T'}{\partial t} + u_b \frac{\partial T'}{\partial x} + w' \frac{\partial T_b}{\partial z} = \kappa\left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial z^2}\right) \quad (42)$$

$$\frac{\partial C'}{\partial t} + u_b \frac{\partial C'}{\partial x} + w' \frac{\partial C_b}{\partial z} = \varpi\left(\frac{\partial^2 C'}{\partial x^2} + \frac{\partial^2 C'}{\partial z^2}\right) \quad (43)$$

$$\left(\frac{\partial}{\partial t} + u_b \frac{\partial}{\partial x}\right)\left(\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial z^2}\right) + w\frac{\partial}{\partial z}\left(\frac{\partial^2 \phi_b}{\partial z^2}\right) + \sigma_b\left(\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial z^2}\right) + \sigma' \frac{\partial^2 \phi_b}{\partial z^2} + \frac{\partial \phi_b}{\partial z}\frac{\partial \sigma'}{\partial z} + \frac{\partial \phi'}{\partial z}\frac{\partial \sigma_b}{\partial z} = 0 \quad (44)$$

These perturbed equations are made dimensionless using the quantities

$$u^* = \frac{u}{u_0}, w^* = \frac{w}{u_0}, z^* = \frac{z}{h}, x^* = \frac{x}{h}, p^* = \frac{p}{\rho u_0^2}, t^* = \frac{t}{\left(\frac{h}{u_0}\right)}, \rho_e^* = \frac{\rho_e}{\varepsilon_0\left(\frac{V}{h^2}\right)}, E_x^* = \frac{E_x}{\left(\frac{V}{h}\right)}, E_z^* = \frac{E_z}{\left(\frac{V}{h}\right)},$$

$$\phi^* = \frac{\phi}{V}, \sigma^* = \frac{\sigma}{\sigma_0}, T^* = \frac{T}{\beta h}, C^* = \frac{C}{\gamma h}, \sigma^* = \frac{\sigma}{\sigma_0} \quad (45)$$

Substituting Eq. (45) into Eqs. (39) to (44) and for simplicity neglecting the asterisks (*) and the primes, we get

$$\frac{\partial u}{\partial t} + u_b \frac{\partial u}{\partial x} + Du_b w = -\frac{\partial p}{\partial x} + \frac{1}{R_e}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\Lambda_c}{R_e}\left(\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial z^2} + \frac{\partial^4 u}{\partial z^4}\right) + W_e \rho_{eb} E_x - \frac{Ta}{R_e} w \quad (46)$$

$$\frac{\partial w}{\partial t} + u_b \frac{\partial w}{\partial x} = -\frac{\partial p}{\partial z} + \frac{1}{R_e}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{\Lambda_c}{R_e}\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial z^2} + \frac{\partial^4 w}{\partial z^4}\right) + W_e\left(\rho_{eb} E_z + \rho_e E_{bz}\right) - \frac{Ta}{R_e} w \quad (47)$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{48}$$

$$\frac{\partial T}{\partial t}+u_b\frac{\partial T}{\partial x}+w'DT_b=\frac{1}{\Pr R_e}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{49}$$

$$\frac{\partial C}{\partial t}+u_b\frac{\partial C}{\partial x}+w'DC_b=\frac{1}{Le\,R_e}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial z^2}\right) \tag{50}$$

$$\left(\frac{\partial}{\partial t}+u_b\frac{\partial}{\partial x}\right)\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}\right)+wD\left(D^2\phi_b\right)+\frac{\tau}{R_e}\left(\sigma_b\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}\right)+\sigma D^2\phi_b+D\phi_b\frac{\partial\sigma}{\partial z}+\frac{\partial\phi}{\partial z}D\sigma_b\right)=0 \tag{51}$$

where $W_e=\dfrac{\varepsilon_0 V^2}{\rho u_0^2 h^2}$ the Electric number, which Physically represents the ratio of electric energy to kinetic energy, $Ta=\dfrac{2\Omega h^2}{\nu}$ the Taylor number, $\Pr=\dfrac{\nu}{\kappa}$, the Prandtl number, $Le=\dfrac{\nu}{\varpi}$,the Lewis number, $\tau=\dfrac{\sigma_0 h^2}{\nu\varepsilon_0}$ the ratio of viscous relaxation time to charge relaxation time.

