Simulation of Coulomb Collisions in Plasma Accelerators for Space Applications

D. D'Andrea[1], W.Maschek[1] and R. Schneider[2]

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[1]Institut for Institute for Nuclear and Energy Technologies - Research Center Karlsruhe
[2]Institut for Pulsed Power and Microwave Technology - Research Center Karlsruhe

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- SIMPLEX – An example of „applied“ accelerator
- Physical and numerical model
- A time scaling analysis for :
  - Intra-species
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Introduction: Space Propulsion

- Spacecraft acceleration generated by propellant discharge: $T = dm/dt\ v_e$
- Rate of expulsion of propellant $W = dm/dt\ g_0$
- Specific impulse: $I_{sp} = T/W = v_e / g_0$
- For constant exhaust velocity: $\Delta v = v_e \ln \frac{m_0}{m_f}$

The Electric Propulsion

Acceleration of propulsion gases by electrical heating and/or by electric and magnetic body forces.

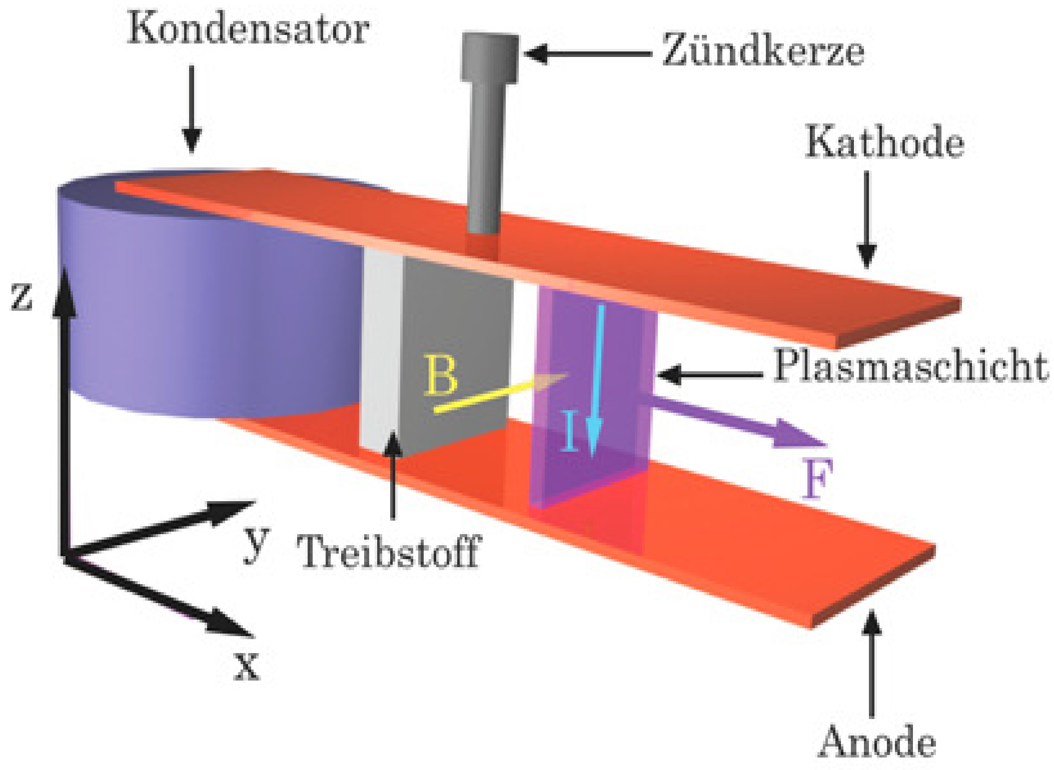
- Electrothermal : electrical heat addition and expansion through nozzle. Resistojets and Arcjets.
- Electrostatic : application of electric fields. Ion Thrusters and Colloid Thrusters
- Electromagnetic : application of electromagnetic fields. Hall Thrusters, Pulsed Plasma Thrusters(PPT) and Magnetoplasmadynamic Thrusters (MPDT).

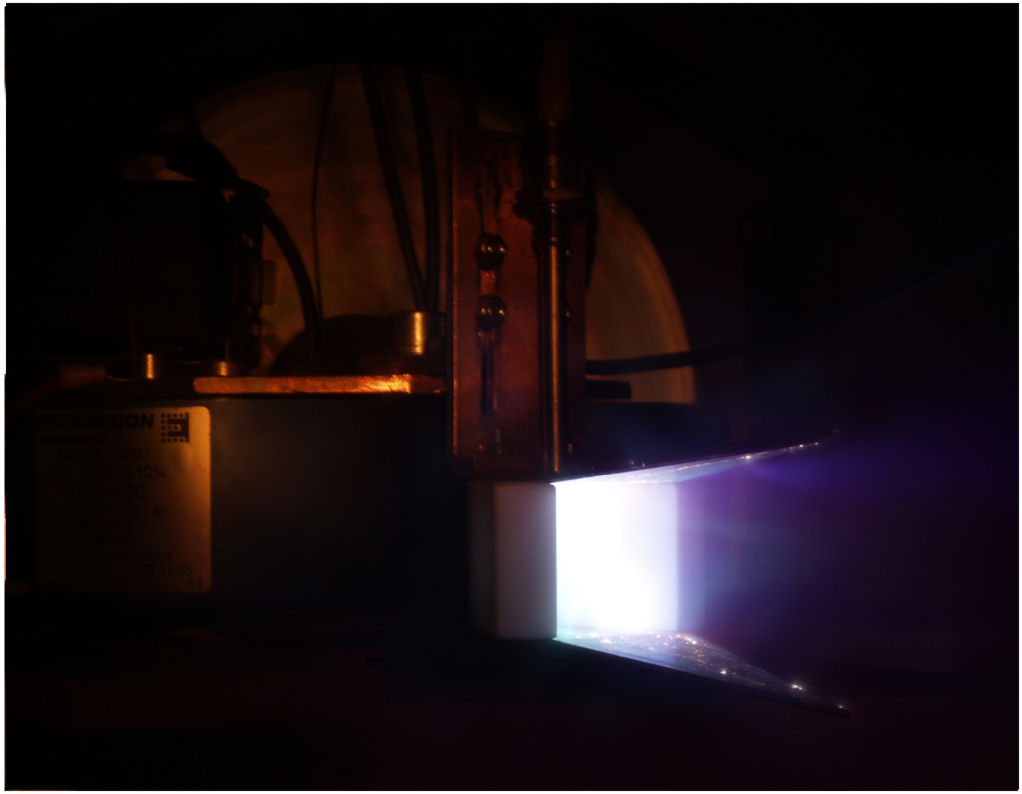
Robert G. Jahn - Physics of Electric Propulsion.(1968)