To discuss the stability of the system Eqs. (36) to (51) we use the normal mode solution of the form

$$\left(function\ of\ z\right)e^{il(x-ct)} \tag{52}$$

where $c=c_r+ic_i$ is the velocity of perturbed quantities, c_r is the phase velocity and c_i is the growth rate and l is the horizontal wave number which is real and positive. If $c_i>0$, the system is unstable and $c_i<0$, the system is stable. Eqs. (38) to (40), using Eq. (41), and after simplification, take the form

$$\left[\left(D^2-l^2\right)-ilR_e\left(u_b-c\right)-\Lambda_c\left(D^2-l^2\right)^2\right]u=ilR_e p+R_e Du_b w-W_e R_e\rho_{eb}E_x+Ta\,w \tag{53}$$

$$\left[\left(D^2-l^2\right)-ilR_e\left(u_b-c\right)-\Lambda_c\left(D^2-l^2\right)^2\right]w=R_e Dp-W_e R_e\left(\rho_{eb}E_z+\rho_e E_{bz}\right)-Ta\,u \tag{54}$$

$$ilu+Dw=0 \tag{55}$$

$$\left[ilR_e\left(u_b-c\right)-\frac{1}{\Pr}\left(D^2-l^2\right)\right]T=DT_b\,w \tag{56}$$

$$\left[ilR_e\left(u_b-c\right)-\frac{1}{Le}\left(D^2-l^2\right)\right]C=DC_b\,w \tag{57}$$

$$\left[ilR_e\left(u_b-c\right)\left(D^2-l^2\right)+\tau\left(\sigma_b\left(D^2-l^2\right)+D\sigma\, D\right)\right]\phi=-R_eD\left(D^2\phi_b\right)w-\tau\left(D^2\phi_b\sigma+D\phi_b\, D\sigma\right) \quad (58)$$

Eliminating the pressure p between Eqs. (53) and (54), by operating D on Eq. (53), multiplying Eq. (54) by il and then subtracting and use the stream function $u=D\psi, w=-il\psi$ and Eq. (8), we get the stability equation

$$lR_e(D^2-l^2)\psi-lR_e\frac{D^2u_b}{(u_b-c)}\psi+i\frac{(D^2-l^2)^2}{(u_b-c)}\psi-i\Lambda_c\frac{(D^2-l^2)^3}{(u_b-c)}\psi+iW_eR_e\frac{\left(D\rho_{eb}\, E_x\right)}{\left(u_b-c\right)}=0 \quad (59)$$

Eq. (59) is the required stability equation which is called the modified form of Orr-Sommerfeld equation (modified in the sense of incorporating the contribution from the electric force, $\rho_e\vec{E}$ and the couple stress fluid). The boundary conditions now take the form

$$\psi=D\psi=D^3\psi=0 \ at\ z=0\ and\ 1 \quad (60)$$

$$T=0\ at\ z=0\ and\ 1 \quad (61)$$

$$C=0\ at\ z=0\ and\ 1 \quad (62)$$

$$\phi=0\ at\ z=0\ and\ 1 \quad (63)$$

4. STABILITY ANALYSIS

To find the conditions for stability or instability of the basic flow, we find, following Shubha et al. (2008), Drazin and Reid (2004), and Rudraiah et al. (2011), the nature of c using the energy method. For this, we multiply Eq. (59) by $\bar{\psi}$, the complex conjugate of ψ, and integrating the resulting equation with respect to z from 0 to 1 and using the boundary conditions (60) and after simplification, we get

$$\left(I_2^2+2l^2I_1^2+l^4I_0^2\right)+\Lambda_c\left(I_3^2+3l^2I_2^2+3l^4I_1^2+l^6I_0^2\right)=-ilR_eQ+ilR_ec(I_1^2+l^2I_0^2) \quad (64)$$

where

$$I_n^2=\int_0^1\left|D^n\psi\right|^2dz \quad (n=0\ to\ 3)$$

$$Q=\int_0^1\left[u_b\left|D\psi\right|^2+(l^2u_b+D^2u_b)\left|\psi\right|^2\right]dz+\int_0^1\bar{\psi}D\psi Du_bdz-W_e\int_0^1\left(D^2-l^2\right)D\phi\bar{\psi}=Q_r+iQ_i$$

$$Q_r=\mathrm{Re}\left(Q\right)=\int_0^1\left\{u_b\left|D\psi\right|^2+\left(l^2u_b+\frac{1}{2}D^2u_b\right)\left|\psi\right|^2-W_e\left(D^2-l^2\right)\left(\psi_rD\phi_r+\psi_iD\phi_i\right)\right\}dz \quad (65)$$

and

$$Q_i = \text{Im}(Q) = \int_0^1 \left\{ \left(\psi_r D\psi_i - \psi_i D\psi_r \right) Du_b + W_e \left(D^2 - l^2 \right) \left(\psi_i D\phi_r - \psi_r D\phi_i \right) \right\} dz \tag{66}$$

The second term on the left hand side of Eq. (64) is the contribution of couple stress fluid and the term involving W_e in the first term on the right hand side of Eq. (64) is the effect of electric field.