To the Moon with SIMPLEX

Stuttgart **I**nstationary **M**agneto**P**lasmadynamic thruster for **L**unar **Ex**ploration

Pictures from Uni-Stuttgart website

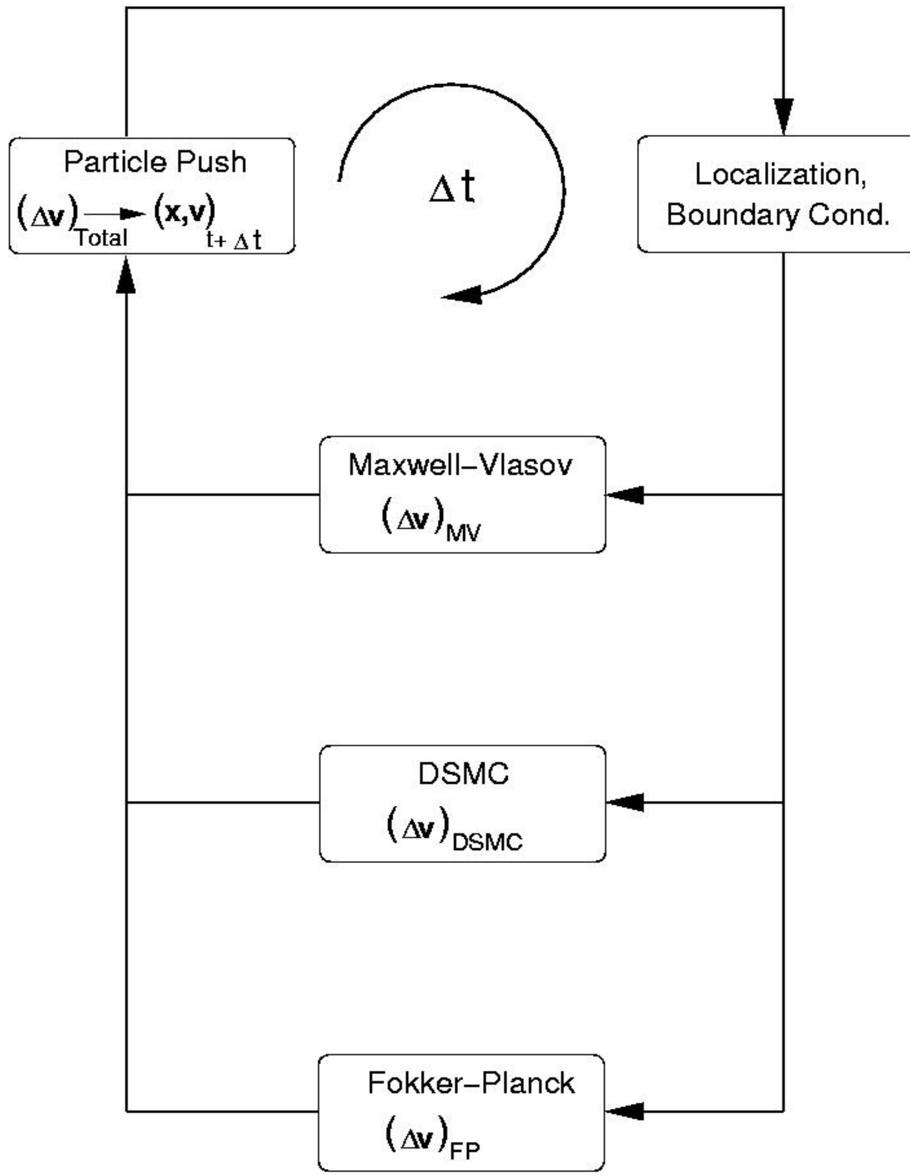




- Pulse time: 8 µs
- Peak current: 40 kA
- Capacitor voltage: 2000 V
- Exhaust velocity: 12 km/s
- Mean thrust: 1,4 mN
- Mass ablated/Bit 160 µg

Physical Mathematical Modelling

- Electrical quasi neutrality conditions
- Non-equilibrium conditions in several degrees of freedom. Failure of continuous models,e.g. Fluid models
- Presence of external and self-induced **E**-**B** fields
- Charged-Neutral collisions and Chemical reactions
- Elastic charged particle interactions



Intra-Species Collision: Governing Equations

$$\left(\frac{\delta f_\alpha}{\delta t}\right)_{Col} = -\frac{\partial}{\partial \mathbf{v}}\left[\mathbf{F}^{(\alpha)} f_\alpha\right] + \frac{1}{2}\frac{\partial^2}{\partial \mathbf{v}\partial \mathbf{v}} : \left[\overleftrightarrow{\mathbf{D}}^{(\alpha)} f_\alpha\right]$$ Fokker-Planck Equation (FP)

KEY QUANTITIES: THE ROSENBLTUH POTENTIALS

$$\mathbf{F}^{(\alpha)} \propto \frac{\partial H^\beta}{\partial \mathbf{v}}$$ Dynamical Friction

$$H^\beta = \frac{m_\alpha}{\mu_{\alpha\beta}} \int_{-\infty}^{+\infty} \frac{f_\beta(\mathbf{x}, \mathbf{w}, t)}{|\mathbf{v} - \mathbf{w}|} d^3 w$$

$$\overleftrightarrow{\mathbf{D}}^{(\alpha)} \propto \frac{\partial^2 G^\beta}{\partial \mathbf{v}\partial \mathbf{v}}$$ Diffusion Tensor

$$G^\beta = \int_{-\infty}^{+\infty} |\mathbf{v} - \mathbf{w}| f_\beta(\mathbf{x}, \mathbf{w}, t)\ d^3 w$$