Equating the real and imaginary parts of Eq. (64) to zero, we respectively get

$$c_r = \frac{Q_r}{\left(I_1^2 + l^2 I_0^2 \right)} \tag{67}$$

$$c_i = \frac{Q_i - \dfrac{1}{lR_e}\left[\Lambda_c I_3^2 + \left(1 + 3l^2\Lambda_c\right) I_2^2 + l^2\left(2 + 3l^2\Lambda_c\right) I_1^2 + l^4\left(1 + l^2\Lambda_c\right) I_0^2 \right]}{\left(I_1^2 + l^2 I_0^2 \right)} \tag{68}$$

We write Eq. (66) in the form

$$\text{Im}(Q) = \frac{i}{2}\int_0^1 \left(\psi D\bar{\psi} - \bar{\psi} D\psi \right) Du_b dz + W_e \int_0^1 \left(D^2 - l^2 \right) \left(\psi_i D\phi_r - \psi_r D\phi_i \right) dz \tag{69}$$

From Eq. (69) it follows that

$$\left| \text{Im}(Q) \right| \le \int_0^1 |\psi| |D\psi| |Du_b| dz + W_e \int_0^1 \left| \left(D^2 - l^2 \right) \left(\psi_i D\phi_r - \psi_r D\phi_i \right) \right| dz$$

and using Schwarz's inequality, we get

$$\left| \text{Im}(Q) \right| \le I_1 I_0 q + B_1$$

where

$$q = \max_{0 \le z \le 1} \left| Du_b \right|$$

$$B_1 = W_e \int_0^1 \left| \left(D^2 - l^2 \right) \left(\psi_i D\phi_r - \psi_r D\phi_i \right) \right| dy$$

This gives the upper bound for c_i

$$c_i \le \frac{\left(qI_0 I_1 + B_1 \right) - \dfrac{1}{lR_e}\left[\Lambda_c I_3^2 + \left(1 + 3l^2\Lambda_c\right) I_2^2 + l^2\left(2 + 3l^2\Lambda_c\right) I_1^2 + l^4\left(1 + l^2\Lambda_c\right) I_0^2 \right]}{\left(I_1^2 + l^2 I_0^2 \right)} \tag{70}$$

From Eq. (70), it follows that a sufficient condition for stability is

$$R_e < \frac{1}{l\left(qI_0 I_1 + B_1 \right)}\left[\Lambda_c I_3^2 + \left(1 + 3l^2\Lambda_c\right) I_2^2 + l^2\left(2 + 3l^2\Lambda_c\right) I_1^2 + l^4\left(1 + l^2\Lambda_c\right) I_0^2 \right] \tag{71}$$

5. Numerical solution

Equations (56) to (59) together with boundary conditions (60) to (63) constitute an eigenvalue problem with c as an eigenvalue. Since the differential equations involve variable coefficients, the resulting eigenvalue problem has to be solved numerically, and note that the Galerkin-Type weighted residual method is more suitable to solve the same. Accordingly, the variables are written in series of basis function as

$$\psi(z)=\sum_{i=1}^{n}A_i\psi_i(z),\ \varphi(z)=\sum_{i=1}^{n}B_i\varphi_i(z),\ T(z)=\sum_{i=1}^{n}D_iT_i(z),\ C(z)=\sum_{i=1}^{n}E_iC_i(z) \tag{72}$$

where A_i, B_i, D_i and E_i are unknown constants are the basis functions $\psi_i(z), \varphi_i(z), T_i(z)$ and $C_i(z)$ are generally chosen that they satisfy the corresponding boundary conditions. Substituting Eq. (72) into Eqs. (56) to (59), multiplying the stability equation by $\psi_j(z)$, the continuity of charges equation by $\varphi_j(z)$, the temperature equation by $T_j(z)$ and the concentration equation by $C_j(z)$, performing the integration by parts with respect to z between $z=0$ and $z=1$ and using the boundary conditions (60) to (63), we obtain the following systems of 4n linear homogeneous algebraic equations in the 4n unknowns A_i, B_i, D_i, E_i ; $i=1,2,.....n$