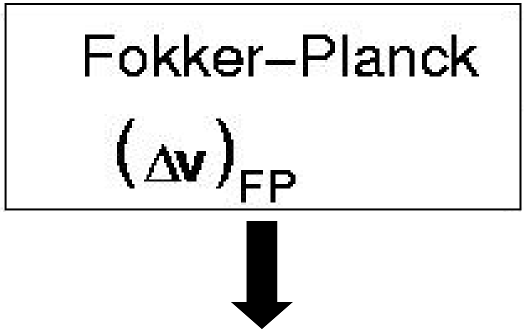
THE STOCHASTIC DIFFERENTIAL EQUATION (SDE)

$\vec{V} = \vec{V}(t)$ Stochastic variable; satisfying the SDE: $d\vec{V}(t) = \vec{F} \cdot dt + \overleftrightarrow{\mathbf{B}} \cdot d\vec{W}(t)$

with transition probability $P_2(\vec{V}, t / \vec{V}_0, t_0)$ $\Rightarrow$ P_2 fullfills a FP equation

Euler scheme $\vec{V}_p^{(n+1)} = \vec{V}_p^{(n)} + \vec{F}^{(n)} \cdot \Delta t + \overleftrightarrow{\mathbf{B}}^{(n)} \cdot \sqrt{\Delta t}\,\vec{\eta}_p^{(n+1)}$ $\overleftrightarrow{\mathbf{D}} = \overleftrightarrow{\mathbf{B}}\,\overleftrightarrow{\mathbf{B}}^T$

Numerical Framework: PIC Scheme

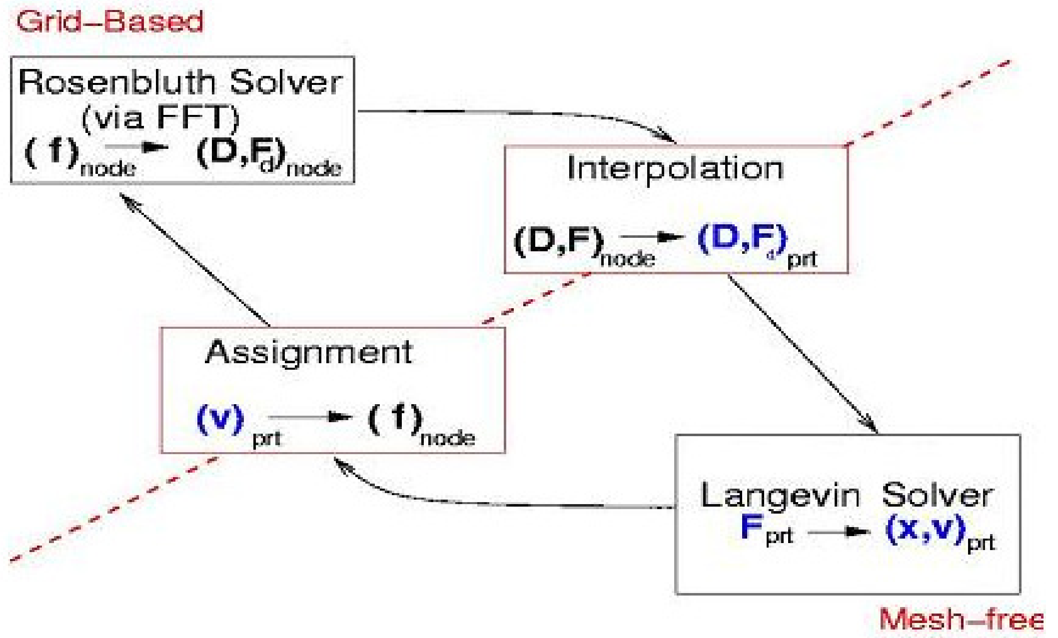


Evaluate the „key-quantities“

on the grid

Interpolate the coefficients

onto the particles



Reconstruct the

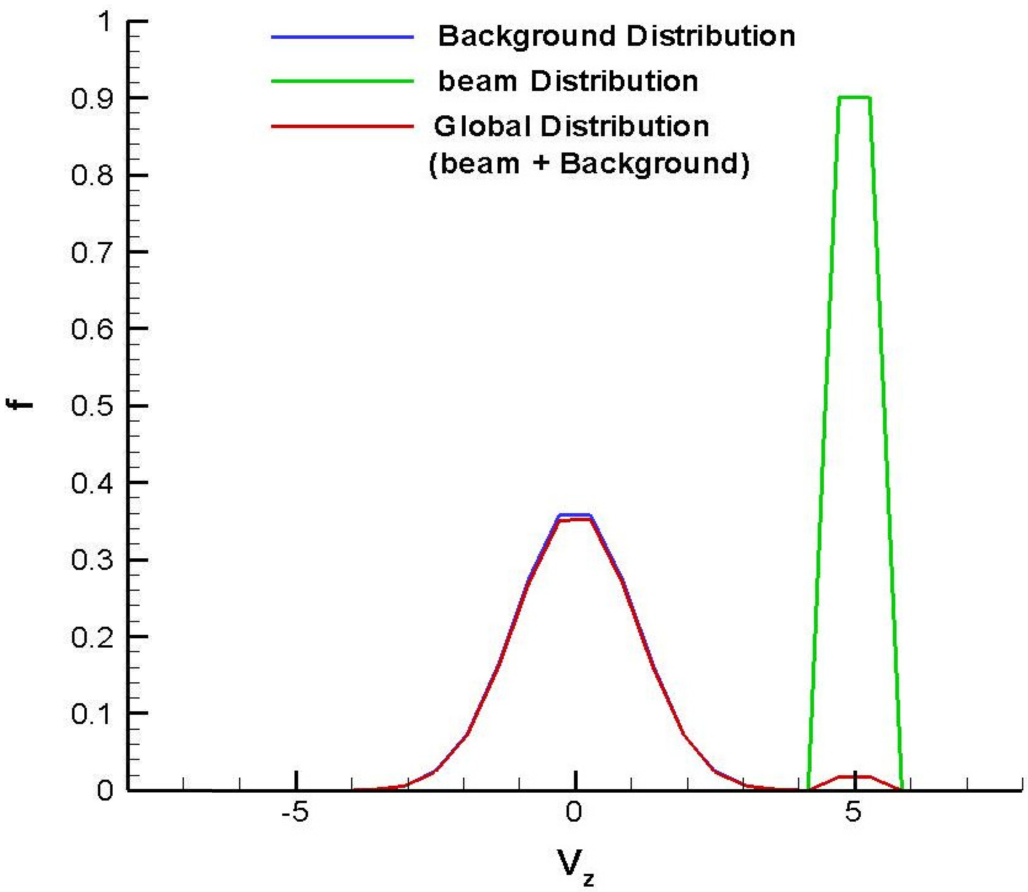
distribution function at $t_{n+1} = t_n + \Delta t$

Push the particles

in velocity space

Time Scaling Analysis

Delta vs. Maxwell



Possible approaches:

- **FULLY SELF-CONSISTENT**

  Mutual Influence of beam and background particles

- **TEST-PARTICLE ANSATZ**

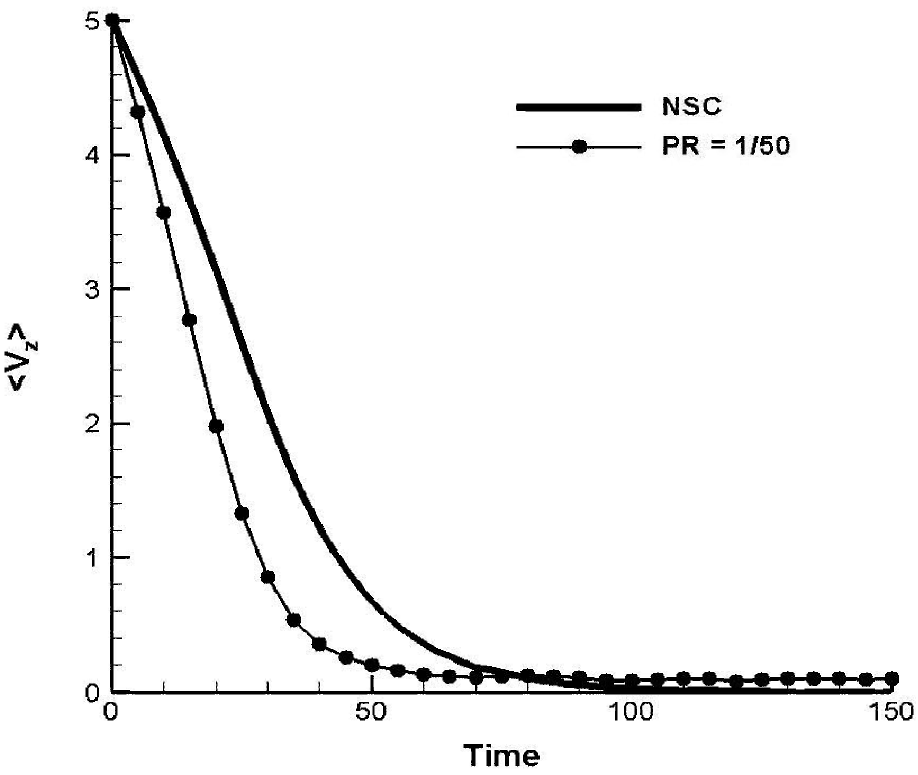
  Separation of friction and diffusion effects $\Rightarrow$ unrealstic

- **NOT SELF-CONSISTENT**

  Stochastical modelling of beam particles evolution with fixed background Maxwell

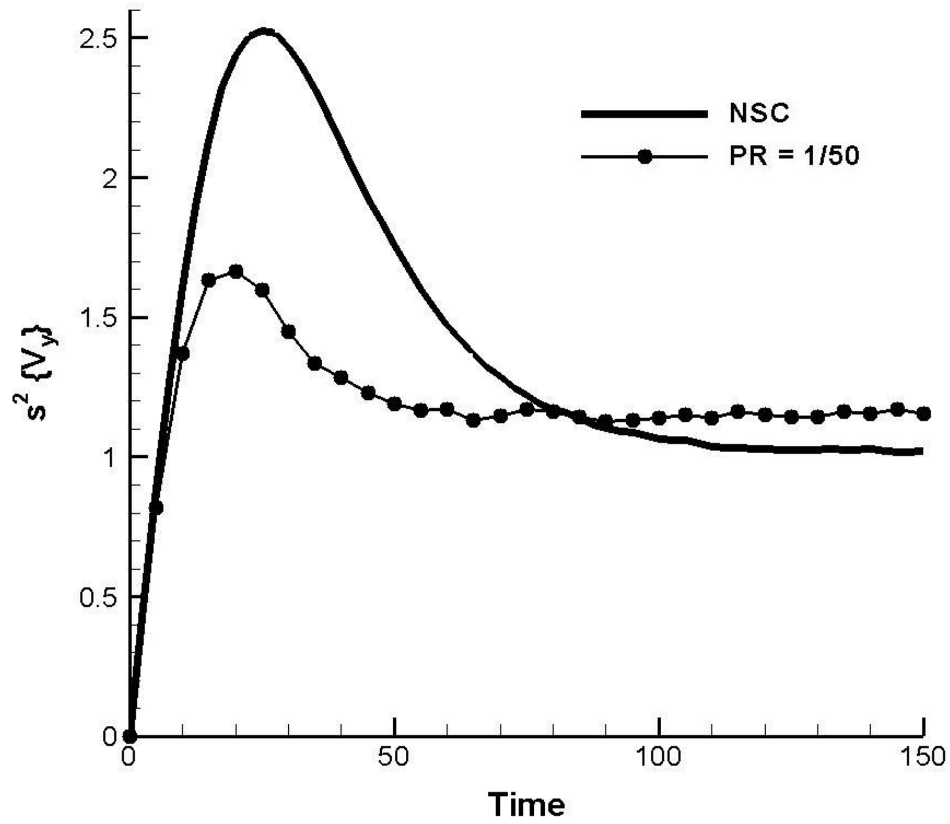
Moments Analysis

Mean value investigation



Mean value time evolution for self-consistent and reference simulation

Variance investigation



Transversal variance time evolution for self-consistent and reference simulation

Inter-Species Collision: Governing Equations

- Preliminaries: $c_e >> w_X \qquad m_e / m_X << 1 \qquad c_e = |\vec{c}_e| = const.$