$$a_{ji}\,A_i+b_{ji}\,B_i=0 \tag{73}$$

$$c_{ji}\,A_i+d_{ji}\,B_i+e_{ji}D_i+f_{ji}E_i=0 \tag{74}$$

$$g_{ji}\,A_i+h_{ji}D_i=0 \tag{75}$$

$$p_{ji}\,A_i+q_{ji}E_i=0 \tag{76}$$

The coefficients a_{ji} to q_{ji} involve the inner products of the basis functions and are given by

$$a_{ji}=lR_e\left(\left\langle u_b\,D^2\psi_j\psi_i-l^2u_b\,\psi_j\psi_i\right\rangle-c\left\langle D^2\psi_j\psi_i-l^2\,\psi_j\psi_i\right\rangle-\left\langle D^2u_b\psi_j\psi_i\right\rangle\right)+$$
$$i\left\langle D^4\psi_j\psi_i+l^4\,\psi_j\psi_i-2l^2D^2\psi_j\psi_i\right\rangle-i\Lambda_c\left\langle D^6\psi_j\psi_i-l^6\,\psi_j\psi_i-3l^2\left(D^4\psi_j\psi_i+l^2D^2\psi_j\psi_i\right)\right\rangle$$

$$b_{ji}=-lWe\,R_e\left\langle D\rho_{eb}\,\varphi_i\psi_j\right\rangle$$

$$c_{ji}=-il\,R_e\left\langle D^3\varphi_b\,\varphi_j\psi_i\right\rangle$$

$$d_{ji}=ilR_e\left(\left\langle u_b\,D^2\varphi_i\varphi_j-l^2u_b\,\varphi_j\varphi_i\right\rangle-c\left\langle D^2\varphi_i\varphi_j-l^2\,\varphi_j\varphi_i\right\rangle\right)+\tau\left(\left\langle\sigma_bD^2\varphi_i\varphi_j-l^2\,\varphi_j\varphi_i\right\rangle+\left\langle D\sigma_bD\varphi_i\varphi_j\right\rangle\right)$$

$$e_{ji} = \tau\alpha_t \left(\left\langle D^2\varphi_b\, \varphi_j T_i \right\rangle + \left\langle D\varphi_b\, DT_i \varphi_j \right\rangle \right)$$

$$f_{ji} = \tau\alpha_c \left(\left\langle D^2\varphi_b\, \varphi_j C_i \right\rangle + \left\langle D\varphi_b\, DC_i \varphi_j \right\rangle \right)$$

$$g_{ji} = ilR_e \left\langle \psi_i T_j \right\rangle$$

$$h_{ji} = ilR_e \left(\left\langle u_b T_i T_j \right\rangle - c \left\langle T_i T_j \right\rangle \right) - \frac{1}{\Pr} \left\langle D^2 T_i T_j - l^2\, T_i T_j \right\rangle$$

$$p_{ji} = ilR_e \left\langle \psi_i C_j \right\rangle$$

$$q_{ji} = ilR_e \left(\left\langle u_b C_i C_j \right\rangle - c \left\langle C_i C_j \right\rangle \right) - \frac{1}{Le} \left\langle D^2 C_i C_j - l^2\, C_i C_j \right\rangle \tag{77}$$

where the inner product is defined as $\langle ----\rangle = \int_0^1 (----)\, dz$.

The above set of homogeneous algebraic equations can have a non-trivial solution if and only if

$$\begin{vmatrix} a_{ji} & b_{ji} & 0 & 0 \\ c_{ji} & d_{ji} & e_{ji} & f_{ji} \\ g_{ji} & 0 & h_{ji} & 0 \\ p_{ji} & 0 & 0 & q_{ji} \end{vmatrix} = 0 \tag{78}$$

The eigenvalue has to be extracted from the above characteristics equation. For this, we select the trial functions as follows:

$$\psi_i = \frac{1}{5}(-1+z)^{i+1} z^{i+1} \left(-1 + 2\left(-1 + z^i\right) z^i\right),\ \varphi_i = z^{i+1} - z^i, T_i = z^{i+1} - z^i, C_i = z^{i+1} - z^i \tag{79}$$

The basis functions chosen above satisfy the boundary conditions automatically. Using these, the inner products were evaluated. The coupled equations (56) to (59) can now be expressed in the form of a generalized eigenvalue problem with c as an eigenvalue.

5. RESULTS AND DISCUSSION

A sufficient condition for stability in terms of Reynolds number, R_e, given by Eq. (71). Equations (56) to (59) together with boundary conditions (60) to (63) is also computed numerically using Galerkin Technique and the results are depicted graphically.