- Friction and Diffusion known from the simple form of:

$$H_X(\vec{c},t) = c^{-1} \quad \text{and} \quad G_X(\vec{c},t) = c$$

- SDE becomes: $d\hat{C}(t) = -\alpha^2 \hat{C}(t)\,dt + \alpha \overleftrightarrow{H}\, d\vec{W}(t)$, where:

$$\alpha^2 = \Gamma_P^{(eX)} n_X c^{-3} \quad \text{and} \quad \overleftrightarrow{H} = \overleftrightarrow{I} - \hat{c}\hat{c}^T$$

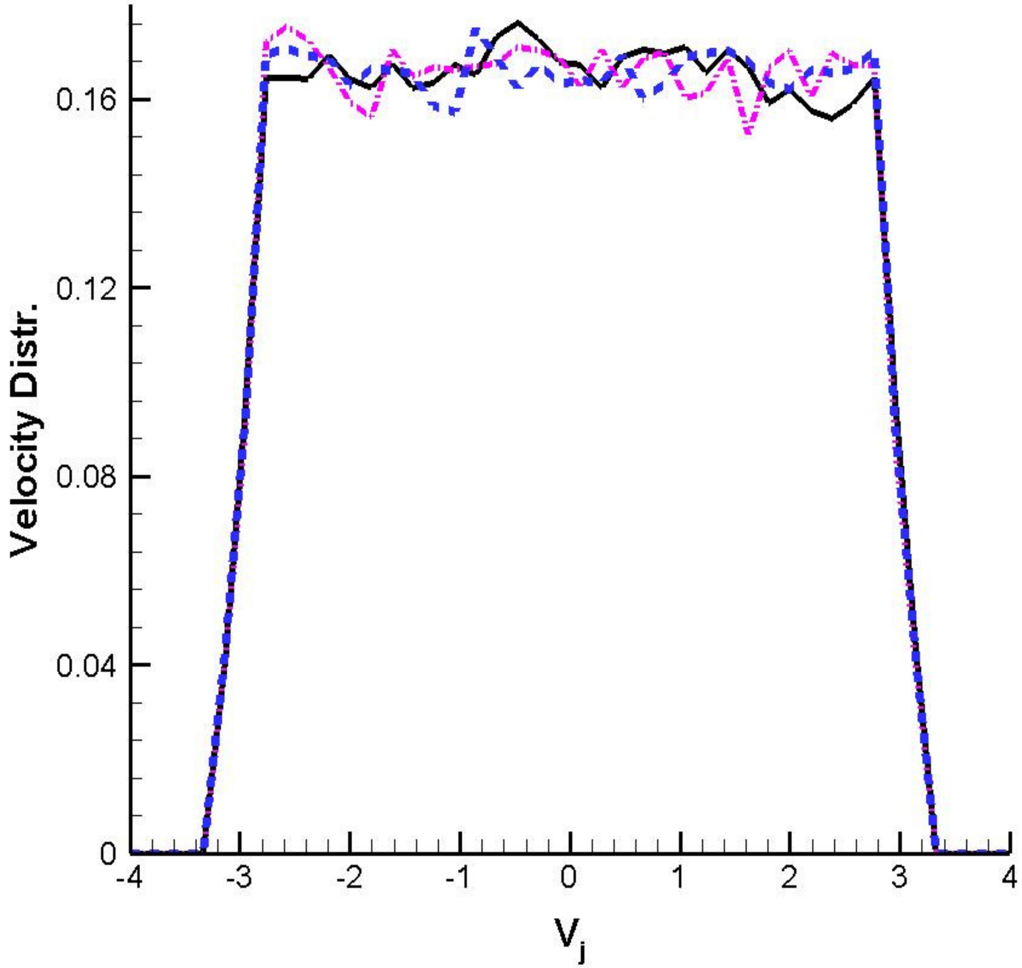
- First and second moment time development :

$$M_i = e^{-\alpha^2 (t-t_0)} M_0 \ ,$$

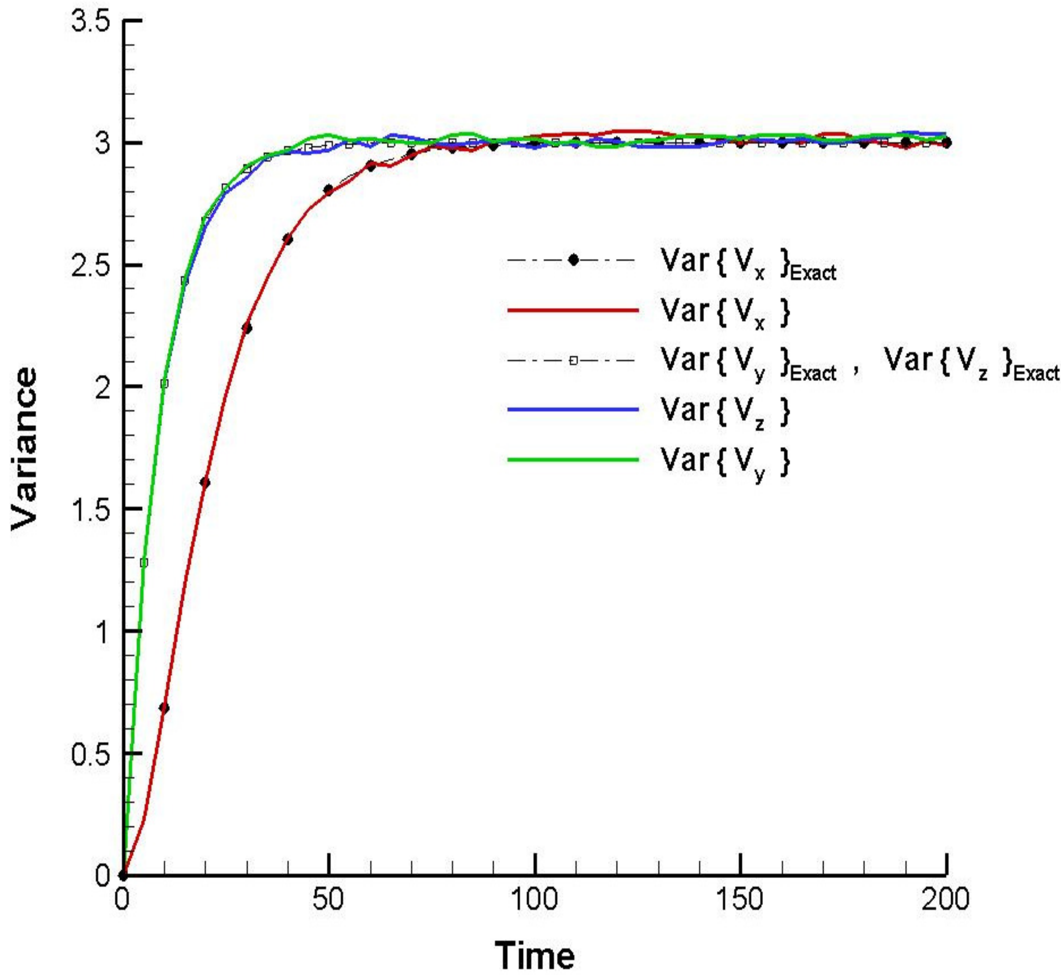
$$P_{ij} = \frac{1}{3}\delta_{ij} + \left[P_{ij}(t_0) - \frac{1}{3}\delta_{ij} \right] \exp\left\{-3\alpha^2 (t-t_0)\right\}$$

(e,X) Collision

Electron beam impinging a background Ion distribution: $V^{(e)}_{x,0} = V_0,\ V^{(e)}_{y,0} = 0,\ V^{(e)}_{z,0} = 0$



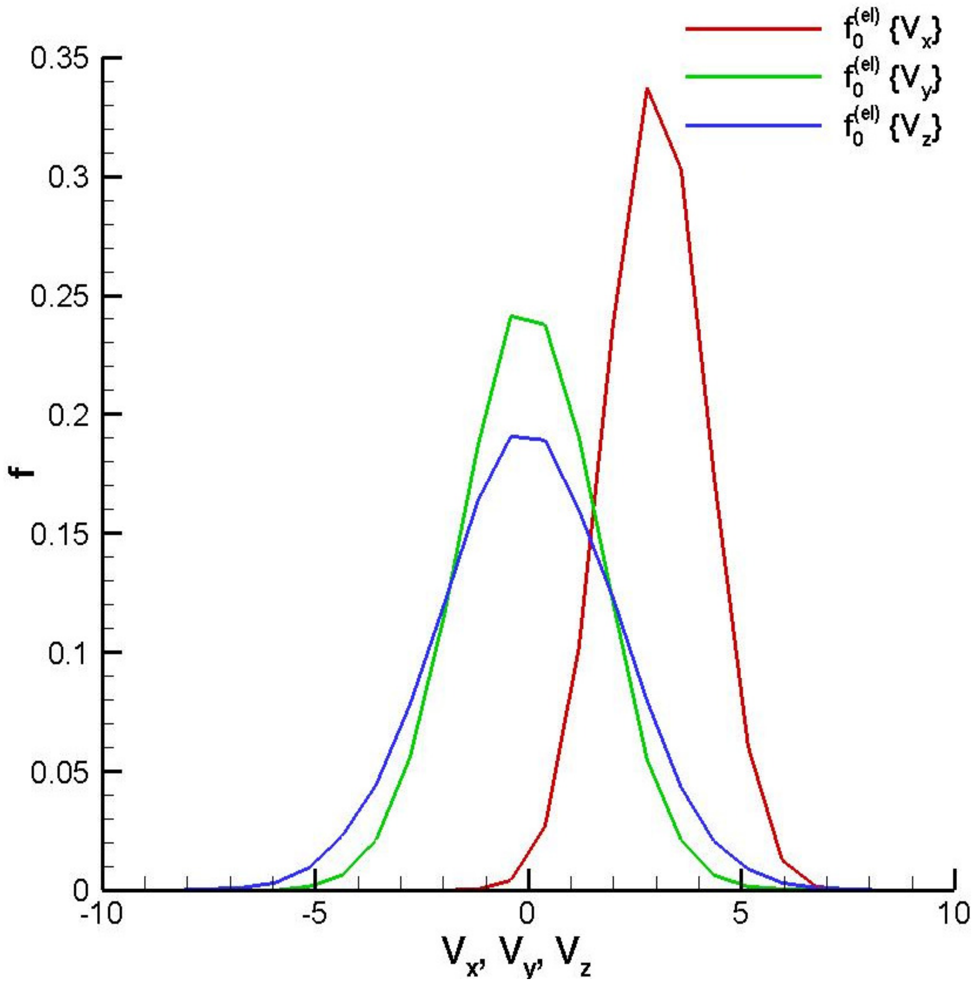
Final electron distribution function



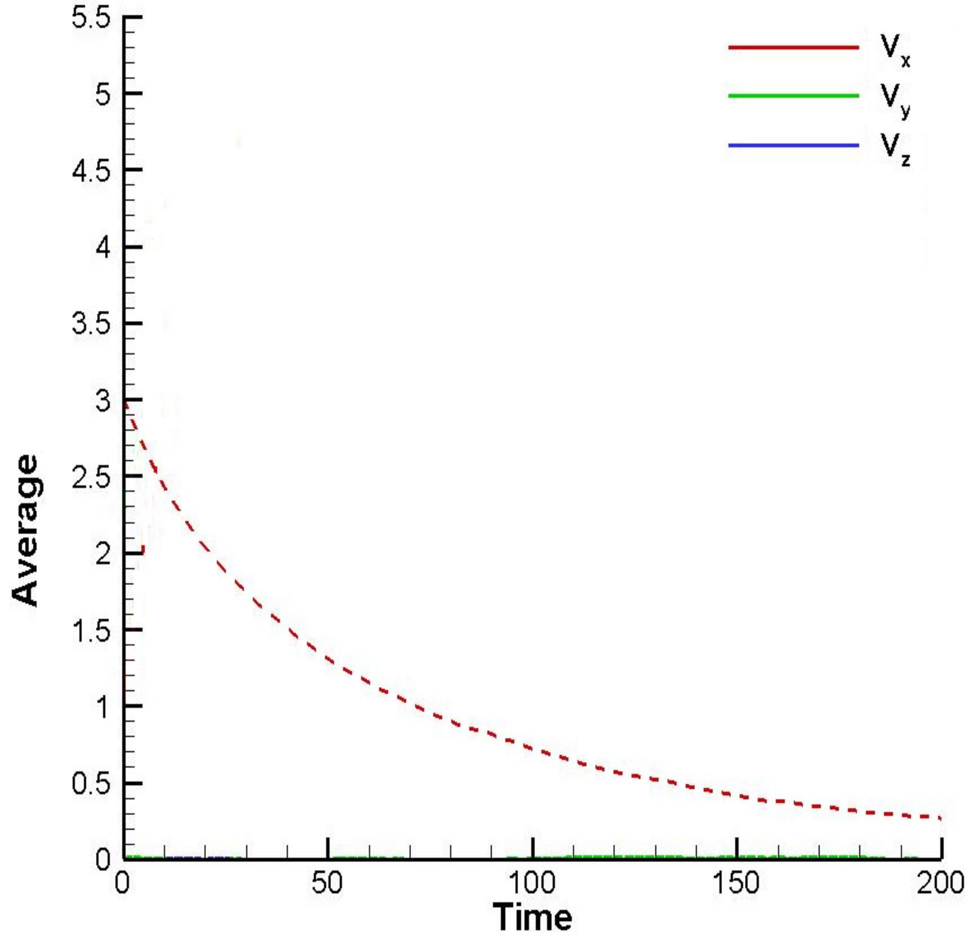
Moment temporal evolution