Figure 2 shows that the plot of growth rate c_i as a function of horizontal wave number l for different values of couple stress parameter, $\Lambda_c = \frac{1}{a^2}$ for fixed values of electric number, $W_e = 10$, Reynolds number, $R_e = 0.8$, Prandtl number, $\Pr = 5$, Lewis number $Le = 20$, $\tau = 2$. From the Fig. 3, it is clear that an increase in the value of a increases the value of c_i because an increase in a is to decrease the viscosity implying decrease in resistance to the flow, which in turn promotes instability much faster.

Figure 3 shows that the plot of c_i with l for different values of W_e for fixed values of $R_e = 0.8, a = 10, \Pr = 5, Le = 20, \tau = 2.$ From this figure, it may be inferred that for an increase in the value of W_e, decreases the value of c_i and thus make the system more stable. The reason being that an increase in W_e is to decrease the kinetic energy and hence makes the system more stable.

6.CONCLUSIONS

It is known that in the stability of classical Poiseuille flow a sufficient condition for stability is the existence of point of inflexion. This stability of Poiseuille flow was extended to electrohydrodynamic stability of an inviscid poorly conducting parallel fluid flow in the presence of an electric field and in the absence of couple stress fluid by Shubha et al (2008). They have shown that the electrohydrodynamic stability is determined in terms of electric number rather than the point of inflexion of the basic velocity profile. In contrast to this, in our paper a sufficient condition for stability is obtained for sufficiently small value of Reynolds number, R_e. From this we found that strengthening or weakening of a sufficient condition for stability depends on the electric number, W_e, the coefficient of couple stress fluid, m, and Prandtl number, $\Pr$, Lewis number , Le, and τ . From these, we conclude that the interaction of electric field with couple stress is more effective in stabilizing a poorly conducting couple stress fluid compare to that in ordinary Newtonian viscous fluid.

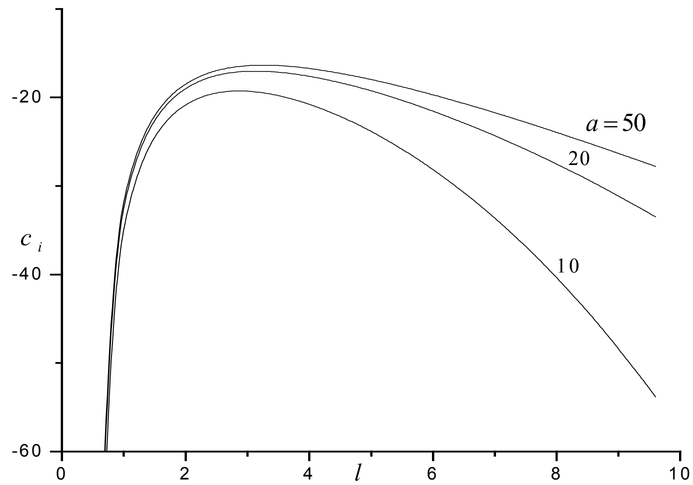

Fig. 2: Variation of c_i with l for different values of a when
$W_e = 10, R_e = 0.8,$ $\Pr = 5,$ $Le = 20$, $\tau = 2.$

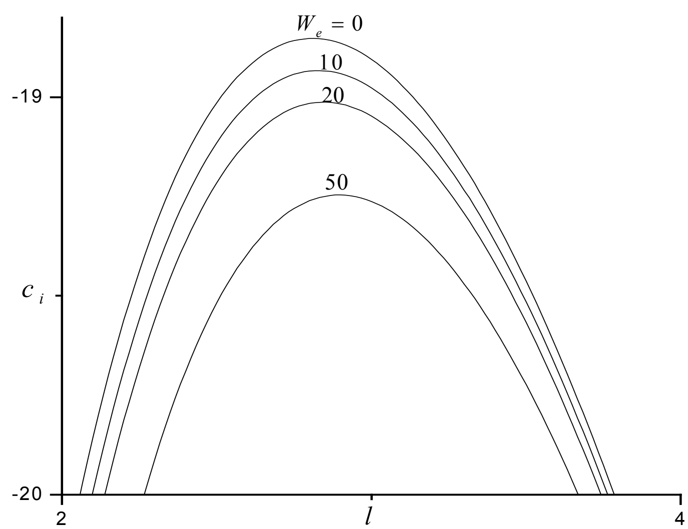

Fig 3: Variation of c_i with l for different values of W_e when
$R_e = 0.8, a = 10, \Pr = 5, Le = 20, \tau = 2.$

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