Coupled Calculations:(e,X) + (e,e) Collision

Non-equilibrium electrons impinging a background Ion distribution:



Initial electron distribution function



Moment temporal evolution

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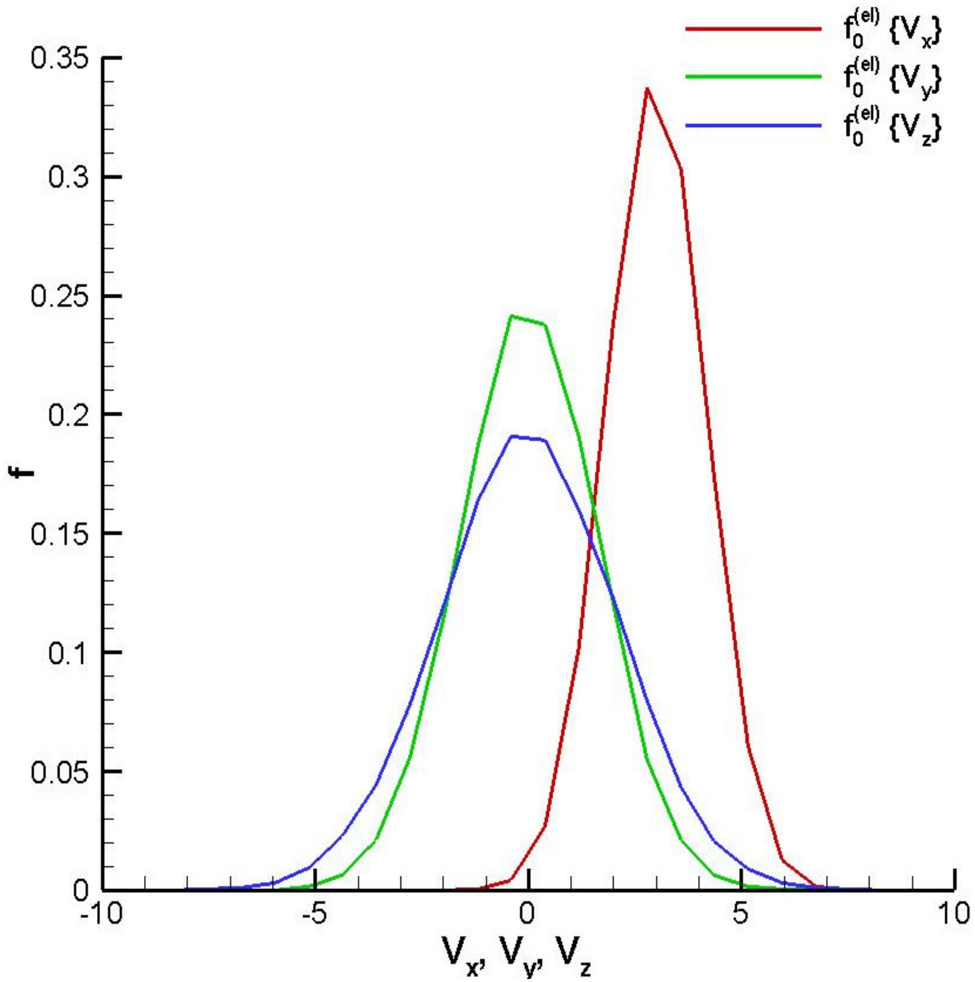

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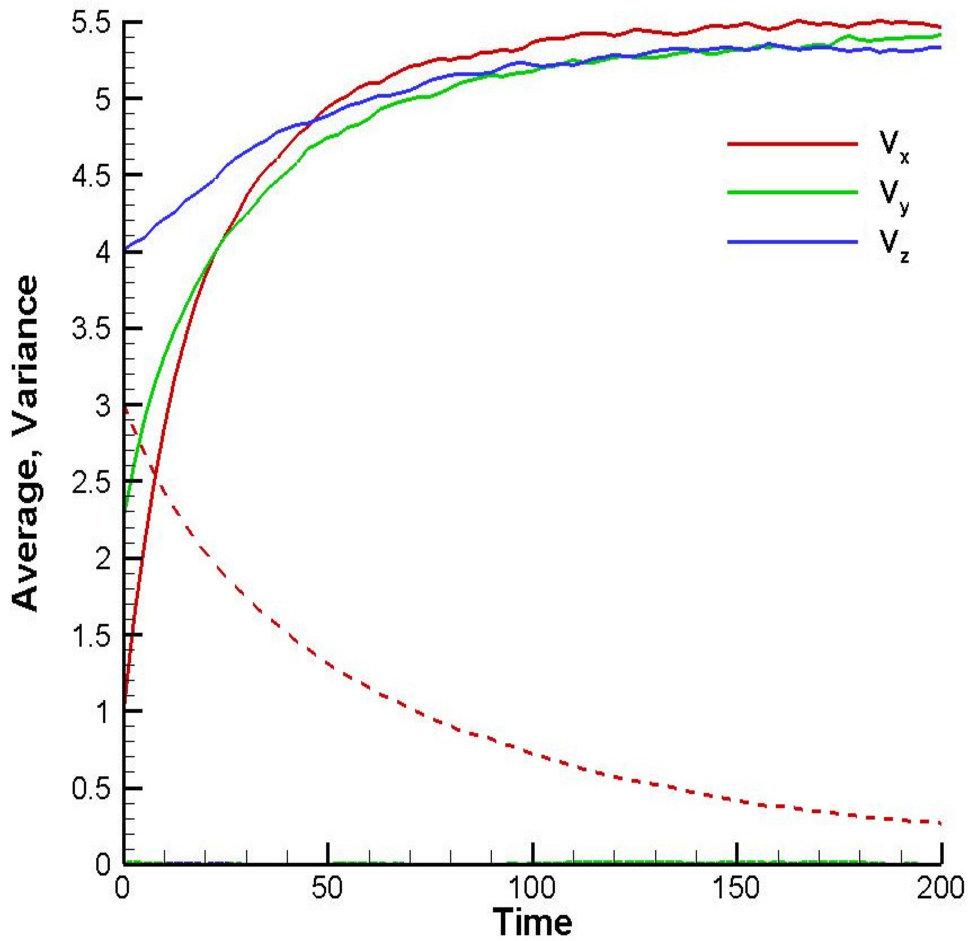
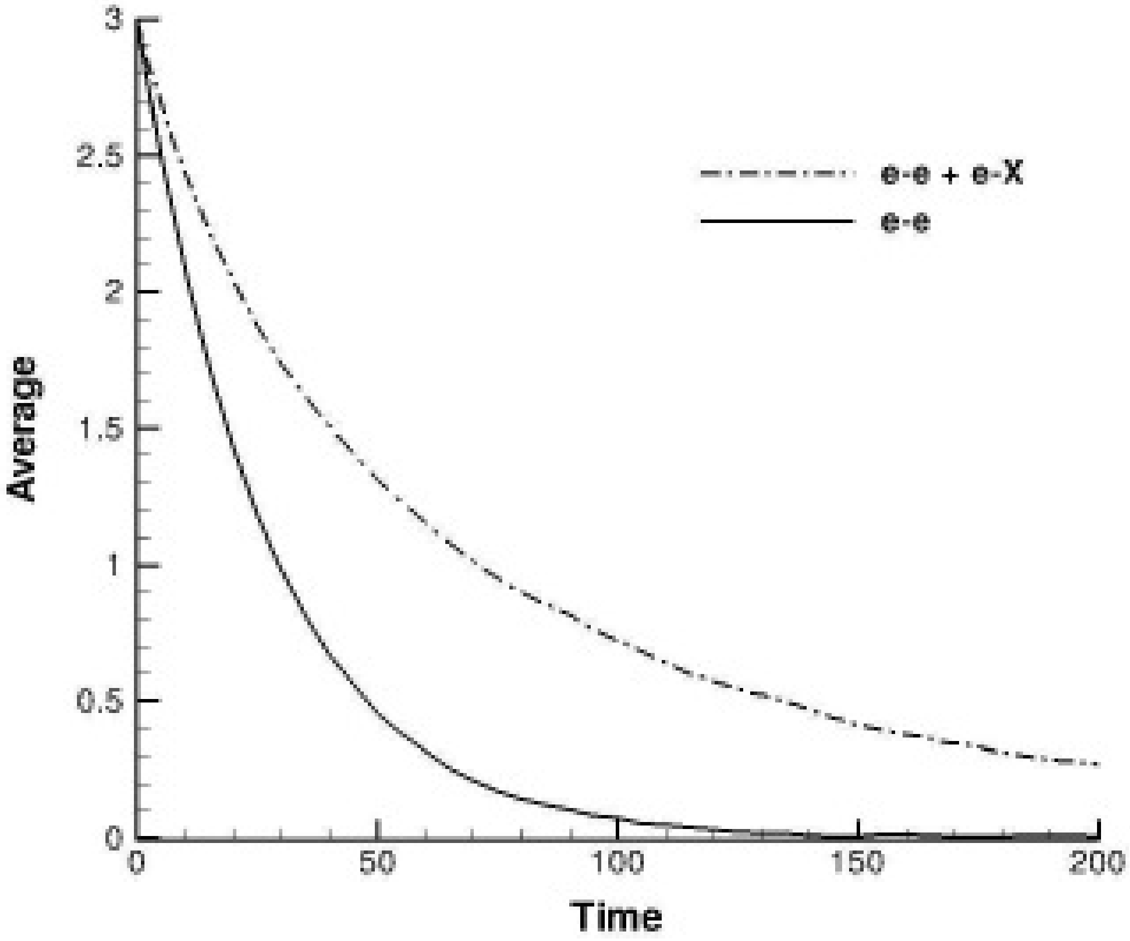


Moments temporal evolution

Comparison



- Comparison between the mean value decay for the x-direction of the velocity in the (e-X) case and the coupled case (e-e)+ (e-X)

- Delay in the coupled case due to non constant α^2

Summary and Conclusions

- Development of a three dimensional, self-consistent code for Coulomb collisions simulations
- Qualitative time scaling analysis performed for intra- and inter-species case
- Comparison with coupled calculation indicate the fundamental role of the momentum transfer collision frequency
- Coupling with a Maxwell-Vlasov solver will give a better insight of the influence of the electromagntic fields, external and self-generated